10 Complexe getallen

Voorkennis De exacte-waarden-cirkel

Bladzijde 40

1 **a** $\sin\left(\frac{3}{4}\pi\right) = \frac{1}{2}\sqrt{2}$

b $\cos\left(-\frac{3}{4}\pi\right) = -\frac{1}{2}\sqrt{2}$

c $\sin\left(-\frac{2}{3}\pi\right) = -\frac{1}{2}\sqrt{3}$

d $\cos\left(-\frac{2}{3}\pi\right) = -\frac{1}{2}$

e $\sin\left(-\frac{1}{3}\pi\right) = -\frac{1}{2}\sqrt{3}$

f $\cos\left(-\frac{5}{6}\pi\right) = -\frac{1}{2}\sqrt{3}$

2 **a** $\sin(\alpha) = \frac{1}{2}\sqrt{3}$ geeft $\alpha = \frac{1}{3}\pi \vee \alpha = \frac{2}{3}\pi$

b $\cos(\alpha) = -\frac{1}{2}$ geeft $\alpha = -\frac{2}{3}\pi \vee \alpha = \frac{2}{3}\pi$

c $\sin(\alpha) = -\frac{1}{2}\sqrt{2}$ geeft $\alpha = -\frac{1}{4}\pi \vee \alpha = -\frac{3}{4}\pi$

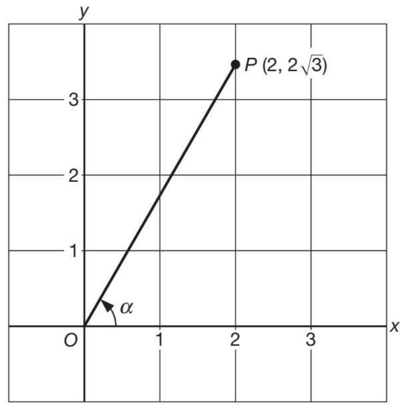
d $\cos(\alpha) = 0$ geeft $\alpha = -\frac{1}{2}\pi \vee \alpha = \frac{1}{2}\pi$

e $\sin(\alpha) = -\frac{1}{2}\sqrt{3}$ geeft $\alpha = -\frac{1}{3}\pi \vee \alpha = -\frac{2}{3}\pi$

f $\cos(\alpha) = \frac{1}{2}\sqrt{2}$ geeft $\alpha = -\frac{1}{4}\pi \vee \alpha = \frac{1}{4}\pi$

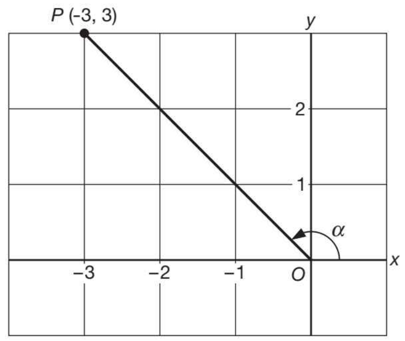
Bladzijde 41

3 **a**



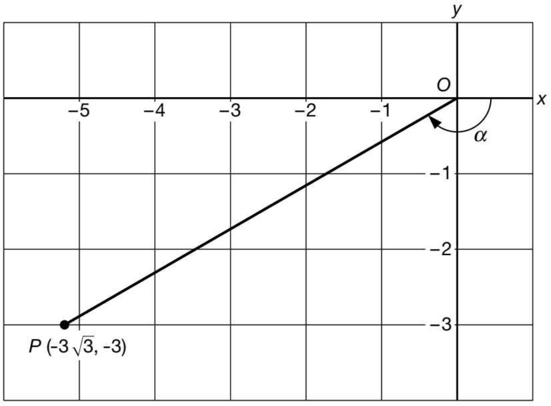
$\tan(\alpha) = \dfrac{2\sqrt{3}}{2} = \sqrt{3}$ geeft $\alpha = \frac{1}{3}\pi$

