8 Vectormeetkunde

Voorkennis Vectoren in het platte vlak

Bladzijde 139

1 **a** $|\vec{a}| = \left|\begin{pmatrix} 4 \\ -3 \end{pmatrix}\right| = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

b $|\vec{b}| = \left|\begin{pmatrix} -\sqrt{5} \\ \sqrt{10} \end{pmatrix}\right| = \sqrt{(-\sqrt{5})^2 + (\sqrt{10})^2} = \sqrt{5 + 10} = \sqrt{15}$

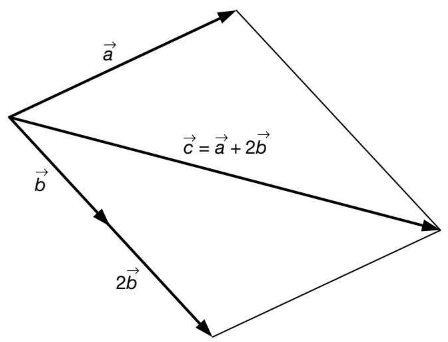
c $|\vec{c}| = 0{,}8 \cdot \left|\begin{pmatrix} -14 \\ -2 \end{pmatrix}\right| = 0{,}8 \cdot \sqrt{(-14)^2 + (-2)^2} = 0{,}8 \cdot \sqrt{196 + 4} = 0{,}8 \cdot \sqrt{200} = 8\sqrt{2}$

2 **a** $\vec{c} = \vec{a} - \vec{b} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

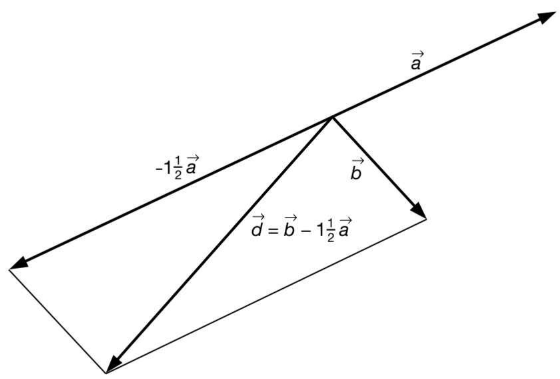
b $\vec{d} = 3\vec{a} - 2\vec{b} = 3 \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix} - 2 \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 13 \end{pmatrix}$

c $\vec{e} = 1\frac{1}{2}\vec{a} + 2\frac{1}{2}\vec{b} = 1\frac{1}{2} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix} + 2\frac{1}{2} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -1\frac{1}{2} \\ 4\frac{1}{2} \end{pmatrix} + \begin{pmatrix} 2\frac{1}{2} \\ -5 \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix}$

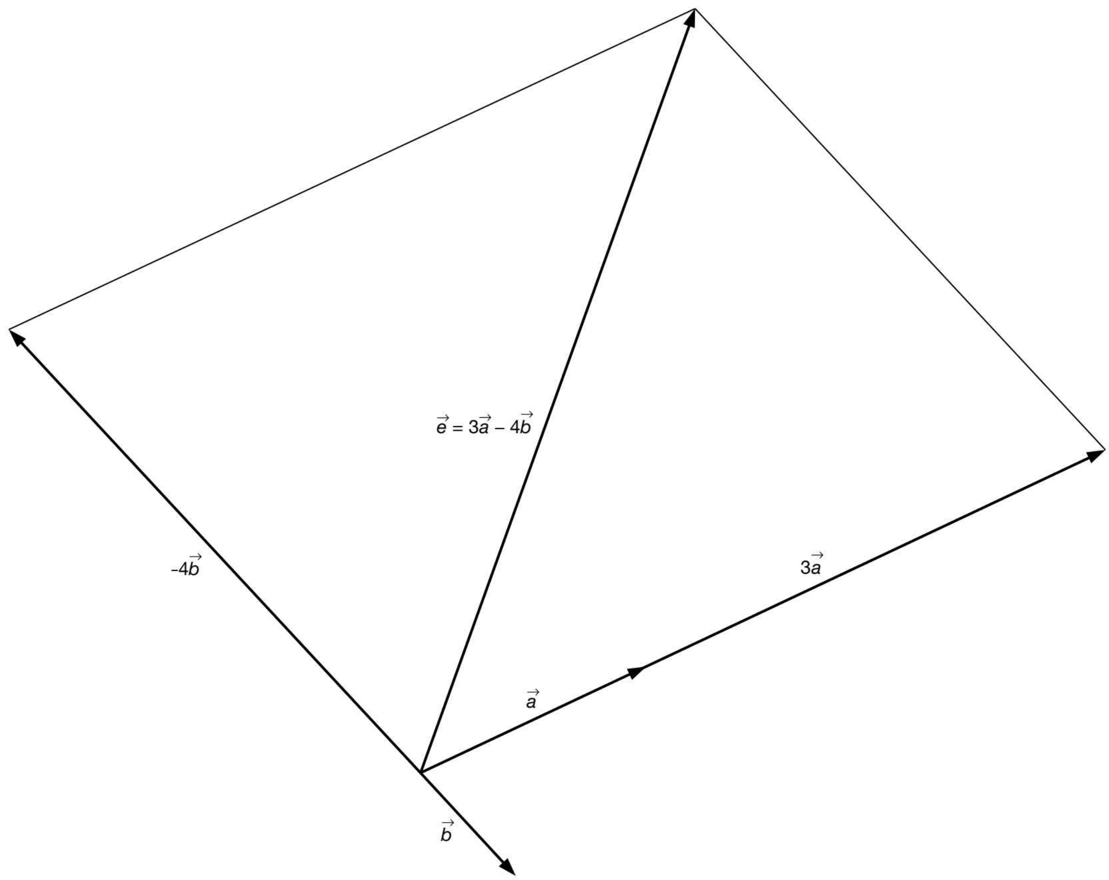
3 **a**



b



c



4 $d(A, B) = |\overrightarrow{AB}| = |\vec{b} - \vec{a}| = \left|\begin{pmatrix}2\\3\end{pmatrix} - \begin{pmatrix}5\\0\end{pmatrix}\right| = \left|\begin{pmatrix}-3\\3\end{pmatrix}\right| = \sqrt{(-3)^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$d(A, C) = |\overrightarrow{AC}| = |\vec{c} - \vec{a}| = \left|\begin{pmatrix}0\\5\end{pmatrix} - \begin{pmatrix}5\\0\end{pmatrix}\right| = \left|\begin{pmatrix}-5\\5\end{pmatrix}\right| = \sqrt{(-5)^2 + 5^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

$d(B, C) = |\overrightarrow{BC}| = |\vec{c} - \vec{b}| = \left|\begin{pmatrix}0\\5\end{pmatrix} - \begin{pmatrix}2\\3\end{pmatrix}\right| = \left|\begin{pmatrix}-2\\2\end{pmatrix}\right| = \sqrt{(-2)^2 + 2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

omtrek $= 3\sqrt{2} + 5\sqrt{2} + 2\sqrt{2} = 10\sqrt{2}$

Bladzijde 140

5 **a** $\vec{a} \cdot \vec{b} = 2 \cdot 3 + 5 \cdot -1 = 6 - 5 = 1$

$|\vec{a}| = \sqrt{2^2 + 5^2} = \sqrt{29}$ en $|\vec{b}| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$

$\cos(\angle(\vec{a}, \vec{b})) = \dfrac{1}{\sqrt{29} \cdot \sqrt{10}} = \dfrac{1}{\sqrt{290}}$

$\angle(\vec{a}, \vec{b}) \approx 86{,}6°$

b $\vec{c} \cdot \vec{d} = -1 \cdot 2 + 4 \cdot -3 = -2 - 12 = -14$

$|\vec{c}| = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$ en $|\vec{d}| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$

$\cos(\angle(\vec{c}, \vec{d})) = \dfrac{-14}{\sqrt{17} \cdot \sqrt{13}} = \dfrac{-14}{\sqrt{221}}$

$\angle(\vec{c}, \vec{d}) \approx 160{,}3°$

6 **a** $\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix}5\\3\end{pmatrix} - \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$ en $\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix}1\\8\end{pmatrix} - \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}-2\\6\end{pmatrix}$

b $\overrightarrow{AB} \cdot \overrightarrow{AC} = 2 \cdot -2 + 1 \cdot 6 = -4 + 6 = 2$

$|\overrightarrow{AB}| = \sqrt{2^2 + 1^2} = \sqrt{5}$ en $|\overrightarrow{AC}| = \sqrt{(-2)^2 + 6^2} = \sqrt{40}$

$\cos(\angle BAC) = \dfrac{2}{\sqrt{5} \cdot \sqrt{40}} = \dfrac{2}{\sqrt{200}}$

$\angle BAC \approx 81{,}9°$

Bladzijde 141

7 **a** Uit $\vec{a} \cdot \vec{b} = 0$ volgt dat $\cos(\angle(\vec{a}, \vec{b})) = 0$, dus $\angle(\vec{a}, \vec{b}) = 90°$.

b $$\left.\begin{aligned} &\vec{a} \cdot \vec{b} = \begin{pmatrix} p \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 6 \end{pmatrix} = 2p - 30 \\ &\vec{a} \cdot \vec{b} = 0 \end{aligned}\right\} \begin{aligned} 2p - 30 &= 0 \\ 2p &= 30 \\ p &= 15 \end{aligned}$$

c $$\left.\begin{aligned} &\overrightarrow{AB} \cdot \overrightarrow{AC} = \begin{pmatrix} p-3 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 14 \end{pmatrix} = -2p + 6 \\ &\overrightarrow{AB} \cdot \overrightarrow{AC} = 0 \end{aligned}\right\} \begin{aligned} -2p + 6 &= 0 \\ -2p &= -6 \\ p &= 3 \end{aligned}$$

Bladzijde 142

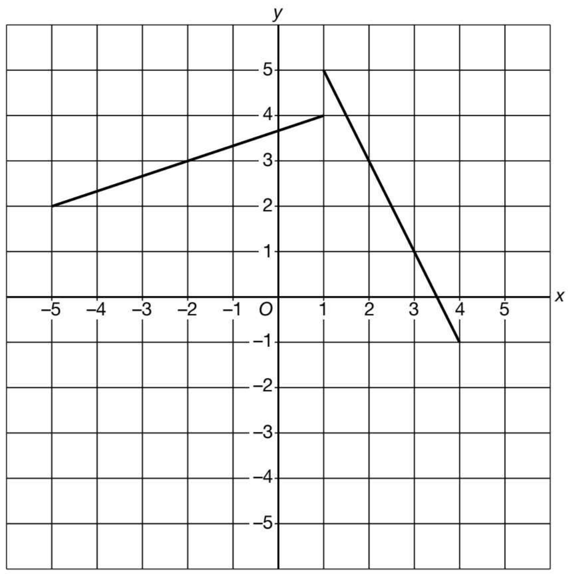
8 **a** $$\left.\begin{aligned} &k\colon \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + \lambda(\vec{b} - \vec{a}) \\ &\vec{a} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \text{ en } \vec{b} - \vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \end{pmatrix} \end{aligned}\right\} k\colon \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 3 \end{pmatrix}$$

b $$\left.\begin{aligned} &l\colon \begin{pmatrix} x \\ y \end{pmatrix} = \vec{c} + \lambda(\vec{d} - \vec{c}) \\ &\vec{c} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \text{ en } \vec{d} - \vec{c} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \end{aligned}\right\} l\colon \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

9

λ	-1	2
punt	(4, -1)	(1, 5)

μ	-1	1
punt	(-5, 2)	(1, 4)



10 Punt $A(-2, 6)$.
$3 - \lambda = -2$
$-\lambda = -5$
$\lambda = 5$
$\lambda = 5$ geeft $(-2, 11)$ dus A ligt niet op k.

Punt $B(0, -7)$.
$3 - \lambda = 0$
$-\lambda = -3$
$\lambda = 3$
$\lambda = 3$ geeft $(0, 7)$ dus B ligt niet op k.

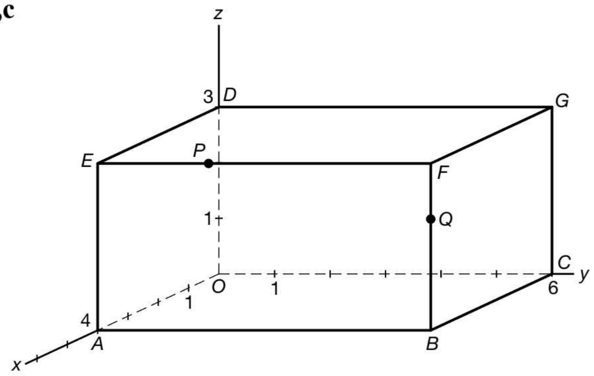
Punt $C(13, -19)$.
$3 - \lambda = 13$
$-\lambda = 10$
$\lambda = -10$
$\lambda = -10$ geeft $(13, -19)$ dus C ligt op k.

8.1 Coördinaten in de ruimte

Bladzijde 143

1 **a** M $\quad 4, 2\frac{1}{2}$ en 3
b N $\quad 2, 5$ en 3
c B $\quad 4, 5$ en 0
d G $\quad 0, 5$ en 3
e E $\quad 4, 0$ en 3
f C $\quad 0, 5$ en 0

2 **a,c**



b $O(0, 0, 0)$, $B(4, 6, 0)$, $E(4, 0, 3)$, $F(4, 6, 3)$ en $G(0, 6, 3)$

d $\vec{m} = \frac{1}{2}(\vec{p} + \vec{r}) = \frac{1}{2}\left(\begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \\ 2 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 8 \\ 8 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 2\frac{1}{2} \end{pmatrix}$

Dus $M\left(4, 4, 2\frac{1}{2}\right)$.

Bladzijde 145

3 **a** $\vec{m} = \frac{1}{2}(\vec{c} + \vec{f}) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 6 \\ 12 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}$

Dus $M(3, 6, 3)$.

b $\vec{p} = \begin{pmatrix} 6 \\ 3 \\ 6 \end{pmatrix}$ en $\vec{q} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}$

$\vec{r} = \frac{1}{2}(\vec{p} + \vec{q}) = \frac{1}{2}\left(\begin{pmatrix} 6 \\ 3 \\ 6 \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 12 \\ 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 6 \\ 1\frac{1}{2} \\ 4\frac{1}{2} \end{pmatrix}$

Dus $R\left(6, 1\frac{1}{2}, 4\frac{1}{2}\right)$.

c $d(C, Q) = \left|\overrightarrow{CQ}\right| = \left|\vec{q} - \vec{c}\right| = \left|\begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}\right| = \left|\begin{pmatrix} 6 \\ -6 \\ 3 \end{pmatrix}\right| = \sqrt{6^2 + (-6)^2 + 3^2} = \sqrt{36 + 36 + 9} = \sqrt{81} = 9$

4 **a** $\vec{m} = \frac{1}{2}(\vec{b} + \vec{t}) = \frac{1}{2}\left(\begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \\ 8 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 9 \\ 9 \\ 8 \end{pmatrix} = \begin{pmatrix} 4\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix}$

Dus $M\left(4\frac{1}{2}, 4\frac{1}{2}, 4\right)$.

b $\vec{p} = \frac{1}{2}(\vec{c} + \vec{m}) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 4\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 4\frac{1}{2} \\ 10\frac{1}{2} \\ 4 \end{pmatrix} = \begin{pmatrix} 2\frac{1}{4} \\ 5\frac{1}{4} \\ 2 \end{pmatrix}$

Dus $P\left(2\frac{1}{4}, 5\frac{1}{4}, 2\right)$.

c $d(M, S) = \left|\overrightarrow{MS}\right| = \left|\vec{s} - \vec{m}\right| = \left|\begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 4\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix}\right| = \left|\begin{pmatrix} -1\frac{1}{2} \\ -1\frac{1}{2} \\ -4 \end{pmatrix}\right| = \sqrt{\left(-1\frac{1}{2}\right)^2 + \left(-1\frac{1}{2}\right)^2 + (-4)^2} = \sqrt{20\frac{1}{2}} = \frac{1}{2}\sqrt{82}$

5 **a** $D(0, 0, 2)$ en $G(0, 2, 2)$, dus $M\left(\frac{0+0}{2}, \frac{0+2}{2}, \frac{2+2}{2}\right) = M(0, 1, 2)$.

$B(2, 2, 0)$ en $C(0, 2, 0)$, dus $N\left(\frac{2+0}{2}, \frac{2+2}{2}, \frac{0+0}{2}\right) = N(1, 2, 0)$.

$$\overrightarrow{MA} = \vec{a} - \vec{m} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$

$$\overrightarrow{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

b $\left|\overrightarrow{MA}\right| = \sqrt{(2)^2 + (-1)^2 + (-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$

$\left|\overrightarrow{MN}\right| = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$

Bladzijde 147

6 De vectoren $\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$ en $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ hebben dezelfde richting én de vectoren $\begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$ en $\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$ hebben dezelfde richting, dus $\angle\left(\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}\right) = \angle\left(\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}\right)$.

De vectoren $\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$ en $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ hebben dezelfde richting maar de vectoren $\begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$ en $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ hebben niet dezelfde richting, dus $\angle\left(\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}\right) \neq \angle\left(\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}\right)$.

7 $d(A, B) = \left|\overrightarrow{AB}\right| = \left|\vec{b} - \vec{a}\right| = \left|\begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{pmatrix}\right| = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2} =$

$\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2 + b_1{}^2 + b_2{}^2 + b_3{}^2 - 2a_1b_1 - 2a_2b_2 - 2a_3b_3}$

$$d(O, A) = \left|\vec{a}\right| = \left|\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}\right| = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}$$

$$d(O, B) = \left|\vec{b}\right| = \left|\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}\right| = \sqrt{b_1{}^2 + b_2{}^2 + b_3{}^2}$$

De cosinusregel in driehoek OAB geeft

$d(A, B)^2 = d(O, A)^2 + d(O, B)^2 - 2 \cdot d(O, A) \cdot d(O, B) \cdot \cos(\varphi)$

$a_1{}^2 + a_2{}^2 + a_3^2 + b_1{}^2 + b_2{}^2 + b_3{}^2 - 2a_1b_1 - 2a_2b_2 - 2a_3b_3 = a_1{}^2 + a_2{}^2 + a_3{}^2 + b_1{}^2 + b_2{}^2 + b_3{}^2 - 2 \cdot \left|\vec{a}\right| \cdot \left|\vec{b}\right| \cdot \cos(\varphi)$

$2 \cdot \left|\vec{a}\right| \cdot \left|\vec{b}\right| \cdot \cos(\varphi) = 2a_1b_1 + 2a_2b_2 + 2a_3b_3$

$$\cos(\varphi) = \frac{2a_1b_1 + 2a_2b_2 + 2a_3b_3}{2 \cdot \left|\vec{a}\right| \cdot \left|\vec{b}\right|} = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\left|\vec{a}\right| \cdot \left|\vec{b}\right|} = \frac{\vec{a} \cdot \vec{b}}{\left|\vec{a}\right| \cdot \left|\vec{b}\right|}$$

8 $\angle ENG = \angle(\overrightarrow{NE}, \overrightarrow{NG})$

$$\overrightarrow{NE} = \vec{e} - \vec{n} = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}$$

$$\overrightarrow{NG} = \vec{g} - \vec{n} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{NE}, \overrightarrow{NG})) = \frac{\begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}\right|} = \frac{-4 + 0 + 16}{6 \cdot \sqrt{20}} = \frac{12}{6\sqrt{20}} = \frac{2}{\sqrt{20}}$$

$\angle ENG = \angle(\overrightarrow{NE}, \overrightarrow{NG}) \approx 63{,}4°$

9 **a** $\overrightarrow{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 3 \end{pmatrix}$

$$\overrightarrow{CE} = \vec{e} - \vec{c} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{BD}, \overrightarrow{CE})) = \frac{\begin{pmatrix} -6 \\ -4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} -6 \\ -4 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}\right|} = \frac{-36 + 16 + 9}{\sqrt{61} \cdot \sqrt{61}} = \frac{-11}{61}$$

$\angle(\overrightarrow{BD}, \overrightarrow{CE}) \approx 100{,}4$

b $\angle BGD = \angle(\overrightarrow{GB}, \overrightarrow{GD})$

$$\overrightarrow{GB} = \vec{b} - \vec{g} = \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ -2 \end{pmatrix}$$

$$\overrightarrow{GD} = \vec{d} - \vec{g} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{GB}, \overrightarrow{GD})) = \frac{\begin{pmatrix} 6 \\ 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} 6 \\ 0 \\ -2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}\right|} = \frac{0 + 0 - 2}{\sqrt{40} \cdot \sqrt{17}} = \frac{-2}{\sqrt{680}}$$

$\angle BGD = \angle(\overrightarrow{GB}, \overrightarrow{GD}) \approx 94{,}4°$

8

Bladzijde 148

10 **a** $A(3, 3, 0)$, $B(-3, 3, 0)$, $C(-3, -3, 0)$, $D(3, -3, 0)$, $E(3, 3, 6)$, $F(-3, 3, 6)$, $G(-3, -3, 6)$, $H(3, -3, 6)$ en $T(0, 0, 10)$.

b $\overrightarrow{TH} = \vec{t} - \vec{h} = \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}$ dus een richtingsvector van de lijn HT is $\begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}$.

$T(0, 0, 10)$ ligt op de lijn HT dus een steunvector is $\begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$.

c Het Oxy-vlak bestaat uit alle punten waarvan de z-coördinaat gelijk is aan 0.
P ligt in dit vlak dus $z_P = 0$.

d $\left.\begin{matrix} z_P = 0 \\ z_P = 10 - 4\lambda \end{matrix}\right\} \begin{aligned} 10 - 4\lambda &= 0 \\ -4\lambda &= -10 \\ \lambda &= 2\tfrac{1}{2} \end{aligned}$

e $\lambda = 2\frac{1}{2}$ geeft $P\left(7\frac{1}{2}, -7\frac{1}{2}, 0\right)$

Bladzijde 149

11 **a** Het Oxz-vlak bestaat uit alle punten waarvan de y-coördinaat gelijk is aan 0.
Q ligt in dit vlak dus $y_Q = 0$.

b Het Oyz-vlak bestaat uit alle punten waarvan de x-coördinaat gelijk is aan 0.
De x-coördinaat van deze punten is dus gelijk aan 0.

12 **a** $\vec{m} = \begin{pmatrix} 4 \\ 0 \\ 1\frac{1}{2} \end{pmatrix}$ en $\vec{p} - \vec{m} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 1\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \\ 1\frac{1}{2} \end{pmatrix}$, dus MP: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 1 \\ 1\frac{1}{2} \end{pmatrix}$.

$z_T = 0$ geeft $3 + 1\frac{1}{2}\lambda = 0$, dus $\lambda = -2$.

$\lambda = -2$ geeft $\vec{t} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} - 2 \cdot \begin{pmatrix} -4 \\ 1 \\ 1\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \\ 0 \end{pmatrix}$

Dus $T(8, -1, 0)$.

b $\vec{p} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$ en $\vec{q} - \vec{p} = \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix}$, dus PQ: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix}$.

$z_U = 0$ geeft $3 - \lambda = 0$, dus $\lambda = 3$.

$\lambda = 3$ geeft $\vec{u} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + 3 \cdot \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 12 \\ 13 \\ 0 \end{pmatrix}$

Dus $U(12, 13, 0)$.

Bladzijde 150

13 **a** $\vec{m} = \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix}$ en $\vec{p} - \vec{m} = \begin{pmatrix} 5\frac{1}{4} \\ 5\frac{1}{4} \\ 2 \end{pmatrix} - \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix} = \begin{pmatrix} 3\frac{3}{4} \\ \frac{3}{4} \\ -2 \end{pmatrix}$, dus MP: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3\frac{3}{4} \\ \frac{3}{4} \\ -2 \end{pmatrix}$.

$z_D = 0$ geeft $4 - 2\lambda = 0$, dus $\lambda = 2$.

$\lambda = 2$ geeft $\vec{d} = \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix} + 2 \cdot \begin{pmatrix} 3\frac{3}{4} \\ \frac{3}{4} \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ 0 \end{pmatrix}$

Dus $D(9, 6, 0)$.

b $\vec{q} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix}$ en $\vec{p} - \vec{q} = \begin{pmatrix} 5\frac{1}{4} \\ 5\frac{1}{4} \\ 2 \end{pmatrix} - \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} = \begin{pmatrix} 1\frac{1}{2} \\ 3 \\ -4 \end{pmatrix}$, dus QP: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1\frac{1}{2} \\ 3 \\ -4 \end{pmatrix}$.

$x_E = 0$ geeft $3\frac{3}{4} + 1\frac{1}{2}\lambda = 0$, dus $\lambda = -2\frac{1}{2}$.

$\lambda = -2\frac{1}{2}$ geeft $\vec{e} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} - 2\frac{1}{2} \cdot \begin{pmatrix} 1\frac{1}{2} \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ -5\frac{1}{4} \\ 16 \end{pmatrix}$

Dus $E\left(0, -5\frac{1}{4}, 16\right)$.

c $\vec{q} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix}$ en $\vec{m} - \vec{q} = \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix} - \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} = \begin{pmatrix} -2\frac{1}{4} \\ 2\frac{1}{4} \\ -2 \end{pmatrix}$, dus QM: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -2\frac{1}{4} \\ 2\frac{1}{4} \\ -2 \end{pmatrix}$.

$y_F = 0$ geeft $2\frac{1}{4} + 2\frac{1}{4}\lambda = 0$, dus $\lambda = -1$.

$\lambda = -1$ geeft $\vec{f} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} - \begin{pmatrix} -2\frac{1}{4} \\ 2\frac{1}{4} \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 8 \end{pmatrix}$

Dus $F(6, 0, 8)$.

8.2 Vergelijkingen van vlakken

Bladzijde 152

14 **a** $A(3, 0, 0)$ invullen geeft $20 \cdot 3 + 15 \cdot 0 + 12 \cdot 0 = 60$ klopt.
$B(0, 4, 0)$ invullen geeft $20 \cdot 0 + 15 \cdot 4 + 12 \cdot 0 = 60$ klopt.
$C(0, 0, 5)$ invullen geeft $20 \cdot 0 + 15 \cdot 0 + 12 \cdot 5 = 60$ klopt.

b Het linker- en rechterlid delen door 60 geeft $\frac{20x}{60} + \frac{15y}{60} + \frac{12z}{60} = \frac{60}{60}$ ofwel $\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 1$.

c Je kunt de coördinaten van de snijpunten met de assen herkennen aan de getallen in de noemers.

8

Bladzijde 154

15 **a** Voor het snijpunt met de x-as geldt $y = 0$ en $z = 0$.
Dit geeft $ax = d$

$$x = \frac{d}{a}$$

Dus het snijpunt is $A\left(\frac{d}{a}, 0, 0\right)$.