b



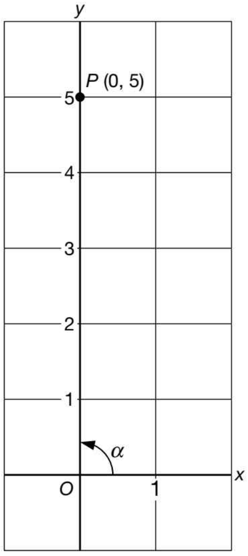
$\tan(\alpha) = \dfrac{3}{-3} = -1$ geeft $\alpha = \frac{3}{4}\pi$

c



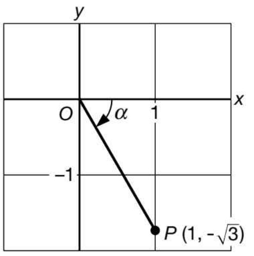
$\tan(\alpha) = \dfrac{-3}{-3\sqrt{3}} = \frac{1}{3}\sqrt{3}$ geeft $\alpha = -\frac{5}{6}\pi$

d



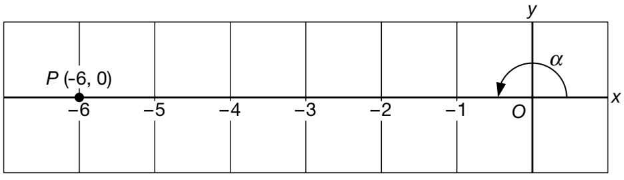
$\alpha = \frac{1}{2}\pi$

e



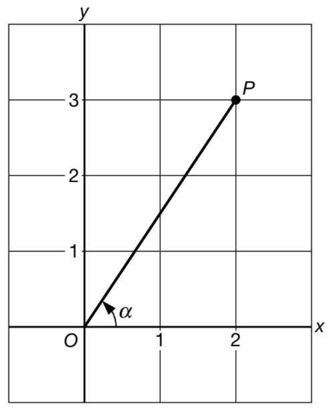
$\tan(\alpha) = \dfrac{-\sqrt{3}}{1} = -\sqrt{3}$ geeft $\alpha = -\frac{1}{3}\pi$

f



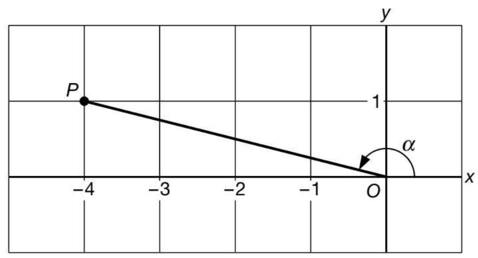
$\alpha = \pi$

4 **a**



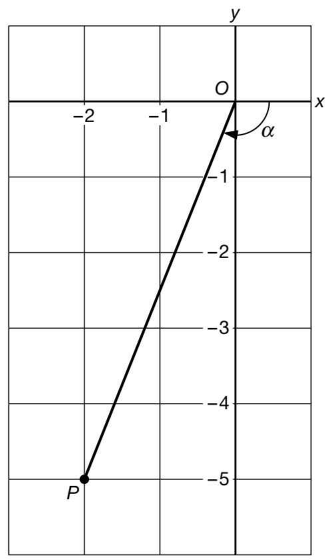
$\tan(\alpha) = \frac{3}{2} = 1\frac{1}{2}$ geeft $\alpha \approx 0{,}983$

b



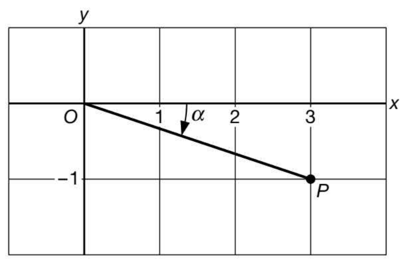
$\tan(\alpha) = \frac{1}{-4} = -\frac{1}{4}$ geeft $\alpha \approx 2{,}897$

c



$\tan(\alpha) = \frac{-5}{-2} = 2\frac{1}{2}$ geeft $\alpha \approx -1{,}951$

d



$\tan(\alpha) = \frac{-1}{3} = -\frac{1}{3}$ geeft $\alpha \approx -0{,}322$

10

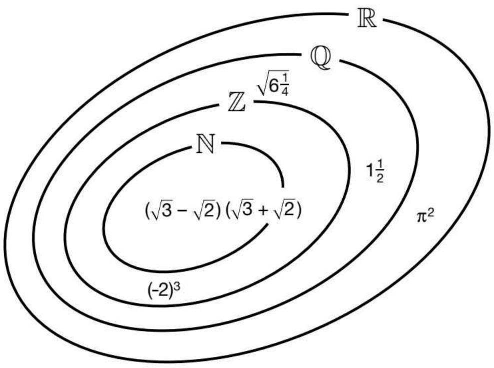
10.1 Rekenen met complexe getallen

Bladzijde 42

1 **a** I $2x + 5 = 13$
$2x = 8$
$x = 4$
II $2x + 5 = 3$
$2x = -2$
$x = -1$
III $2x + 5 = 8$
$2x = 3$
$x = 1\frac{1}{2}$
IV $x^2 + 5 = 8$
$x^2 = 3$
$x = \sqrt{3} \vee x = -\sqrt{3}$
b Bij III en IV.

Bladzijde 43

2



3 **a** $x^2 + x = 6$
$x^2 + x - 6 = 0$
$(x - 2)(x + 3) = 0$
$x = 2 \vee x = -3$
b $x^2 + x = 4$
$x^2 + x - 4 = 0$
$D = 1^2 - 4 \cdot 1 \cdot -4 = 17$
geen oplossing in ℚ
c $x^3 - x = 0$
$x(x^2 - 1) = 0$
$x = 0 \vee x^2 = 1$
$x = 0 \vee x = 1 \vee x = -1$
d $x^3 - 2x = 0$
$x(x^2 - 2) = 0$
$x = 0 \vee x^2 = 2$
$x = 0$
e $x^4 - 9x = 0$
$x(x^3 - 9) = 0$
$x = 0 \vee x^3 = 9$
$x = 0$
f $x^4 - 9x^2 = 0$
$x^2(x^2 - 9) = 0$
$x^2 = 0 \vee x^2 = 9$
$x = 0 \vee x = 3 \vee x = -3$

4 **a** $(x + 3)^2 = 7$
$x + 3 = \sqrt{7} \vee x + 3 = -\sqrt{7}$
$x = -3 + \sqrt{7} \vee x = -3 - \sqrt{7}$
b $(x + 3)^2 = -7$ heeft geen oplossing in ℝ omdat $(x + 3)^2$ niet negatief kan zijn.

Bladzijde 45

5 **a** $3x + 5i + 3 = 2i - x$
$4x = -3 - 3i$
$x = -\frac{3}{4} - \frac{3}{4}i$
b $2x^2 + 10 = 0$
$2x^2 = -10$
$x^2 = -5$
$x^2 = 5i^2$
$x = i\sqrt{5} \vee x = -i\sqrt{5}$
c $(x + 2)^2 + 10 = 0$
$(x + 2)^2 = -10$
$(x + 2)^2 = 10i^2$
$x + 2 = i\sqrt{10} \vee x + 2 = -i\sqrt{10}$
$x = -2 + i\sqrt{10} \vee x = -2 - i\sqrt{10}$
d $x^2 - 10x + 40 = 0$
$(x - 5)^2 - 25 + 40 = 0$
$(x - 5)^2 = -15$
$(x - 5)^2 = 15i^2$
$x - 5 = i\sqrt{15} \vee x - 5 = -i\sqrt{15}$
$x = 5 + i\sqrt{15} \vee x = 5 - i\sqrt{15}$
e $x^2 + 8x + 14 = 0$
$(x + 4)^2 - 16 + 14 = 0$
$(x + 4)^2 = 2$
$x + 4 = \sqrt{2} \vee x + 4 = -\sqrt{2}$
$x = -4 + \sqrt{2} \vee x = -4 - \sqrt{2}$
f $(x + 3)^2 = -16$
$(x + 3)^2 = 16i^2$
$x + 3 = 4i \vee x + 3 = -4i$
$x = -3 + 4i \vee x = -3 - 4i$