Voor het snijpunt met de y-as geldt $x = 0$ en $z = 0$.
Dit geeft $by = d$

$$y = \frac{d}{b}$$

Dus het snijpunt is $B\left(0, \frac{d}{b}, 0\right)$.

Voor het snijpunt met de z-as geldt $x = 0$ en $y = 0$.
Dit geeft $cz = d$

$$z = \frac{d}{c}$$

Dus het snijpunt is $C\left(0, 0, \frac{d}{c}\right)$.

b $\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 0 \\ \frac{d}{b} \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{d}{a} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{d}{a} \\ \frac{d}{b} \\ 0 \end{pmatrix}$

$\overrightarrow{AB} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -\frac{d}{a} \\ \frac{d}{b} \\ 0 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = -d + d + 0 = 0$, dus $\overrightarrow{AB} \perp \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

$\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 0 \\ 0 \\ \frac{d}{c} \end{pmatrix} - \begin{pmatrix} \frac{d}{a} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{d}{a} \\ 0 \\ \frac{d}{c} \end{pmatrix}$

$\overrightarrow{AC} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -\frac{d}{a} \\ 0 \\ \frac{d}{c} \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = -d + 0 + d = 0$, dus $\overrightarrow{AC} \perp \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

c $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ staat loodrecht op $\overrightarrow{AB}$ en op $\overrightarrow{AC}$ dus ook loodrecht op de snijdende lijnen *AB* en *AC*.

Dan moet $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ loodrecht op *V* staan omdat *AB* en *AC* in *V* liggen, en is $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ dus een normaalvector van *V*.

Bladzijde 155

16 **a** *ACD* snijdt de *x*-as in *A*(4, 0, 0), de *y*-as in *C*(0, 4, 0) en de *z*-as in *D*(0, 0, 4),

dus *ACD*: $\frac{x}{4} + \frac{y}{4} + \frac{z}{4} = 1$ ofwel $x + y + z = 4$.

Dus $\vec{n}_{ACD} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

b Van *C* naar *M* ga je 4 in de *x*-richting en −2 in de *y*-richting.
Ga je vanuit *M*(4, 2, 0) opnieuw 4 in de *x*-richting en −2 in de *y*-richting,
dan kom je in het punt (8, 0, 0).
De lijn *CM* en dus ook het vlak *CMD* snijdt de *x*-as in het punt (8, 0, 0).
Dus *CMD* snijdt de *x*-as in (8, 0, 0), de *y*-as in *C*(0, 4, 0) en de *z*-as in *D*(0, 0, 4),

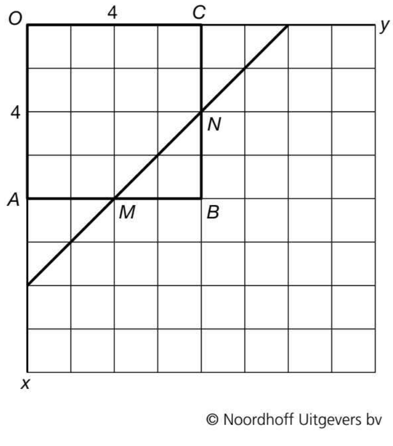
dus *CMD*: $\frac{x}{8} + \frac{y}{4} + \frac{z}{4} = 1$ ofwel *CMD*: $x + 2y + 2z = 8$.

Dus $\vec{n}_{CMD} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$.

c Zie de figuur: *MN* snijdt de *x*-as in (6, 0, 0) en de *y*-as in (0, 6, 0).
MND snijdt de *x*-as in (6, 0, 0), de *y*-as in (0, 6, 0) en de *z*-as in *D*(0, 0, 4),

dus *MND*: $\frac{x}{6} + \frac{y}{6} + \frac{z}{4} = 1$ ofwel *MND*: $2x + 2y + 3z = 12$.

Dus $\vec{n}_{MND} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$.



d *MNF* snijdt de x-as in (6, 0, 0) en de y-as in (0, 6, 0), dus MNF: $\frac{x}{6} + \frac{y}{6} + \frac{z}{r} = 1$.

$F(4, 4, 4)$ invullen in $\frac{x}{6} + \frac{y}{6} + \frac{z}{r} = 1$ geeft $\frac{4}{6} + \frac{4}{6} + \frac{4}{r} = 1$

$$\frac{2}{3} + \frac{2}{3} + \frac{4}{r} = 1$$
$$\frac{4}{r} = -\frac{1}{3}$$
$$r = -12$$

MNF: $\frac{x}{6} + \frac{y}{6} - \frac{z}{12} = 1$ ofwel MNF: $2x + 2y - z = 12$

Dus $\vec{n}_{MNF} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$.

e $\vec{d} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$ en $\vec{b} - \vec{d} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -4 \end{pmatrix}$, dus BD: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

Substitutie van $x = \lambda, y = \lambda$ en $z = 4 - \lambda$ in $2x + 2y - z = 12$ geeft
$2\lambda + 2\lambda - (4 - \lambda) = 12$
$2\lambda + 2\lambda - 4 + \lambda = 12$
$5\lambda = 16$
$\lambda = 3\frac{1}{5}$

Dus het snijpunt is $\left(3\frac{1}{5}, 3\frac{1}{5}, \frac{4}{5}\right)$.

17 **a** *ACD* snijdt de x-as in $A(5, 0, 0)$, de y-as in $C(0, 6, 0)$ en de z-as in $D(0, 0, 4)$,
dus ACD: $\frac{x}{5} + \frac{y}{6} + \frac{z}{4} = 1$ ofwel $12x + 10y + 15z = 60$.

b Van C naar M ga je 5 in de x-richting en −3 in de y-richting.
Ga je vanuit $M(5, 3, 0)$ opnieuw 5 in de x-richting en −3 in de y-richting, dan kom je in het punt (10, 0, 0).
De lijn CM en dus ook het vlak CMD snijdt de x-as in het punt (10, 0, 0).
Dus CMD snijdt de x-as in (10, 0, 0), de y-as in $C(0, 6, 0)$ en de z-as in $D(0, 0, 4)$,
dus CMD: $\frac{x}{10} + \frac{y}{6} + \frac{z}{4} = 1$ ofwel CMD: $6x + 10y + 15z = 60$.

Dus $\vec{n}_{CMD} = \begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix}$.

c $\vec{d} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$ en $\vec{b} - \vec{d} = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ -4 \end{pmatrix}$, dus BD: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 6 \\ -4 \end{pmatrix}$.

ACF snijdt de x-as in (5, 0, 0), de y-as in (0, 6, 0) en de z-as in (0, 0, −4),
dus ACF: $\frac{x}{5} + \frac{y}{6} - \frac{z}{4} = 1$ ofwel ACF: $12x + 10y - 15z = 60$.
Substitutie van $x = 5\lambda, y = 6\lambda$ en $z = 4 - 4\lambda$ in $12x + 10y - 15z = 60$ geeft
$12 \cdot 5\lambda + 10 \cdot 6\lambda - 15(4 - 4\lambda) = 60$
$60\lambda + 60\lambda - 60 + 60\lambda = 60$
$180\lambda = 120$
$\lambda = \frac{2}{3}$

Dus het snijpunt is $\left(3\frac{1}{3}, 4, 1\frac{1}{3}\right)$.

8

18 **a** ACP snijdt de x-as in $A(4, 0, 0)$ en de y-as in $C(0, 6, 0)$, dus ACP: $\frac{x}{4} + \frac{y}{6} + \frac{z}{r} = 1$.

$P\left(1, 1\frac{1}{2}, 2\frac{1}{2}\right)$ invullen in $\frac{x}{4} + \frac{y}{6} + \frac{z}{r} = 1$ geeft $\frac{1}{4} + \frac{1\frac{1}{2}}{6} + \frac{2\frac{1}{2}}{r} = 1$

$$\frac{1}{4} + \frac{1}{4} + \frac{2\frac{1}{2}}{r} = 1$$

$$\frac{2\frac{1}{2}}{r} = \frac{1}{2}$$

$$r = 5$$

ACP: $\frac{x}{4} + \frac{y}{6} + \frac{z}{5} = 1$ ofwel ACP: $15x + 10y + 12z = 60$.

b MNP snijdt de x-as in $(6, 0, 0)$ en de y-as in $(0, 9, 0)$, dus MNP: $\frac{x}{6} + \frac{y}{9} + \frac{z}{r} = 1$.

$P\left(1, 1\frac{1}{2}, 2\frac{1}{2}\right)$ invullen in $\frac{x}{6} + \frac{y}{9} + \frac{z}{r} = 1$ geeft $\frac{1}{6} + \frac{1\frac{1}{2}}{9} + \frac{2\frac{1}{2}}{r} = 1$

$$\frac{1}{6} + \frac{1}{6} + \frac{2\frac{1}{2}}{r} = 1$$

$$\frac{2\frac{1}{2}}{r} = \frac{2}{3}$$

$$2r = 7\frac{1}{2}$$

$$r = 3\frac{3}{4}$$

MNP: $\frac{x}{6} + \frac{y}{9} + \frac{z}{3\frac{3}{4}} = 1$ ofwel MNP: $15x + 10y + 24z = 90$.

Dus $\vec{n}_{MNP} = \begin{pmatrix} 15 \\ 10 \\ 24 \end{pmatrix}$.

c ANT snijdt de x-as in $(4, 0, 0)$ en de y-as in $(0, 12, 0)$, dus ANT: $\frac{x}{4} + \frac{y}{12} + \frac{z}{r} = 1$.

$T(2, 3, 5)$ invullen in $\frac{x}{4} + \frac{y}{12} + \frac{z}{r} = 1$ geeft $\frac{2}{4} + \frac{3}{12} + \frac{5}{r} = 1$

$$\frac{1}{2} + \frac{1}{4} + \frac{5}{r} = 1$$

$$\frac{5}{r} = \frac{1}{4}$$

$$r = 20$$

ANT: $\frac{x}{4} + \frac{y}{12} + \frac{z}{20} = 1$ ofwel ANT: $15x + 5y + 3z = 60$.

Dus $\vec{r}_l = \vec{n}_{ANT} = \begin{pmatrix} 15 \\ 5 \\ 3 \end{pmatrix}$ en l: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1\frac{1}{2} \\ 2\frac{1}{2} \end{pmatrix} + \lambda \begin{pmatrix} 15 \\ 5 \\ 3 \end{pmatrix}$.

l: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1\frac{1}{2} \\ 2\frac{1}{2} \end{pmatrix} + \lambda \begin{pmatrix} 15 \\ 5 \\ 3 \end{pmatrix}$ snijden met ANT: $15x + 5y + 3z = 60$ geeft

$15(1 + 15\lambda) + 5(1\frac{1}{2} + 5\lambda) + 3(2\frac{1}{2} + 3\lambda) = 60$
$15 + 225\lambda + 7\frac{1}{2} + 25\lambda + 7\frac{1}{2} + 9\lambda = 60$
$259\lambda = 30$
$\lambda = \frac{30}{259}$

Het snijpunt is $\left(2\frac{191}{259}, 2\frac{41}{518}, 2\frac{439}{518}\right)$.

19 **a** $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} = a_1(a_2b_3 - a_3b_2) + a_2(a_3b_1 - a_1b_3) + a_3(a_1b_2 - a_2b_1) =$

$a_1a_2b_3 - a_1a_3b_2 + a_2a_3b_1 - a_1a_2b_3 + a_1a_3b_2 - a_2a_3b_1 = 0$

b $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \cdot \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} = b_1(a_2b_3 - a_3b_2) + b_2(a_3b_1 - a_1b_3) + b_3(a_1b_2 - a_2b_1) =$

$a_2b_1b_3 - a_3b_1b_2 + a_3b_1b_2 - a_1b_2b_3 + a_1b_2b_3 - a_2b_1b_3 = 0$

Bladzijde 157

20 **a**
$$\begin{array}{cc l} 3 & 1 & \\ 1 & 2 & 1 \cdot 3 - 2 \cdot 2 = -1 \\ 2 & 3 & 2 \cdot 1 - 3 \cdot 3 = -7 \\ 3 & 1 & 3 \cdot 2 - 1 \cdot 1 = 5 \\ 1 & 2 & \\ 2 & 3 & \end{array}$$

Dus $\vec{a} \times \vec{b} = \begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix}$.

b
$$\begin{array}{cc l} 3 & 12 & \\ 1 & 4 & 1 \cdot 8 - 2 \cdot 4 = 0 \\ 2 & 8 & 2 \cdot 12 - 3 \cdot 8 = 0 \\ 3 & 12 & 3 \cdot 4 - 1 \cdot 12 = 0 \\ 1 & 4 & \\ 2 & 8 & \end{array}$$

Dus $\vec{a} \times \vec{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

c
$$\begin{array}{cc l} 1 & -3 & \\ 2 & -1 & 2 \cdot -2 - 3 \cdot -1 = -1 \\ 3 & -2 & 3 \cdot -3 - 1 \cdot -2 = -7 \\ 1 & -3 & 1 \cdot -1 - 2 \cdot -3 = 5 \\ 2 & -1 & \\ 3 & -2 & \end{array}$$

Dus $\vec{b} \times \vec{d} = \begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix}$.

d
$$\begin{array}{cc l} 1 & 2 & \\ 2 & 4 & 2 \cdot 6 - 3 \cdot 4 = 0 \\ 3 & 6 & 3 \cdot 2 - 1 \cdot 6 = 0 \\ 1 & 2 & 1 \cdot 4 - 2 \cdot 2 = 0 \\ 2 & 4 & \\ 3 & 6 & \end{array}$$

Dus $\vec{b} \times \vec{e} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

e
$$\begin{array}{cc l} 1 & 3 & \\ 2 & 1 & 2 \cdot 2 - 3 \cdot 1 = 1 \\ 3 & 2 & 3 \cdot 3 - 1 \cdot 2 = 7 \\ 1 & 3 & 1 \cdot 1 - 2 \cdot 3 = -5 \\ 2 & 1 & \\ 3 & 2 & \end{array}$$

Dus $\vec{b} \times \vec{a} = \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix}$.

f
$$\begin{array}{cc l} -3 & 1 & \\ -1 & 2 & -1 \cdot 3 - -2 \cdot 2 = 1 \\ -2 & 3 & -2 \cdot 1 - -3 \cdot 3 = 7 \\ -3 & 1 & -3 \cdot 2 - -1 \cdot 1 = -5 \\ -1 & 2 & \\ -2 & 3 & \end{array}$$

Dus $\vec{d} \times \vec{b} = \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix}$.

21 **a** $\vec{a} \cdot \vec{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3$

$\vec{b} \cdot \vec{a} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = b_1a_1 + b_2a_2 + b_3a_3 = a_1b_1 + a_2b_2 + a_3b_3$

Dus $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ is waar voor elk tweetal vectoren.

b Bij 20c vond je $\vec{b} \times \vec{d} = \begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix}$.

Bij 20f vond je $\vec{d} \times \vec{b} = \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix}$.

Dus $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$ is niet waar voor elk tweetal vectoren.

Bladzijde 158

22 **a** $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \cdot 1 - 2 \cdot -2 \\ 2 \cdot 4 - 3 \cdot 1 \\ 3 \cdot -2 - -1 \cdot 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$

b $\begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \cdot 1 - 1 \cdot 0 \\ 1 \cdot 2 - 0 \cdot 1 \\ 0 \cdot 0 - 5 \cdot 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ -10 \end{pmatrix}$

c $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \cdot 0 - 1 \cdot 1 \\ 1 \cdot 0 - 1 \cdot 0 \\ 1 \cdot 1 - 1 \cdot 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

d $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \cdot 0 - 0 \cdot 1 \\ 0 \cdot 0 - 1 \cdot 0 \\ 1 \cdot 1 - 0 \cdot 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

23 $\vec{r}_k \times \vec{r}_m = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \cdot 1 - 2 \cdot 1 \\ 2 \cdot 1 - 2 \cdot 1 \\ 2 \cdot 1 - -1 \cdot 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}$, dus $\vec{r}_l = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$.

$l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + v\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

24 **a** $C(0, 2, 0)$ en $G(0, 2, 2)$, dus $M(0, 2, 1)$.

$\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$

$\overrightarrow{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$

$\overrightarrow{AB} \times \overrightarrow{BM} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 - 0 \cdot 0 \\ 0 \cdot -2 - 0 \cdot 1 \\ 0 \cdot 0 - 2 \cdot -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$

De vector $\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ staat loodrecht op $\overrightarrow{AB}$ en $\overrightarrow{BM}$, dus de vector $\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ staat loodrecht op het vlak waar $\overrightarrow{AB}$ en $\overrightarrow{BM}$ richtingsvectoren van zijn. Dat is het vlak ABM.

Dus de vector $\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ staat loodrecht op het vlak ABM en dus $\vec{n}_{ABM} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$.

b $\vec{n}_{ABM} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$, dus ABM: $2x + 4z = d$.

$A(2, 0, 0)$ invullen geeft $d = 4$.
Dus ABM: $2x + 4z = 4$ ofwel $x + 2z = 2$.

Bladzijde 160

25 **a** $\overrightarrow{EG} = \vec{g} - \vec{e} = \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$\overrightarrow{EP} = \vec{p} - \vec{e} = \begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$

$\vec{n}_{EGP} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ -5 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}$

$\left.\begin{array}{l} EGP\text{: } 2x + 2y + 5z = d \\ \text{door } E(5, 0, 5) \end{array}\right\} d = 10 + 0 + 25 = 35$

Dus EGP: $2x + 2y + 5z = 35$.

b $AD \perp OEF$, dus $\overrightarrow{AD} = \vec{n}_{OEF}$.

$\overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 5 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$\left.\begin{array}{l} OEF\text{: } x - z = d \\ \text{door } O(0, 0, 0) \end{array}\right\} d = 0$

Dus OEF: $x - z = 0$.

c $\overrightarrow{DE} = \vec{e} - \vec{d} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\overrightarrow{EP} = \vec{p} - \vec{e} = \begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$

$\vec{n}_{DEP} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix}$

$\left.\begin{array}{l} DEP\text{: } 2y + 5z = d \\ \text{door } D(0, 0, 5) \end{array}\right\} d = 25$

Dus DEP: $2y + 5z = 25$.

26 **a** $BD \perp ACT$, dus $\overrightarrow{BD} = \vec{n}_{ACT}$.

$\overrightarrow{BD} = \vec{d} - \vec{b} = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\left.\begin{array}{l} ACT\text{: } x + y = d \\ \text{door } A(2, -2, 0) \end{array}\right\} d = 2 - 2 = 0$

Dus ACT: $x + y = 0$.

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b $C(-2, 2, 0)$ en $T(0, 0, 6)$, dus $M(-1, 1, 3)$.

$$\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\overrightarrow{AM} = \vec{m} - \vec{a} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{n}_{ABM} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\left.\begin{array}{l} ABM\colon x + z = d \\ \text{door } B(2, 2, 0) \end{array}\right\} d = 2$$

Dus ABM: $x + z = 2$.

c $$\overrightarrow{BD} = \vec{d} - \vec{b} = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\overrightarrow{BM} = \vec{m} - \vec{b} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix}$$

$$\vec{n}_{BDM} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$$

$$\left.\begin{array}{l} BDM\colon 3x - 3y + 2z = d \\ \text{door } B(2, 2, 0) \end{array}\right\} d = 6 - 6 + 0 = 0$$

Dus BDM: $3x - 3y + 2z = 0$.

27 **a** $$\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 6 \\ 0 \end{pmatrix}$$

$$\overrightarrow{CG} = \vec{g} - \vec{c} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{n}_{ACG} = \begin{pmatrix} -5 \\ 6 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \\ 0 \end{pmatrix}$$

$$\left.\begin{array}{l} ACG\colon 6x + 5y = d \\ \text{door } A(5, 0, 0) \end{array}\right\} d = 30$$

Dus ACG: $6x + 5y = 30$.

b $$\overrightarrow{BE} = \vec{e} - \vec{b} = \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$$

$$\overrightarrow{BG} = \vec{g} - \vec{b} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 4 \end{pmatrix}$$

$$\vec{n}_{BEG} = \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} \times \begin{pmatrix} -5 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -12 \\ -10 \\ -15 \end{pmatrix} \triangleq \begin{pmatrix} 12 \\ 10 \\ 15 \end{pmatrix}$$

8.3 Hoeken in de ruimte

Bladzijde 162

28 **a** $\overrightarrow{MA} = \vec{a} - \vec{m} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$

$\overrightarrow{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

$$\cos(\angle(\overrightarrow{MA}, \overrightarrow{MN})) = \frac{\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}\right|} = \frac{2 - 1 + 4}{\sqrt{9} \cdot \sqrt{6}} = \frac{5}{3\sqrt{6}}$$

$\angle(\overrightarrow{MA}, \overrightarrow{MN}) \approx 47{,}1°$

b De hoek tussen de vectoren is een scherpe hoek waarvan de benen langs de lijnen AM en MN liggen. Dit is juist de hoek tussen de lijnen AM en MN.

Bladzijde 163

29 **a** $\vec{r}_{CE} = \vec{e} - \vec{c} = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$\vec{r}_{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

$$\cos(\angle(CE, MN)) = \frac{\left|\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}\right|}{\left|\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}\right|} = \frac{|1 - 1 - 2|}{\sqrt{3} \cdot \sqrt{6}} = \frac{2}{\sqrt{18}}$$

Dus $\angle(CE, MN) \approx 61{,}9°$.

b $\vec{r}_{AG} = \vec{g} - \vec{a} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$\vec{r}_{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$

$$\cos(\angle(AG, BM)) = \frac{\left|\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}\right|}{\left|\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}\right|} = \frac{|2 - 1 + 2|}{\sqrt{3} \cdot 3} = \frac{1}{\sqrt{3}}$$

Dus $\angle(AG, CE) \approx 54{,}7°$.

c $\angle AMC = \angle(\overrightarrow{MA}, \overrightarrow{MC})$

$$\overrightarrow{MA} = \vec{a} - \vec{m} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$

$$\overrightarrow{MC} = \vec{c} - \vec{m} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{MA}, \overrightarrow{MC})) = \frac{\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}\right|} = \frac{0 - 1 + 4}{3 \cdot \sqrt{5}} = \frac{1}{\sqrt{5}}$$

$\angle AMC = \angle(\overrightarrow{MA}, \overrightarrow{MC}) \approx 63{,}4°$

Bladzijde 164

30 **a** $D(0, 0, 3)$ en $G(0, 4, 2)$, dus $M\left(0, 2, 2\frac{1}{2}\right)$.

$$\overrightarrow{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 0 \\ 2 \\ 2\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \\ 2\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} -12 \\ 4 \\ 5 \end{pmatrix}$$

$$\overrightarrow{CE} = \vec{e} - \vec{c} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}$$

$$\cos(\angle(AM, CE)) = \frac{\left|\begin{pmatrix} -12 \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}\right|}{\left|\begin{pmatrix} -12 \\ 4 \\ 5 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}\right|} = \frac{|-72 - 16 + 15|}{\sqrt{185} \cdot \sqrt{61}} = \frac{73}{\sqrt{11285}}$$

Dus $\angle(AM, CE) \approx 46{,}6°$.

b $$\overrightarrow{AF} = \vec{f} - \vec{a} = \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$\overrightarrow{EG} = \vec{g} - \vec{e} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}$$

$$\cos(\angle(AF, EG)) = \frac{\left|\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}\right|}{\left|\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}\right|} = \frac{|0 + 8 - 1|}{\sqrt{5} \cdot \sqrt{53}} = \frac{7}{\sqrt{265}}$$

Dus $\angle(AF, EG) \approx 64{,}5°$.

c $\angle EMC = \angle(\overrightarrow{ME}, \overrightarrow{MC})$

$$\overrightarrow{ME} = \vec{e} - \vec{m} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 2\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ \frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}$$

$$\overrightarrow{MC} = \vec{c} - \vec{m} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 2\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -2\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 4 \\ -5 \end{pmatrix}$$

$$\cos(\angle(AM, CE)) = \frac{\begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 4 \\ -5 \end{pmatrix}}{\left|\begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 0 \\ 4 \\ -5 \end{pmatrix}\right|} = \frac{0 - 16 - 5}{\sqrt{161} \cdot \sqrt{41}} = \frac{-21}{\sqrt{6601}}$$

Dus $\angle EMC = \angle(\overrightarrow{ME}, \overrightarrow{MC}) \approx 105{,}0°$.

31 **a** $C(0, 4, 0)$ en $T(2, 2, 6)$, dus $M(1, 3, 3)$.

$$\overrightarrow{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\overrightarrow{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix}$$

$$\cos(\angle(AM, BM)) = \frac{\left|\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix}\right|}{\left|\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix}\right|} = \frac{|3 - 1 + 3|}{\sqrt{3} \cdot \sqrt{19}} = \frac{5}{\sqrt{57}}$$

Dus $\angle(AM, BM) \approx 48{,}5°$.

b $$\vec{r}_{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{r}_{BT} = \vec{t} - \vec{b} = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$$

$$\cos(\angle(AM, BT)) = \frac{\left|\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}\right|}{\left|\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}\right|} = \frac{|1 - 1 + 3|}{\sqrt{3} \cdot \sqrt{11}} = \frac{3}{\sqrt{33}}$$

Dus $\angle(AM, BT) \approx 58{,}5°$.

c $\angle OMB = \angle(\overrightarrow{MO}, \overrightarrow{MB})$

$$\overrightarrow{MO} = \vec{o} - \vec{m} = \begin{pmatrix} -1 \\ -3 \\ -3 \end{pmatrix}$$

$$\overrightarrow{MB} = \vec{b} - \vec{m} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -3 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{MO}, \overrightarrow{MB})) = \frac{\begin{pmatrix} -1 \\ -3 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ -3 \end{pmatrix}}{\left|\begin{pmatrix} -1 \\ -3 \\ -3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 3 \\ 1 \\ -3 \end{pmatrix}\right|} = \frac{-3 - 3 + 9}{\sqrt{19} \cdot \sqrt{19}} = \frac{3}{19}$$

$\angle OMB = \angle(\overrightarrow{MO}, \overrightarrow{MB}) \approx 80{,}9°$

32 **a** $\vec{r}_{OT} = \vec{t} = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ dus OT: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$.

$\lambda = 0$ geeft $O(0, 0, 0)$ en $\lambda = 2$ geeft $T(2, 2, 6)$.
Dus $P(\lambda, \lambda, 3\lambda)$ met $0 \leq \lambda \leq 2$ is voor elke λ een punt op het lijnstuk OT.

b $\vec{r}_{BP} = \vec{p} - \vec{b} = \begin{pmatrix} \lambda \\ \lambda \\ 3\lambda \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda - 4 \\ \lambda - 4 \\ 3\lambda \end{pmatrix}$ en $\vec{r}_{OT} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$.