6 **a** $\frac{1}{3}x + 10 + 2i = \frac{1}{4}x + 12 - 5i$
$\frac{1}{12}x = 2 - 7i$
$x = 24 - 84i$
b $(2x + 3)^2 + 10 = 0$
$(2x + 3)^2 = -10$
$(2x + 3)^2 = 10i^2$
$2x + 3 = i\sqrt{10} \vee 2x + 3 = -i\sqrt{10}$
$2x = -3 + i\sqrt{10} \vee 2x = -3 - i\sqrt{10}$
$x = -1\frac{1}{2} + \frac{1}{2}i\sqrt{10} \vee x = -1\frac{1}{2} - \frac{1}{2}i\sqrt{10}$
c $(x - 3)^2 + 2x = 0$
$x^2 - 6x + 9 + 2x = 0$
$x^2 - 4x + 9 = 0$
$(x - 2)^2 - 4 + 9 = 0$
$(x - 2)^2 = -5$
$(x - 2)^2 = 5i^2$
$x - 2 = i\sqrt{5} \vee x - 2 = -i\sqrt{5}$
$x = 2 + i\sqrt{5} \vee x = 2 - i\sqrt{5}$
d $4x^2 + 4x + 7 = 0$
$(2x + 1)^2 - 1 + 7 = 0$
$(2x + 1)^2 = -6$
$(2x + 1)^2 = 6i^2$
$2x + 1 = i\sqrt{6} \vee 2x + 1 = -i\sqrt{6}$
$2x = -1 + i\sqrt{6} \vee 2x = -1 - i\sqrt{6}$
$x = -\frac{1}{2} + \frac{1}{2}i\sqrt{6} \vee 2x = -\frac{1}{2} - \frac{1}{2}i\sqrt{6}$

7 **a** $(2 + i) \cdot (10 - 5i) = 20 - 10i + 10i - 5i^2 = 20 + 5 = 25$
b $(2 + i) \cdot (2 - i) = 4 - i^2 = 4 + 1 = 5$
$(3 + 2i) \cdot (3 - 2i) = 9 - 4i^2 = 9 + 4 = 13$
c $(a + bi) \cdot (c + di) = ac + adi + bci + bdi^2 = ac + adi + bci - bd = (ac - bd) + (ad + bc)i$

Bladzijde 46

8 **a** $(2 + 5i) + (4 - 6i) = 6 - i$
b $(5 - i)(5 + i) = 25 - i^2 = 25 + 1 = 26$
c $(2 + i)^2 = 4 + 4i + i^2 = 4 + 4i - 1 = 3 + 4i$
d $i(6 + 7i) = 6i + 7i^2 = 6i - 7 = -7 + 6i$
e $i^5 = i^2 \cdot i^2 \cdot i = -1 \cdot -1 \cdot i = i$
f $(1 + i)(6 - i) + (3 - i)(3 + 2i) = 6 - i + 6i - i^2 + 9 + 6i - 3i - 2i^2 = 6 - i + 6i + 1 + 9 + 6i - 3i + 2 = 18 + 8i$

9 **a** $\frac{2}{1+i} = \frac{2}{1+i} \cdot \frac{1-i}{1-i} = \frac{2-2i}{1-i^2} = \frac{2-2i}{1+1} = \frac{2-2i}{2} = 1 - i$

b $\frac{2+3i}{i} = \frac{2+3i}{i} \cdot \frac{i}{i} = \frac{2i+3i^2}{i^2} = \frac{2i-3}{-1} = -2i + 3 = 3 - 2i$

c $\frac{2+i}{2-i} = \frac{2+i}{2-i} \cdot \frac{2+i}{2+i} = \frac{4+4i+i^2}{4-i^2} = \frac{4+4i-1}{4+1} = \frac{3+4i}{5} = \frac{3}{5} + \frac{4}{5}i$