$BP \perp OT$, dus $\vec{r}_{BP} \cdot \vec{r}_{OT} = 0$

$$\begin{pmatrix} \lambda - 4 \\ \lambda - 4 \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = 0$$

$$\lambda - 4 + \lambda - 4 + 9\lambda = 0$$
$$11\lambda = 8$$
$$\lambda = \tfrac{8}{11}$$

Dus $P\left(\tfrac{8}{11}, \tfrac{8}{11}, 2\tfrac{2}{11}\right)$.

c De hoek tussen BP en OT is maximaal 90° voor $P\left(\tfrac{8}{11}, \tfrac{8}{11}, 2\tfrac{2}{11}\right)$.
De hoek tussen BP en OT is minimaal als $P = O$ of als $P = T$.
Bereken dus $\angle BOT$ en $\angle BTO$.
$\tan(\angle BOT) = \frac{TS}{OS} = \frac{6}{\sqrt{8}}$, dus $\angle BOT \approx 64{,}8°$.
$\angle BTO = 180° - 2\angle BOT \approx 50{,}5°$
Dus $\angle(BP, OT)$ kan alle waarden aannemen van 50,5° tot en met 90°.

d $\angle(BP, OT) = 55°$

$$\cos(55°) = \frac{\left|\begin{pmatrix} \lambda - 4 \\ \lambda - 4 \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}\right|}{\left|\begin{pmatrix} \lambda - 4 \\ \lambda - 4 \\ 3\lambda \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}\right|}$$

$$\cos(55°) = \frac{|\lambda - 4 + \lambda - 4 + 9\lambda|}{\sqrt{(\lambda - 4)^2 + (\lambda - 4)^2 + 9\lambda^2} \cdot \sqrt{11}}$$

$$\cos(55°) = \frac{|11\lambda - 8|}{\sqrt{22(\lambda - 4)^2 + 99\lambda^2}}$$

Voer in $y_1 = \dfrac{\text{abs}(11x-8)}{\sqrt{22(x-4)^2+99x^2}}$ en $y_2 = \cos(55°)$.
Intersect geeft $x = 1{,}8075...$
Dus $P(1{,}81;\ 1{,}81;\ 5{,}42)$.

Bladzijde 166

33 **a** Lijn n is een normaal van ACD.

$\vec{r}_{OF} = \vec{f} = \begin{pmatrix}4\\3\\2\end{pmatrix}$ en $\vec{r}_n = \begin{pmatrix}3\\4\\6\end{pmatrix}$.

$$\cos(\angle(OF, n)) = \frac{\left|\begin{pmatrix}4\\3\\2\end{pmatrix}\cdot\begin{pmatrix}3\\4\\6\end{pmatrix}\right|}{\left|\begin{pmatrix}4\\3\\2\end{pmatrix}\right|\cdot\left|\begin{pmatrix}3\\4\\6\end{pmatrix}\right|} = \frac{|12+12+12|}{\sqrt{29}\cdot\sqrt{61}} = \frac{36}{\sqrt{1769}}, \text{ dus } \angle(OF, n) \approx 31{,}1°.$$

Dus $\angle(OF, ACD) \approx 90° - 31{,}1° = 58{,}9°$.

b $\vec{r}_{CE} = \vec{e} - \vec{c} = \begin{pmatrix}4\\0\\2\end{pmatrix} - \begin{pmatrix}0\\3\\0\end{pmatrix} = \begin{pmatrix}4\\-3\\2\end{pmatrix}$

Lijn n is een normaal van OBG.

$\vec{r}_{OB} = \vec{b} = \begin{pmatrix}4\\3\\0\end{pmatrix}$ en $\vec{r}_{OG} = \vec{g} = \begin{pmatrix}0\\3\\2\end{pmatrix}$

$\vec{r}_n = \vec{n}_{OBG} = \vec{r}_{OB} \times \vec{r}_{OG} = \begin{pmatrix}4\\3\\0\end{pmatrix} \times \begin{pmatrix}0\\3\\2\end{pmatrix} = \begin{pmatrix}6\\-8\\12\end{pmatrix} \triangleq \begin{pmatrix}3\\-4\\6\end{pmatrix}$

$$\cos(\angle(CE, n)) = \frac{\left|\begin{pmatrix}4\\-3\\2\end{pmatrix}\cdot\begin{pmatrix}3\\-4\\6\end{pmatrix}\right|}{\left|\begin{pmatrix}4\\-3\\2\end{pmatrix}\right|\cdot\left|\begin{pmatrix}3\\-4\\6\end{pmatrix}\right|} = \frac{|12+12+12|}{\sqrt{29}\cdot\sqrt{61}} = \frac{36}{\sqrt{1769}}, \text{ dus } \angle(CE, n) \approx 31{,}1°.$$

Dus $\angle(CE, OBG) \approx 90° - 31{,}1° = 58{,}9°$.

34 **a** Kies het assenstelsel zo, dat je krijgt $A(4, 0, 0)$, $C(0, 4, 0)$ en $D(0, 0, 4)$.

$\vec{r}_{AG} = \vec{g} - \vec{a} = \begin{pmatrix}0\\4\\4\end{pmatrix} - \begin{pmatrix}4\\0\\0\end{pmatrix} = \begin{pmatrix}-4\\4\\4\end{pmatrix} \triangleq \begin{pmatrix}-1\\1\\1\end{pmatrix}$

Lijn n is een normaal van EBG.

$OF \perp EBG$, dus $\vec{r}_n = \overrightarrow{OF} = \vec{f} = \begin{pmatrix}4\\4\\4\end{pmatrix} \triangleq \begin{pmatrix}1\\1\\1\end{pmatrix}$.

$$\cos(\angle(AG, n)) = \frac{\left|\begin{pmatrix}-1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}\right|}{\left|\begin{pmatrix}-1\\1\\1\end{pmatrix}\right|\cdot\left|\begin{pmatrix}1\\1\\1\end{pmatrix}\right|} = \frac{|-1+1+1|}{\sqrt{3}\cdot\sqrt{3}} = \tfrac{1}{3}, \text{ dus } \angle(AG, n) \approx 70{,}5°.$$

Dus $\angle(AG, EBG) \approx 90° - 70{,}5° = 19{,}5°$.

b $\vec{r}_{DM} = \vec{m} - \vec{d} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

Lijn n is een normaal van ACD.

$OF \perp ACD$, dus $\vec{r}_n = \overrightarrow{OF} = \vec{f} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

$$\cos(\angle(DM, n)) = \frac{\left|\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right|}{\left|\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right|} = \frac{|2 + 1 - 2|}{3 \cdot \sqrt{3}} = \frac{1}{3\sqrt{3}}, \text{ dus } \angle(DM, n) \approx 78{,}9°.$$

Dus $\angle(DM, ACD) \approx 90° - 78{,}9° = 11{,}1°$.

c $\vec{r}_{GM} = \vec{m} - \vec{g} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$

Lijn n is een normaal van CDM.

$\vec{r}_{CD} = \vec{d} - \vec{c} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

$\vec{r}_{CM} = \vec{m} - \vec{c} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$

$\vec{r}_n = \vec{n}_{CDM} = \vec{r}_{CD} \times \vec{r}_{CM} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

$$\cos(\angle(GM, n)) = \frac{\left|\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}\right|}{\left|\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}\right|} = \frac{|2 - 2 - 4|}{3 \cdot 3} = \tfrac{4}{9}, \text{ dus } \angle(GM, n) \approx 63{,}6°.$$

Dus $\angle(GM, CDM) \approx 90° - 63{,}6° = 26{,}4°$.

35 Voor de hellingshoek φ geldt $\cos(\varphi) = \dfrac{3{,}0}{3{,}8}$, dus $\varphi \approx 37{,}9°$.

Bladzijde 167

36 Het vlak MND snijdt de assen in (6, 0, 0), (0, 6, 0) en (0, 0, 4).

MND: $\dfrac{x}{6} + \dfrac{y}{6} + \dfrac{z}{4} = 1$ ofwel MND: $2x + 2y + 3z = 12$

Dus $\vec{r}_m = \vec{n}_{MND} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$.

37 **a** Lijn m is een normaal van MNF en lijn n is een normaal van OND.

$$\vec{r}_m = \vec{n}_{MNF} = \overrightarrow{MN} \times \overrightarrow{MF} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{OND} = \overrightarrow{ON} \times \overrightarrow{OD} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 16 \\ -8 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \right|} = \frac{|4 - 2 + 0|}{3 \cdot \sqrt{5}} = \frac{2}{3\sqrt{5}}$$

Dus $\angle(MNF, OND) = \angle(m, n) \approx 72{,}7°$.

b Lijn m is een normaal van MND en lijn n is een normaal van OMD.

$$\vec{r}_m = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \text{ en } \vec{r}_n = \vec{n}_{OMD} = \overrightarrow{OM} \times \overrightarrow{OD} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ -16 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \right|} = \frac{|2 - 4 + 0|}{\sqrt{17} \cdot \sqrt{5}} = \frac{2}{\sqrt{85}}$$

Dus $\angle(MND, OMD) = \angle(m, n) \approx 77{,}5°$.

Bladzijde 168

38 **a** Lijn m is een normaal van AMN en lijn n is een normaal van OAM.
MN snijdt de y-as in $(0, 9, 0)$ en de z-as in $(0, 0, 6)$.

$$AMN: \frac{x}{5} + \frac{y}{9} + \frac{z}{6} = 1$$
$$18x + 10y + 15z = 90$$

Dus $\vec{r}_n = \vec{n}_{AMN} = \begin{pmatrix} 18 \\ 10 \\ 15 \end{pmatrix}$.

$$\vec{r}_n = \vec{n}_{OAM} = \overrightarrow{OA} \times \overrightarrow{OM} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -20 \\ 15 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 18 \\ 10 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix} \right|}{\left| \begin{pmatrix} 18 \\ 10 \\ 15 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix} \right|} = \frac{|0 - 40 + 45|}{\sqrt{649} \cdot 5} = \frac{1}{\sqrt{649}}$$

Dus $\angle(AMN, OAM) = \angle(m, n) \approx 87{,}8°$.

b Lijn m is een normaal van EMN en lijn n is een normaal van BCD.

EMN: $\frac{x}{r} + \frac{y}{9} + \frac{z}{6} = 1$

$E(5, 0, 4)$ invullen in $\frac{x}{r} + \frac{y}{9} + \frac{z}{6} = 1$ geeft $\frac{5}{r} + \frac{0}{9} + \frac{4}{6} = 1$

$$\frac{5}{r} + \frac{2}{3} = 1$$

$$\frac{5}{r} = \frac{1}{3}$$

$$r = 15$$

EMN: $\frac{x}{15} + \frac{y}{9} + \frac{z}{6} = 1$

$6x + 10y + 15z = 90$

Dus $\vec{r}_m = \vec{n}_{EMN} = \begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix}$.

$$\vec{r}_n = \vec{n}_{BCD} = \vec{r}_{BC} \times \vec{r}_{DC} = \begin{pmatrix} -5 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ -20 \\ -30 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left|\begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}\right|}{\left|\begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}\right|} = \frac{|0 + 20 + 45|}{19 \cdot \sqrt{13}} = \frac{65}{19\sqrt{13}}$$

Dus $\angle(EMN, BCD) = \angle(m, n) \approx 18{,}4°$.

39 **a** $\vec{r}_{DT} = \vec{t} - \vec{d} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$

Lijn n is een normaal van ABN.