d $\frac{3+5i}{12+5i} = \frac{3+5i}{12+5i} \cdot \frac{12-5i}{12-5i} = \frac{36-15i+60i-25i^2}{144-25i^2} = \frac{36+45i+25}{144+25} = \frac{61+45i}{169} = \frac{61}{169} + \frac{45}{169}i$

e $\frac{2i}{1+3i} = \frac{2i}{1+3i} \cdot \frac{1-3i}{1-3i} = \frac{2i-6i^2}{1-9i^2} = \frac{2i+6}{1+9} = \frac{2i+6}{10} = \frac{1}{5}i + \frac{3}{5} = \frac{3}{5} + \frac{1}{5}i$

f $(2+3i) \cdot \frac{3}{2+i} = \frac{6+9i}{2+i} \cdot \frac{2-i}{2-i} = \frac{12-6i+18i-9i^2}{4-i^2} = \frac{12+12i+9}{4+1} = \frac{21+12i}{5} = 4\frac{1}{5} + 2\frac{2}{5}i$

10 **a** $(3 + 4i)^2 = 9 + 24i + 16i^2 = 9 + 24i - 16 = -7 + 24i$

b $\frac{3+i}{3+4i} = \frac{3+i}{3+4i} \cdot \frac{3-4i}{3-4i} = \frac{9-12i+3i-4i^2}{9-16i^2} = \frac{9-9i+4}{9+16} = \frac{13-9i}{25} = \frac{13}{25} - \frac{9}{25}i$

c $(2 + i)^2 - (2 - i)^2 = 4 + 4i + i^2 - (4 - 4i + i^2) = 4 + 4i + i^2 - 4 + 4i - i^2 = 8i$

d $i^2(i^3 - i^4 - i^5) = -1 \cdot (i^2 \cdot i - i^2 \cdot i^2 - i^2 \cdot i^2 \cdot i) = -1 \cdot (-i - 1 - i) = -1 \cdot (-1 - 2i) = 1 + 2i$

e $\frac{5}{2+3i} = \frac{5}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{10-15i}{4-9i^2} = \frac{10-15i}{4+9} = \frac{10-15i}{13} = \frac{10}{13} - 1\frac{2}{13}i$

f $i + i^2 + i^3 + ... + i^{10} = i - 1 - i + 1 + i - 1 - i + 1 + i - 1 = -1 + i$

10

Bladzijde 47

11 $\dfrac{a+bi}{c+di} = \dfrac{a+bi}{c+di} \cdot \dfrac{c-di}{c-di} = \dfrac{ac-adi+bci-bdi^2}{c^2-d^2i^2} = \dfrac{ac-adi+bci+bd}{c^2+d^2} = \dfrac{ac+bd-(ad-bc)i}{c^2+d^2}$

12 **a** $(a+bi)(a-bi) = a^2 - b^2i^2 = a^2+b^2$ is een reëel getal.
b $(3+4i)(3-4i) = 3^2+4^2 = 25$
$(100-200i)(100+200i) = 100^2+200^2 = 50\,000$

13 **a** $\dfrac{z+\bar{z}}{2} = \dfrac{a+bi+a-bi}{2} = \dfrac{2a}{2} = a = \text{Re}(z)$
b $\dfrac{z-\bar{z}}{2i} = \dfrac{a+bi-(a-bi)}{2i} = \dfrac{a+bi-a+bi}{2i} = \dfrac{2bi}{2i} = b = \text{Im}(z)$
c $\bar{\bar{z}} = \overline{\overline{a+bi}} = \overline{a-bi} = a+bi = z$