$C(-2, -2, 0)$ en $T(0, 0, 3)$, dus $N(-1, 1, 1\frac{1}{2})$.

$$\vec{r}_n = \vec{n}_{ABN} = \vec{r}_{AN} \times \vec{r}_{AB} = \begin{pmatrix} -3 \\ 3 \\ 1\frac{1}{2} \end{pmatrix} \times \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \\ -12 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\cos(\angle(DT, n)) = \frac{\left|\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}\right|}{\left|\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}\right|} = \frac{|2 + 0 + 6|}{\sqrt{17} \cdot \sqrt{5}} = \frac{8}{\sqrt{85}}, \text{ dus } \angle(DT, n) \approx 29{,}8°.$$

Dus $\angle(DT, ABN) \approx 90° - 29{,}8° = 60{,}2°$.

b $A(2, -2, 0)$ en $D(-2, -2, 0)$, dus $M(0, -2, 0)$.

$$\vec{r}_{MN} = \vec{n} - \vec{m} = \begin{pmatrix} -1 \\ 1 \\ 1\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 1\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix}$$

$$\vec{r}_{CT} = \vec{t} - \vec{c} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$$

$$\cos(\angle(MN, CT)) = \frac{\left|\begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}\right|}{\left|\begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}\right|} = \frac{|-4 - 12 + 9|}{7 \cdot \sqrt{17}} = \frac{7}{7\sqrt{17}} = \frac{1}{\sqrt{17}}, \text{ dus } \angle(MN, CT) \approx 76{,}0°.$$

c Lijn m is een normaal van ABN en lijn n is een normaal van ACT.

$$\vec{r}_m = \vec{n}_{ABN} = \vec{r}_{AN} \times \vec{r}_{AB} = \begin{pmatrix}1\\0\\2\end{pmatrix} \text{ (zie a)}$$

$$\vec{r}_n = \vec{n}_{ACT} = \vec{r}_{AC} \times \vec{r}_{AT} = \begin{pmatrix}-4\\4\\0\end{pmatrix} \times \begin{pmatrix}-2\\2\\3\end{pmatrix} = \begin{pmatrix}12\\12\\0\end{pmatrix} \triangleq \begin{pmatrix}1\\1\\0\end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left|\begin{pmatrix}1\\0\\2\end{pmatrix} \cdot \begin{pmatrix}1\\1\\0\end{pmatrix}\right|}{\left|\begin{pmatrix}1\\0\\2\end{pmatrix}\right| \cdot \left|\begin{pmatrix}1\\1\\0\end{pmatrix}\right|} = \frac{|1+0+0|}{\sqrt{5}\cdot\sqrt{2}} = \frac{1}{\sqrt{10}}$$

Dus $\angle(ABN, ACT) = \angle(m, n) \approx 71{,}6°$.

d $\angle MBN = \angle(\overrightarrow{BM}, \overrightarrow{BN})$

$$\overrightarrow{BM} = \vec{m} - \vec{b} = \begin{pmatrix}0\\-2\\0\end{pmatrix} - \begin{pmatrix}2\\2\\0\end{pmatrix} = \begin{pmatrix}-2\\-4\\0\end{pmatrix}$$

$$\overrightarrow{BN} = \vec{n} - \vec{b} = \begin{pmatrix}-1\\1\\1\frac{1}{2}\end{pmatrix} - \begin{pmatrix}2\\2\\0\end{pmatrix} = \begin{pmatrix}-3\\-1\\1\frac{1}{2}\end{pmatrix}$$

$$\cos(\angle(\overrightarrow{BM}, \overrightarrow{BN})) = \frac{\begin{pmatrix}-2\\-4\\0\end{pmatrix} \cdot \begin{pmatrix}-3\\-1\\1\frac{1}{2}\end{pmatrix}}{\left|\begin{pmatrix}-2\\-4\\0\end{pmatrix}\right| \cdot \left|\begin{pmatrix}-3\\-1\\1\frac{1}{2}\end{pmatrix}\right|} = \frac{6+4+0}{\sqrt{20}\cdot\sqrt{12\frac{1}{4}}} = \frac{10}{\sqrt{245}}$$

$\angle MBN = \angle(\overrightarrow{BM}, \overrightarrow{BN}) \approx 50{,}3°$

8.4 Afstanden in de ruimte

Bladzijde 170

40 **a** $\overrightarrow{MN} = \vec{n} - \vec{m} = \begin{pmatrix}6\\0\\12\end{pmatrix} - \begin{pmatrix}12\\12\\6\end{pmatrix} = \begin{pmatrix}-6\\-12\\6\end{pmatrix} \triangleq \begin{pmatrix}1\\2\\-1\end{pmatrix}$

V is het vlak door P, loodrecht op MN.

$$\left.\begin{array}{l} V\colon x + 2y - z = d \\ \text{door } P(0, 9, 12) \end{array}\right\} d = 0 + 18 - 12 = 6$$

Dus $V\colon x + 2y - z = 6$.

b $MN\colon \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}12\\12\\6\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-1\end{pmatrix}$ snijden met $V\colon x + 2y - z = 6$ geeft $12 + \lambda + 2\cdot(12 + 2\lambda) - (6 - \lambda) = 6$

$$12 + \lambda + 24 + 4\lambda - 6 + \lambda = 6$$
$$6\lambda = -24$$
$$\lambda = -4$$

Het snijpunt is $Q(8, 4, 10)$.

c $d(P, Q) = |\overrightarrow{PQ}| = |\vec{q} - \vec{p}| = \left|\begin{pmatrix}8\\4\\10\end{pmatrix} - \begin{pmatrix}0\\9\\12\end{pmatrix}\right| = \left|\begin{pmatrix}8\\-5\\-2\end{pmatrix}\right| = \sqrt{8^2 + (-5)^2 + (-2)^2} = \sqrt{93}$

Omdat $\overrightarrow{PQ} \perp \overrightarrow{MN}$ is dit ook $d(P, MN)$.

Bladzijde 171

41 **a** $C(0, 4, 0)$ en $G(0, 4, 2)$, dus $M(0, 4, 1)$.

$$\overrightarrow{MP} = \vec{p} - \vec{m} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

$$d(M, P) = |\overrightarrow{MP}| = \left|\begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}\right| = \sqrt{26}$$

b $$\vec{r}_{DG} = \vec{g} - \vec{d} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

V is het vlak door B, loodrecht op DG.

$$\left.\begin{array}{l} V\colon y = d \\ \text{door } B(4, 4, 0) \end{array}\right\} d = 4$$

Dus V: $y = 4$.

$$DG\colon \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$ snijden met V geeft $\lambda = 4$.

Het snijpunt is $G(0, 4, 2)$.

$$d(B, DG) = d(B, G) = |\overrightarrow{BG}| = \left|\begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix}\right| = \sqrt{20} = 2\sqrt{5}$$

c $$\vec{r}_{MP} = \vec{p} - \vec{m} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

V is het vlak door F, loodrecht op MP.

$$\left.\begin{array}{l} V\colon 3x - 4y + z = d \\ \text{door } F(4, 4, 2) \end{array}\right\} d = 12 - 16 + 2 = -2$$

Dus V: $3x - 4y + z = -2$.

$$MP\colon \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$ snijden met V geeft $3 \cdot 3\lambda - 4 \cdot (4 - 4\lambda) + (1 + \lambda) = -2$

$9\lambda - 16 + 16\lambda + 1 + \lambda = -2$

$26\lambda = 13$

$\lambda = \frac{1}{2}$

Het snijpunt is $S(1\frac{1}{2}, 2, 1\frac{1}{2})$.

$$d(F, MP) = d(F, S) = |\overrightarrow{FS}| = \left|\begin{pmatrix} -2\frac{1}{2} \\ -2 \\ -\frac{1}{2} \end{pmatrix}\right| = \sqrt{10\frac{1}{2}} = \frac{1}{2}\sqrt{42}$$

42 **a** V is het vlak door P, loodrecht op k.

$$\left.\begin{array}{l} V\colon y + 2z = d \\ \text{door } P(8, 10, 0) \end{array}\right\} d = 10 + 0 = 10$$

Dus V: $y + 2z = 10$.

$$k\colon \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$ snijden met V geeft $\lambda + 2 \cdot 2\lambda = 10$

$5\lambda = 10$

$\lambda = 2$

Het snijpunt is $S(4, 2, 4)$.

$$d(P, k) = d(P, S) = |\overrightarrow{PS}| = \left|\begin{pmatrix} -4 \\ -8 \\ 4 \end{pmatrix}\right| = \sqrt{96} = 4\sqrt{6}$$

b V is het vlak door Q, loodrecht op l.

$$\left.\begin{array}{l} V: -x + 3z = d \\ \text{door } Q(1, 5, 6) \end{array}\right\} d = -1 + 18 = 17$$

Dus V: $-x + 3z = 17$.

$$l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} \text{ snijden met } V \text{ geeft } -(-\mu) + 3(-1 + 3\mu) = 17$$

$$10\mu = 20$$

$$\mu = 2$$

Het snijpunt is $S(-2, 1, 5)$.

$$d(Q, l) = d(Q, S) = |\overrightarrow{QS}| = \begin{pmatrix} -3 \\ -4 \\ -1 \end{pmatrix} = \sqrt{26}$$

c V is het vlak door R, loodrecht op m.

$$\left.\begin{array}{l} V: x + 6y + z = d \\ \text{door } R(5, 4, 9) \end{array}\right\} d = 5 + 24 + 9 = 38$$

Dus V: $x + 6y + z = 38$.

$$m: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = v \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix} \text{ snijden met } V \text{ geeft } v + 6 \cdot 6v + v = 38$$

$$38v = 38$$

$$v = 1$$

Het snijpunt is $S(1, 6, 1)$.

$$d(R, m) = d(R, S) = |\overrightarrow{RS}| = \left|\begin{pmatrix} -4 \\ 2 \\ -8 \end{pmatrix}\right| = \sqrt{84} = 2\sqrt{21}$$

43 **a** $\vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix}$

V is het vlak door A, loodrecht op BC.

$$\left.\begin{array}{l} V: -3x + 2z = d \\ \text{door } A(-3, 0, 3) \end{array}\right\} d = 9 + 6 = 15$$

Dus V: $-3x + 2z = 15$.

$$BC: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix} \text{ snijden met } V \text{ geeft } -3 \cdot -3\lambda + 2(1 + 2\lambda) = 15$$

$$9\lambda + 2 + 4\lambda = 15$$

$$13\lambda = 13$$

$$\lambda = 1$$

Het snijpunt is $S(-3, 1, 3)$.

$$d(A, BC) = d(A, S) = \left|\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right| = \sqrt{1} = 1$$

b $O(\triangle ABC) = \frac{1}{2} \cdot d(B, C) \cdot d(A, BC) = \frac{1}{2} \cdot \sqrt{(-3)^2 + 0^2 + 2^2} \cdot 1 = \frac{1}{2}\sqrt{13}$

8

Bladzijde 172

44 $A(8, 0, 0)$ en $E(8, 0, 8)$ dus $M(8, 0, 4)$.
$E(8, 0, 8)$ en $F(8, 8, 8)$ dus $N(8, 4, 8)$.

$$\vec{r}_{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 8 \\ 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

V is het vlak door $C(0, 8, 0)$, loodrecht op MN.

$$\left.\begin{array}{l} V\colon y + z = d \\ \text{door } C(0, 8, 0) \end{array}\right\} d = 8$$

Dus $V\colon y + z = 8$.

$$MN\colon \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \text{ snijden met } V \text{ geeft } \begin{array}{l} \lambda + 4 + \lambda = 8 \\ 2\lambda = 4 \\ \lambda = 2 \end{array}$$

Het snijpunt is $S(8, 2, 6)$.

$$d(C, MN) = d(C, S) = |\overrightarrow{CS}| = \left|\begin{pmatrix} 8 \\ -6 \\ 6 \end{pmatrix}\right| = \sqrt{136} = 2\sqrt{34}$$

$$O(\triangle CMN) = \tfrac{1}{2} \cdot d(C, MN) \cdot d(M, N) = \tfrac{1}{2} \cdot 2\sqrt{34} \cdot \sqrt{4^2 + 4^2} = \sqrt{34} \cdot \sqrt{32} = \sqrt{17} \cdot \sqrt{64} = 8\sqrt{17}$$

45 **a** $$\vec{r}_{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$$

$$\vec{r}_{AG} = \vec{g} - \vec{a} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{n}_{AMG} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$\left.\begin{array}{l} AMG\colon 2x + y + z = d \\ \text{door } A(4, 0, 0) \end{array}\right\} d = 8$$

Dus $AMG\colon 2x + y + z = 8$.

b $B(4, 4, 0)$ en $F(4, 4, 4)$, dus $P(4, 4, 2)$.
k is de lijn door P, loodrecht op AMG.

$$\vec{s}_k = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \text{ en } \vec{r}_k = \vec{n}_{AMG} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \text{ dus } k\colon \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

c k snijden met AMG geeft
$2(4 + 2\lambda) + 4 + \lambda + 2 + \lambda = 8$
$8 + 4\lambda + 4 + \lambda + 2 + \lambda = 8$
$6\lambda = -6$
$\lambda = -1$
Het snijpunt is $Q(2, 3, 1)$.

d $$d(P, Q) = |\overrightarrow{PQ}| = \left|\begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix}\right| = \sqrt{(-2)^2 + (-1)^2 + (-1)^2} = \sqrt{6}$$

$d(P, Q)$ is $d(P, AMG)$ omdat PQ loodrecht op AMG staat en Q in het vlak AMG ligt.

Bladzijde 173

46 **a** $\vec{s}_k = \vec{p} = \begin{pmatrix} x_P \\ y_P \\ z_P \end{pmatrix}$ en $\vec{r}_k = \vec{n}_V = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, dus k: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_P \\ y_P \\ z_P \end{pmatrix} + \lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

b k snijden met V geeft

$a(x_P + a\lambda) + b(y_P + b\lambda) + c(z_P + c\lambda) = d$
$ax_P + a^2\lambda + by_P + b^2\lambda + cz_P + c^2\lambda = d$
$\lambda(a^2 + b^2 + c^2) = d - ax_P - by_P - cz_P$
$\lambda = \dfrac{d - ax_P - by_P - cz_P}{a^2 + b^2 + c^2}$

c $d(P, Q) = |\overrightarrow{PQ}| = \left| \begin{pmatrix} a\lambda \\ b\lambda \\ c\lambda \end{pmatrix} \right| = \sqrt{a^2\lambda^2 + b^2\lambda^2 + c^2\lambda^2} = \sqrt{\lambda^2} \cdot \sqrt{a^2 + b^2 + c^2} = |\lambda|\sqrt{a^2 + b^2 + c^2}$

d $d(P, V) = d(P, Q) = |\lambda|\sqrt{a^2 + b^2 + c^2} = \left| \dfrac{d - ax_P - by_P - cz_P}{a^2 + b^2 + c^2} \right| \cdot \sqrt{a^2 + b^2 + c^2} =$

$$\frac{\sqrt{a^2 + b^2 + c^2} \cdot |d - ax_P - by_P - cz_P|}{a^2 + b^2 + c^2} = \frac{|d - ax_P - by_P - cz_P|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|ax_P + by_P + cz_P - d|}{\sqrt{a^2 + b^2 + c^2}}$$

Bladzijde 174

47 $\vec{r}_{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$

$\vec{r}_{AG} = \vec{g} - \vec{a} = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ 3 \end{pmatrix}$

$\vec{n}_{AMG} = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} \times \begin{pmatrix} -4 \\ 6 \\ 3 \end{pmatrix} = \begin{pmatrix} -18 \\ -6 \\ -12 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

$\left.\begin{array}{l} AMG\text{: } 3x + y + 2z = d \\ \text{door } A(4, 0, 0) \end{array}\right\} d = 12$

Dus AMG: $3x + y + 2z = 12$.

$N(4, 6, 1\frac{1}{2})$, dus $d(N, AMG) = \dfrac{|12 + 6 + 3 - 12|}{\sqrt{3^2 + 1^2 + 2^2}} = \dfrac{9}{\sqrt{14}} \approx 2{,}41$.

48 $\vec{r}_{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\vec{r}_{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$\vec{n}_{ABM} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\left.\begin{array}{l} ABM\text{: } x + z = d \\ \text{door } A(4, 0, 0) \end{array}\right\} d = 4$

Dus ABM: $x + z = 4$.

$N(3, 3, 3)$, dus $d(N, ABM) = \dfrac{|3 + 0 + 3 - 4|}{\sqrt{1^2 + 0^2 + 1^2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$.

8

49 Breng een *Oxyz*-assenstelsel aan zo, dat *O* het punt (0, 0, 0) is, *A* het punt (4, 0, 0), *C* het punt (0, 4, 0) en *D* het punt (0, 0, 4).

$$\vec{r}_{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\vec{n}_{BMD} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\left.\begin{array}{l} BMD\colon x + y + 2z = d \\ \text{door } B(4, 4, 0) \end{array}\right\} d = 4 + 4 + 0 = 8$$

Dus $BMD\colon x + y + 2z = 8$.

$F(4, 4, 4)$, dus $d(F, BMD) = \dfrac{|4 + 4 + 8 - 8|}{\sqrt{1^2 + 1^2 + 2^2}} = \dfrac{8}{\sqrt{6}} = 1\frac{1}{3}\sqrt{6}$.

50 $$\vec{r}_{AF} = \vec{f} - \vec{a} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\vec{n}_V = \vec{r}_{AF} \times \vec{r}_{BD} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

Bladzijde 176

51 **a** $$\vec{r}_{OB} = \vec{b} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\vec{r}_{GM} = \vec{m} - \vec{g} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

V is het vlak door *GM* en evenwijdig met *OB*.

$$\vec{n}_V = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

$$\left.\begin{array}{l} V\colon x - y + 4z = d \\ \text{door } G(0, 4, 4) \end{array}\right\} d = 0 - 4 + 16 = 12$$

Dus $V\colon x - y + 4z = 12$.

$d(OB, GM) = d(O, V) = \dfrac{|0 - 0 + 0 - 12|}{\sqrt{1^2 + (-1)^2 + 4^2}} = \dfrac{12}{\sqrt{18}} = 2\sqrt{2}$

b $\vec{r}_{BG} = \vec{g} - \vec{b} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

$\vec{r}_{CM} = \vec{m} - \vec{c} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

V is het vlak door CM en evenwijdig met BG.

$\vec{n}_V = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$

$\left.\begin{array}{l} V\text{: } 2x + 3y + 2z = d \\ \text{door } C(0, 4, 0) \end{array}\right\} d = 12$

Dus V: $2x + 3y + 2z = 12$.

$d(BG, CM) = d(B, V) = \dfrac{|8 + 12 + 0 - 12|}{\sqrt{2^2 + 3^2 + 2^2}} = \dfrac{8}{\sqrt{17}} = \frac{8}{17}\sqrt{17}$

52 V is het vlak door l en evenwijdig met m.

$\vec{n}_V = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

$\left.\begin{array}{l} V\text{: } x - 2y + 2z = d \\ \text{door } (-1, -2, 0) \end{array}\right\} d = -1 + 4 + 0 = 3$

Dus V: $x - 2y + 2z = 3$.

$P(4, 2, 3)$ ligt op m.

$d(l, m) = d(P, V) = \dfrac{|4 - 4 + 6 - 3|}{\sqrt{1^2 + (-2)^2 + 2^2}} = \dfrac{3}{\sqrt{9}} = 1$

53 **a** $\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix}$

$\vec{r}_{EG} = \vec{g} - \vec{e} = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$

V is het vlak door BD en evenwijdig met EG.

$\vec{n}_V = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} \times \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -9 \\ -6 \\ -24 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ 2 \\ 8 \end{pmatrix}$

$\left.\begin{array}{l} V\text{: } 3x + 2y + 8z = d \\ \text{door } B(4, 6, 0) \end{array}\right\} d = 12 + 12 + 0 = 24$

Dus V: $3x + 2y + 8z = 24$.

$d(BD, EG) = d(E, V) = \dfrac{|12 + 0 + 24 - 24|}{\sqrt{3^2 + 2^2 + 8^2}} = \dfrac{12}{\sqrt{77}} = \frac{12}{77}\sqrt{77}$

b Van de kleinste driehoek AFP is AF de basis en $d(BD, AF)$ de hoogte.

$$\vec{r}_{BD} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} \text{ en } \vec{r}_{AF} = \vec{f} - \vec{a} = \begin{pmatrix} 4 \\ 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

V is het vlak door BD en evenwijdig met AF.

$$\vec{n}_V = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -12 \\ 4 \\ -8 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

$$\left.\begin{array}{l} V\colon 3x - y + 2z = d \\ \text{door } B(4, 6, 0) \end{array}\right\} d = 12 - 6 + 0 = 6$$

Dus V: $3x - y + 2z = 6$.

$$d(BD, AF) = d(A, V) = \frac{|12 - 0 + 0 - 6|}{\sqrt{3^2 + (-1)^2 + 2^2}} = \frac{6}{\sqrt{14}} = \tfrac{3}{7}\sqrt{14}$$

$$d(A, F) = |\vec{AF}| = \left|\begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix}\right| = \sqrt{0^2 + 6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$$

Oppervlakte kleinste driehoek is $\frac{1}{2} \cdot d(A, F) \cdot d(BD, AF) = \frac{1}{2} \cdot 3\sqrt{5} \cdot \frac{3}{7}\sqrt{14} = \frac{9}{14}\sqrt{70}$.

54 **a** Kies het assenstelsel zo, dat je de volgende punten krijgt: $S(0, 0, 0)$, $A(2, -2, 0)$, $B(2, 2, 0)$, $C(-2, 2, 0)$, $D(-2, -2, 0)$ en $T = (0, 0, 3)$.

$$\vec{r}_{AT} = \vec{t} - \vec{a} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}$$

$$\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

V is het vlak door AT en evenwijdig met BD.

$$\vec{n}_V = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ -3 \\ 4 \end{pmatrix}$$

$$\left.\begin{array}{l} V\colon 3x - 3y + 4z = d \\ \text{door } A(2, -2, 0) \end{array}\right\} d = 6 + 6 + 0 = 12$$

Dus V: $3x - 3y + 4z = 12$.

$$d(AT, BD) = d(B, V) = \frac{|6 - 6 + 0 - 12|}{\sqrt{3^2 + (-3)^2 + 4^2}} = \frac{12}{\sqrt{34}} = \tfrac{6}{17}\sqrt{34}$$

b Van de kleinste driehoek BCP is BC de basis en $d(AT, BC)$ de hoogte.

$$\vec{r}_{AT} = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} \text{ en } \vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

V is het vlak door AT en evenwijdig met BC.

$$\vec{n}_V = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix}$$

$$\left.\begin{array}{l} V\colon 3y - 2z = d \\ \text{door } A(2, -2, 0) \end{array}\right\} d = -6$$

Dus V: $3y - 2z = -6$.

$$d(AT, BC) = d(B, V) = \frac{|6 + 0 + 6|}{\sqrt{0^2 + 3^2 + (-2)^2}} = \frac{12}{\sqrt{13}} = \tfrac{12}{13}\sqrt{13}$$

$d(B,C) = 4$

Oppervlakte kleinste driehoek is $\frac{1}{2} \cdot d(B, C) \cdot d(AT, BC) = \frac{1}{2} \cdot 4 \cdot \frac{12}{13}\sqrt{13} = 1\frac{11}{13}\sqrt{13}$

55 Vliegtuig 1 vliegt volgens de lijn

$$l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 270 \\ 313 \\ 9{,}588 \end{pmatrix} + \lambda \begin{pmatrix} 120 - 270 \\ 487 - 313 \\ -0{,}012 - 9{,}588 \end{pmatrix} \text{ ofwel } l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 270 \\ 313 \\ 9{,}588 \end{pmatrix} + \lambda \begin{pmatrix} -150 \\ 174 \\ -9{,}6 \end{pmatrix}$$

Vliegtuig 2 vliegt volgens de lijn

$$m: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 87 \\ 313 \\ 10{,}202 \end{pmatrix} + \mu \begin{pmatrix} 230 - 87 \\ 560 - 313 \\ 0{,}002 - 10{,}202 \end{pmatrix} \text{ ofwel } m: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 87 \\ 313 \\ 10{,}202 \end{pmatrix} + \mu \begin{pmatrix} 143 \\ 247 \\ -10{,}2 \end{pmatrix}$$

V is het vlak door m en evenwijdig met l.

$$\vec{n}_V = \begin{pmatrix} -150 \\ 174 \\ -9{,}6 \end{pmatrix} \times \begin{pmatrix} 143 \\ 247 \\ -10{,}2 \end{pmatrix} = \begin{pmatrix} 596{,}4 \\ -2902{,}8 \\ -61\,932 \end{pmatrix}$$

$$\left.\begin{array}{l} V: 596{,}4x - 2902{,}8y - 61\,932z = d \\ \text{door } (87;\ 313;\ 10{,}202) \end{array}\right\} d = -1\,488\,519{,}864$$

Dus V: $596{,}4x - 2902{,}8y - 61\,932z = -1\,488\,519{,}864$.

$$d(l, m) = d((270;\ 313;\ 9{,}588),\ V) = \frac{|596{,}4 \cdot 270 - 2902{,}8 \cdot 313 - 61\,932 \cdot 9{,}588 + 1\,488\,519{,}864|}{\sqrt{596{,}4^2 + (-2902{,}8)^2 + (-61\,932)^2}} \approx 2{,}3736$$

Dus de kortst mogelijke afstand tussen de twee vliegtuigen is 2374 meter.

Diagnostische toets

Bladzijde 178

1 **a** $M\left(\frac{0+0}{2}, \frac{6+6}{2}, \frac{0+6}{2}\right) = M(0, 6, 3)$, dus $N\left(\frac{0+4}{2}, \frac{6+4}{2}, \frac{3+0}{2}\right) = N\left(2, 5, 1\tfrac{1}{2}\right)$.

b $d(M, P) = |\overrightarrow{MP}| = \left|\begin{pmatrix} 4 \\ -2 \\ -3 \end{pmatrix}\right| = \sqrt{4^2 + (-2)^2 + (-3)^2} = \sqrt{16 + 4 + 9} = \sqrt{29}$

c $\angle DPG = \angle(\overrightarrow{PD}, \overrightarrow{PG})$

$$\overrightarrow{PD} = \vec{d} - \vec{p} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 5 \end{pmatrix}$$

$$\overrightarrow{PG} = \vec{g} - \vec{p} = \begin{pmatrix} 0 \\ 6 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 5 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{PD}, \overrightarrow{PG})) = \frac{\begin{pmatrix} -4 \\ -4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 2 \\ 5 \end{pmatrix}}{\left|\begin{pmatrix} -4 \\ -4 \\ 5 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -4 \\ 2 \\ 5 \end{pmatrix}\right|} = \frac{16 - 8 + 25}{\sqrt{57} \cdot \sqrt{45}} = \frac{33}{\sqrt{2565}}$$

$\angle DPG = \angle(\overrightarrow{PD}, \overrightarrow{PG}) \approx 49{,}3°$

2 **a** $A(4, 0, 0)$ en $T(0, 0, 6)$ geeft $M(2, 0, 3)$.

$$\vec{n} = \vec{b} + \tfrac{1}{4}\overrightarrow{BT} = \vec{b} + \tfrac{1}{4}(\vec{t} - \vec{b}) = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} + \tfrac{1}{4}\left(\begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} + \tfrac{1}{4}\begin{pmatrix} -4 \\ -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 1\frac{1}{2} \end{pmatrix}$$

Dus $N\left(3, 3, 1\tfrac{1}{2}\right)$.

$$\vec{p} = \vec{c} + \tfrac{3}{4}\overrightarrow{CT} = \vec{c} + \tfrac{3}{4}(\vec{t} - \vec{c}) = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} + \tfrac{3}{4}\left(\begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} + \tfrac{3}{4}\begin{pmatrix} 0 \\ -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 4\frac{1}{2} \end{pmatrix}$$

Dus $P\left(0, 1, 4\tfrac{1}{2}\right)$.

b $\vec{m} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$ en $\vec{n} - \vec{m} = \begin{pmatrix} 3 \\ 3 \\ 1\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -1\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$, dus MN: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$.

$x_D = 0$ geeft $2 + 2\lambda = 0$, dus $\lambda = -1$.

$\lambda = -1$ geeft $\vec{d} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 6 \end{pmatrix}$.

Dus $D(0, -6, 6)$.

c $\vec{m} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$ en $\vec{p} - \vec{m} = \begin{pmatrix} 0 \\ 1 \\ 4\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix}$, dus MP: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix}$.

$z_E = 0$ geeft $3 + 3\lambda = 0$, dus $\lambda = -1$.