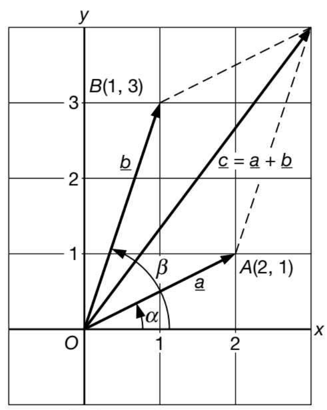
14 **a** $\overline{z_1+z_2} = \overline{a+bi+c+di} = \overline{a+c+(b+d)i} = a+c-(b+d)i = a-bi+c-di = \overline{z_1}+\overline{z_2}$
b $\overline{z_1 \cdot z_2} = \overline{(a+bi)\cdot(c+di)} = \overline{ac+adi+bci+bdi^2} = \overline{ac-bd+(ad+bc)i} = ac-bd-(ad+bc)i$ ... (1)
$\overline{z_1}\cdot\overline{z_2} = \overline{a+bi}\cdot\overline{c+di} = (a-bi)\cdot(c-di) = ac-adi-bci+bdi^2 = ac-bd-(ad+bc)i$ ... (2)
Uit (1) en (2) volgt dat $\overline{z_1 \cdot z_2} = \overline{z_1}\cdot\overline{z_2}$.
c $\overline{\left(\dfrac{z_1}{z_2}\right)} = \overline{\left(\dfrac{a+bi}{c+di}\right)} = \overline{\left(\dfrac{ac+bd-(ad-bc)i}{c^2+d^2}\right)} = \overline{\left(\dfrac{ac+bd}{c^2+d^2} - \dfrac{ad-bc}{c^2+d^2}i\right)} = \dfrac{ac+bd}{c^2+d^2} + \dfrac{ad-bc}{c^2+d^2}i$ ... (1)
$\dfrac{\overline{z_1}}{\overline{z_2}} = \dfrac{\overline{a+bi}}{\overline{c+di}} = \dfrac{a-bi}{c-di}\cdot\dfrac{c+di}{c+di} = \dfrac{ac+adi-bci-bdi^2}{c^2+d^2} = \dfrac{ac+bd+(ad-bc)i}{c^2+d^2} = \dfrac{ac+bd}{c^2+d^2} + \dfrac{ad-bc}{c^2+d^2}i$ ... (2)
Uit (1) en (2) volgt dat $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\overline{z_1}}{\overline{z_2}}$.

15 **a** $\overline{(4+3i)^2} = \overline{16+24i+9i^2} = \overline{16+24i-9} = \overline{7+24i} = 7-24i$
b $\overline{\left(\dfrac{4+3i}{2+i}\right)} = \dfrac{\overline{4+3i}}{\overline{2+i}} = \dfrac{4-3i}{2-i}\cdot\dfrac{2+i}{2+i} = \dfrac{8+4i-6i-3i^2}{4-i^2} = \dfrac{8-2i+3}{4+1^2} = \dfrac{11-2i}{5} = 2\tfrac{1}{5} - \tfrac{2}{5}i$
c $\dfrac{\overline{3+4i}}{3-4i} = \dfrac{3-4i}{3-4i} = 1$

10.2 Het complexe vlak

Bladzijde 49

16 **a**



$\vec{c} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

b $|\vec{a}| = \sqrt{2^2+1^2} = \sqrt{5}$
$|\vec{b}| = \sqrt{1^2+3^2} = \sqrt{10}$
$|\vec{c}| = \sqrt{3^2+4^2} = 5$
$|\vec{a}| + |\vec{b}| = \sqrt{5}+\sqrt{10}$ en $|\vec{c}| = |\vec{a}+\vec{b}| = 5$, dus er geldt niet $|\vec{a}| + |\vec{b}| = |\vec{a}+\vec{b}|$.

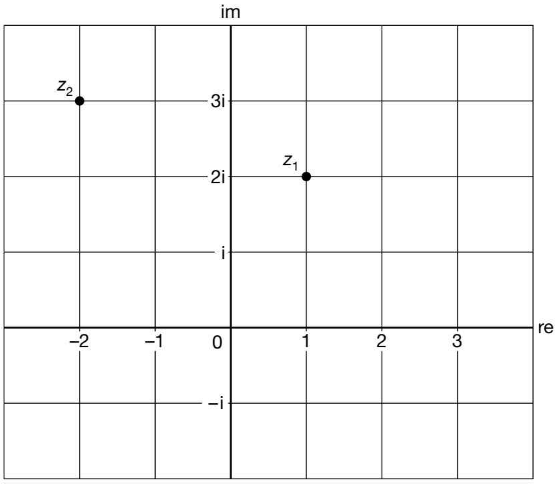
c $\tan(\alpha) = \frac{1}{2}$ geeft $\alpha \approx 0{,}46$

$\tan(\beta) = \dfrac{3}{1} = 3$ geeft $\beta \approx 1{,}25$

$\tan(\angle(\vec{c}, \text{positieve } x\text{-as})) = \frac{4}{3}$ geeft $\angle(\vec{c}, \text{positieve } x\text{-as}) \approx 0{,}93$

Bladzijde 51

17 **a**

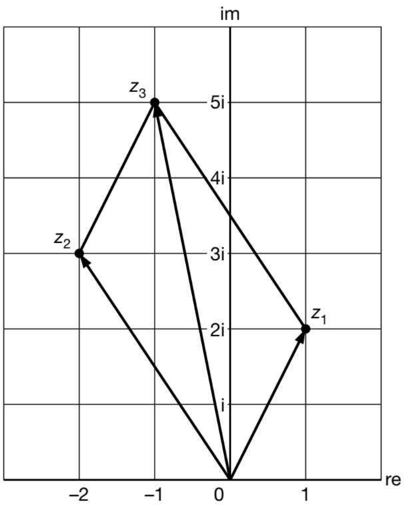


$|z_1| = \sqrt{1^2 + 2^2} = \sqrt{5}$

$|z_2| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$

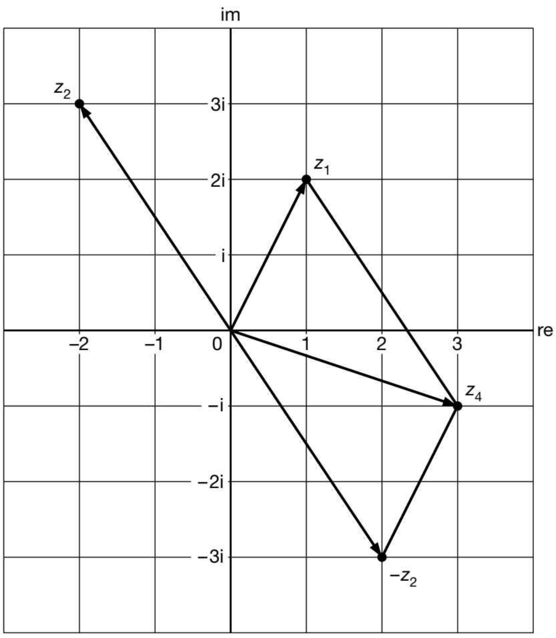
b $z_3 = z_1 + z_2 = 1 + 2i - 2 + 3i = -1 + 5i$

c



d $z_4 = z_1 - z_2 = 1 + 2i - (-2 + 3i) = 1 + 2i + 2 - 3i = 3 - i$

e



f $z_5 = z_1 \cdot z_2 = (1 + 2i) \cdot (-2 + 3i) = -2 + 3i - 4i + 6i^2 = -2 + 3i - 4i - 6 = -8 - i$

g $|z_1 \cdot z_2| = |-8 - i| = \sqrt{(-8)^2 + (-1)^2} = \sqrt{65}$

$|z_1| = \sqrt{5}$ en $|z_2| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$ geeft $|z_1| \cdot |z_2| = \sqrt{5} \cdot \sqrt{13} = \sqrt{65}$.

Dus $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$.