$\lambda = -1$ geeft $\vec{e} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix}$.

Dus $E(6, -2, 0)$.

3 **a** ACD snijdt de x-as in $A(6, 0, 0)$, de y-as in $C(0, 8, 0)$ en de z-as in $D(0, 0, 5)$,
dus ACD: $\frac{x}{6} + \frac{y}{8} + \frac{z}{5} = 1$ ofwel $20x + 15y + 24z = 120$.

b CDF snijdt de y-as in $(0, 8, 0)$ en de z-as in $(0, 0, 5)$, dus CDF: $\frac{x}{r} + \frac{y}{8} + \frac{z}{5} = 1$.

$F(6, 8, 5)$ invullen in $\frac{x}{r} + \frac{y}{8} + \frac{z}{5} = 1$ geeft $\frac{6}{r} + \frac{8}{8} + \frac{5}{5} = 1$

$$\frac{6}{r} + 1 + 1 = 1$$
$$\frac{6}{r} = -1$$
$$r = -6$$

CDF: $\frac{x}{-6} + \frac{y}{8} + \frac{z}{5} = 1$ ofwel CDF: $-20x + 15y + 24z = 120$.

Dus $\vec{n}_{CDF} = \begin{pmatrix} -20 \\ 15 \\ 24 \end{pmatrix}$.

c OF is de lijn $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \begin{pmatrix} 6 \\ 8 \\ 5 \end{pmatrix}$.

CMN snijdt de y-as in $(0, 8, 0)$ en de z-as in $(0, 0, 10)$, dus CMN: $\frac{x}{r} + \frac{y}{8} + \frac{z}{10} = 1$.

$N\left(6, 0, 2\frac{1}{2}\right)$ invullen in $\frac{x}{r} + \frac{y}{8} + \frac{z}{10} = 1$ geeft $\frac{6}{r} + \frac{0}{8} + \frac{2\frac{1}{2}}{10} = 1$

$$\frac{6}{r} + 0 + \frac{1}{4} = 1$$
$$\frac{6}{r} = \frac{3}{4}$$
$$3r = 24$$
$$r = 8$$

CMN: $\frac{x}{8} + \frac{y}{8} + \frac{z}{10} = 1$ ofwel CMN: $5x + 5y + 4z = 40$.
Substitutie van $x = 6\lambda, y = 8\lambda$ en $z = 5\lambda$ in $5x + 5y + 4z = 40$ geeft
$5 \cdot 6\lambda + 5 \cdot 8\lambda + 4 \cdot 5\lambda = 40$
$30\lambda + 40\lambda + 20\lambda = 40$
$90\lambda = 40$
$\lambda = \frac{4}{9}$
Dus het snijpunt is $S\left(2\frac{2}{3},\ 3\frac{5}{9},\ 2\frac{2}{9}\right)$.

8

4 **a** $\overrightarrow{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\overrightarrow{BN} = \vec{n} - \vec{b} = \begin{pmatrix} 6 \\ 0 \\ 2\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -8 \\ 2\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -16 \\ 5 \end{pmatrix}$

$\vec{n}_{BCN} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -16 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \\ -16 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 5 \\ 16 \end{pmatrix}$

$\left.\begin{array}{l} BCN\text{: } 5y + 16z = d \\ \text{door } C(0, 8, 0) \end{array}\right\} d = 40$

Dus BCN: $5y + 16z = 40$.

b $\overrightarrow{BF} = \vec{f} - \vec{b} = \begin{pmatrix} 6 \\ 8 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\overrightarrow{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -8 \\ 5 \end{pmatrix}$

$\vec{n}_{BFD} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} -6 \\ -8 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}$

$\left.\begin{array}{l} BFD\text{: } 4x - 3y = d \\ \text{door } B(6, 8, 0) \end{array}\right\} d = 24 - 24 = 0$

Dus BFD: $4x - 3y = 0$.

c $\overrightarrow{OM} = \vec{m} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}$ en $\overrightarrow{ON} = \vec{n} = \begin{pmatrix} 6 \\ 0 \\ 2\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 12 \\ 0 \\ 5 \end{pmatrix}$

$\vec{n}_{OMN} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix} \times \begin{pmatrix} 12 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 20 \\ 60 \\ -48 \end{pmatrix} \triangleq \begin{pmatrix} 5 \\ 15 \\ -12 \end{pmatrix}$

$\left.\begin{array}{l} OMN\text{: } 5x + 15y - 12z = d \\ \text{door } O(0, 0, 0) \end{array}\right\} d = 0$

Dus OMN: $5x + 15y - 12z = 0$.

Bladzijde 179

5 $\vec{r}_l = \vec{r}_k \times \vec{r}_m = \begin{pmatrix} 0 \\ -3 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ en $\vec{s}_l = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$

$l\text{: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} + v\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$

8

6 **a** $\vec{r}_{OF} = \vec{f} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$

$\vec{r}_{AG} = \vec{g} - \vec{a} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$

$$\cos(\angle(OF, AG)) = \frac{\left|\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}\right|}{\left|\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}\right|} = \frac{|-4+9+4|}{\sqrt{17} \cdot \sqrt{17}} = \frac{9}{17}$$

Dus $\angle(OF, AG) \approx 58{,}0°$.

b $\vec{r}_{DF} = \vec{f} - \vec{d} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}$

$\vec{r}_{AG} = \begin{pmatrix} -4 \\ 6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$

$$\cos(\angle(DF, AG)) = \frac{\left|\begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}\right|}{\left|\begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}\right|} = \frac{|-8+18+2|}{\sqrt{53} \cdot \sqrt{17}} = \frac{12}{\sqrt{901}}$$

Dus $\angle(DF, AG) \approx 66{,}4°$.

c $\angle ADG = \angle(\overrightarrow{DA}, \overrightarrow{DG})$

$\overrightarrow{DA} = \vec{a} - \vec{d} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$

$\overrightarrow{DG} = \vec{g} - \vec{d} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$

$$\cos(\angle(\overrightarrow{DA}, \overrightarrow{DG})) = \frac{\begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}\right|} = \frac{0+0-3}{\sqrt{25} \cdot \sqrt{37}} = \frac{-3}{5\sqrt{37}}$$

$\angle ADG = \angle(\overrightarrow{DA}, \overrightarrow{DG}) \approx 95{,}7°$

7 **a** $\vec{r}_{CE} = \vec{e} - \vec{c} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix}$

Lijn n is een normaal van BDF.

$$\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix}$$

$$\vec{r}_{BF} = \vec{f} - \vec{b} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{BDF} = \vec{r}_{BD} \times \vec{r}_{BF} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}$$

$$\cos(\angle(CE, n)) = \frac{\left| \begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} \right|} = \frac{|-12 - 12 + 0|}{\sqrt{61} \cdot \sqrt{13}} = \frac{24}{\sqrt{793}}$$

Dus $\angle(CE, n) \approx 31{,}5°$ en $\angle(CE, BDF) \approx 90° - 31{,}5° = 58{,}5°$.

b Lijn m is een normaal van DEF en lijn n is een normaal van AFC.

$$\vec{r}_{DE} = \vec{e} - \vec{d} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{r}_{DF} = \vec{f} - \vec{d} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}$$

$$\vec{r}_m = \vec{n}_{DEF} = \vec{r}_{DE} \times \vec{r}_{DF} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 6 \end{pmatrix}$$

$$\vec{r}_{AF} = \vec{f} - \vec{a} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$$

$$\vec{r}_{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{AFC} = \vec{r}_{AF} \times \vec{r}_{AC} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} \times \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 0 \\ -1 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix} \right|}{\left| \begin{pmatrix} 0 \\ -1 \\ 6 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix} \right|} = \frac{|0 + 2 + 18|}{\sqrt{37} \cdot \sqrt{22}} = \frac{20}{\sqrt{814}}$$

Dus $\angle(DEF, AFC) = \angle(m, n) \approx 45{,}5°$.

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c Lijn m is een normaal van BGD en lijn n is een normaal van OBE.

$$\vec{r}_{BG} = \vec{g} - \vec{b} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix}$$

$$\vec{r}_m = \vec{n}_{BGD} = \vec{r}_{BG} \times \vec{r}_{BD} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \\ 6 \end{pmatrix}$$

$$\vec{r}_{OB} = \vec{b} = \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$$\vec{r}_{OE} = \vec{e} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{OBE} = \vec{r}_{OB} \times \vec{r}_{OE} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ -6 \\ -12 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 6 \\ -1 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix} \right|}{\left| \begin{pmatrix} 6 \\ -1 \\ 6 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix} \right|} = \frac{|18 + 2 - 24|}{\sqrt{73} \cdot \sqrt{29}} = \frac{4}{\sqrt{2117}}$$

Dus $\angle(BGD, OBE) = \angle(m, n) \approx 85{,}0°$.

8 **a** $\vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 9 \\ 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix}$

V is het vlak door A, loodrecht op BC.

$\left.\begin{array}{l} V\text{: } 4x + 5y = d \\ \text{door } A(-4, 0, 1) \end{array}\right\} d = -16$

Dus V: $4x + 5y = -16$.

BC: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix}$ snijden met V geeft $4(5 + 4\lambda) + 5(1 + 5\lambda) = -16$

$20 + 16\lambda + 5 + 25\lambda = -16$

$41\lambda = -41$

$\lambda = -1$

Het snijpunt is $S(1, -4, 3)$.

$$d(A, BC) = d(A, S) = |\vec{AS}| = \left| \begin{pmatrix} 5 \\ -4 \\ 2 \end{pmatrix} \right| = \sqrt{45} = 3\sqrt{5}$$

b $O(\triangle ABC) = \frac{1}{2} \cdot d(B, C) \cdot d(A, BC) = \frac{1}{2} \cdot \sqrt{4^2 + 5^2 + 0^2} \cdot 3\sqrt{5} = \frac{1}{2} \cdot \sqrt{41} \cdot 3\sqrt{5} = 1\frac{1}{2}\sqrt{205}$

9 **a** $\overrightarrow{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 5 \end{pmatrix}$

$d(B, M) = |\overrightarrow{BM}| = \left| \begin{pmatrix} -6 \\ -4 \\ 5 \end{pmatrix} \right| = \sqrt{(-6)^2 + (-4)^2 + 5^2} = \sqrt{36 + 16 + 25} = \sqrt{77} \approx 8{,}77$

b $\vec{r}_{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 6 \\ 0 \\ 2\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \\ -2\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 12 \\ -8 \\ -5 \end{pmatrix}$

$\vec{r}_{OF} = \vec{f} = \begin{pmatrix} 6 \\ 8 \\ 5 \end{pmatrix}$

V is het vlak door MN en evenwijdig met OF.

$\vec{n}_V = \begin{pmatrix} 12 \\ -8 \\ -5 \end{pmatrix} \times \begin{pmatrix} 6 \\ 8 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ -90 \\ 144 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -5 \\ 8 \end{pmatrix}$

$\left.\begin{array}{l} V\text{: } -5y + 8z = d \\ \text{door } M(0, 4, 5) \end{array}\right\} d = -20 + 40 = 20$

Dus V: $-5y + 8z = 20$.

$d(MN, OF) = d(O, V) = \dfrac{|0 + 0 - 20|}{\sqrt{(-5)^2 + 8^2}} = \dfrac{20}{\sqrt{89}} \approx 2{,}12$

c $\vec{r}_{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix}$

V is het vlak door D, loodrecht op AC.

$\left.\begin{array}{l} V\text{: } -3x + 4y = d \\ \text{door } D(0, 0, 5) \end{array}\right\} d = 0$

Dus V: $-3x + 4y = 0$.

AC: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix}$ snijden met $-3x + 4y = 0$ geeft $-3(6 - 3\lambda) + 4 \cdot 4\lambda = 0$

$-18 + 9\lambda + 16\lambda = 0$

$25\lambda = 18$

$\lambda = \frac{18}{25}$

Het snijpunt is $S\left(3\frac{21}{25}, 2\frac{22}{25}, 0\right)$.

$d(D, AC) = d(D, S) = |\overrightarrow{DS}| = \left| \begin{pmatrix} 3\frac{21}{25} \\ 2\frac{22}{25} \\ -5 \end{pmatrix} \right| = \sqrt{\left(3\frac{21}{25}\right)^2 + \left(2\frac{22}{25}\right)^2 + (-5)^2} = \sqrt{48{,}04} \approx 6{,}93$

d $\vec{r}_{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 5 \end{pmatrix}$

$\vec{r}_{BN} = \vec{n} - \vec{b} = \begin{pmatrix} 6 \\ 0 \\ 2\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -8 \\ 2\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -16 \\ 5 \end{pmatrix}$

$\vec{n}_{BMN} = \begin{pmatrix} -6 \\ -4 \\ 5 \end{pmatrix} \times \begin{pmatrix} 0 \\ -16 \\ 5 \end{pmatrix} = \begin{pmatrix} 60 \\ 30 \\ 96 \end{pmatrix} \triangleq \begin{pmatrix} 10 \\ 5 \\ 16 \end{pmatrix}$

$\left.\begin{array}{l} BMN\text{: } 10x + 5y + 16z = d \\ \text{door } B(6, 8, 0) \end{array}\right\} d = 60 + 40 + 0 = 100$

Dus BMN: $10x + 5y + 16z = 100$.

$d(A, BMN) = \dfrac{|60 + 0 + 0 - 100|}{\sqrt{10^2 + 5^2 + 16^2}} = \dfrac{40}{\sqrt{381}} \approx 2{,}05$

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