18 **a,b,c,d**

$\text{Re}(z) = \text{Im}(z)$

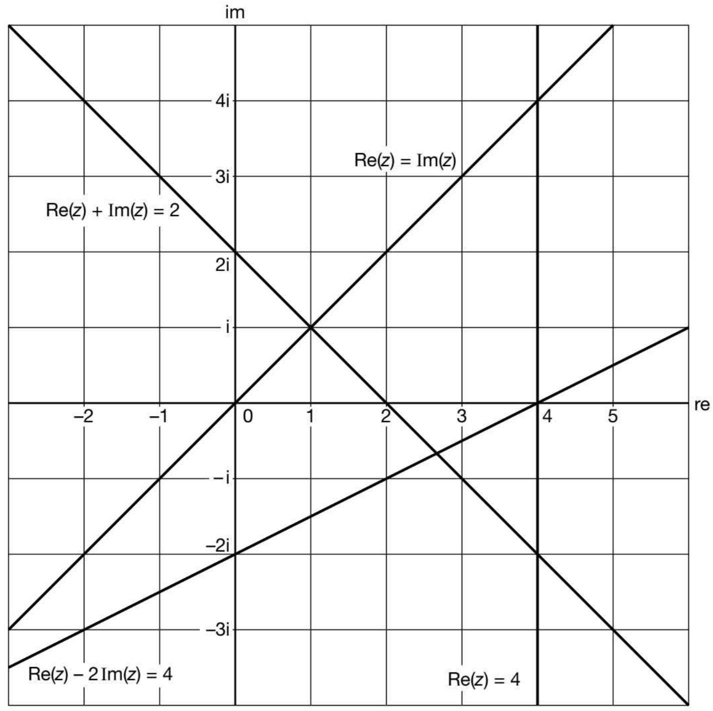
Re(z)	0	1
Im(z)	0	1

$\text{Re}(z) + \text{Im}(z) = 2$

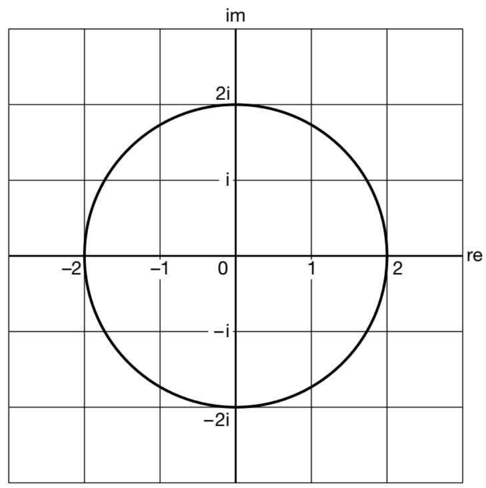
Re(z)	0	2
Im(z)	2	0

$\text{Re}(z) - 2\,\text{Im}(z) = 4$

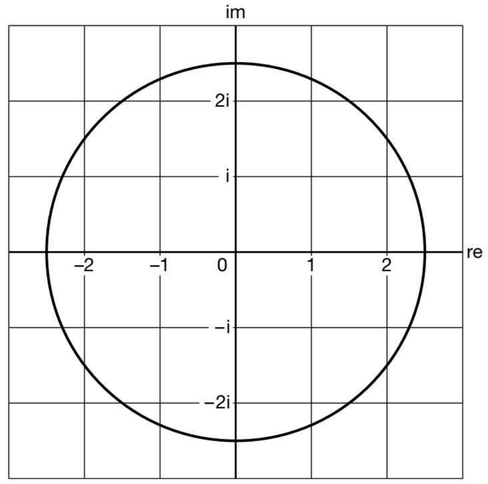
Re(z)	0	4
Im(z)	− 2	0



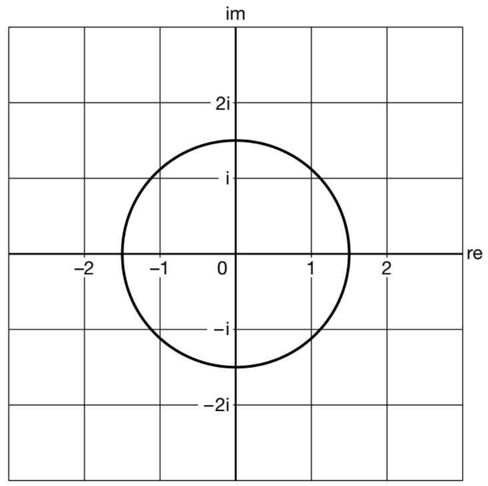
19 **a** $(\text{Re}(z))^2 + (\text{Im}(z))^2 = 4$ dus $|z| = 2$.
Dit is de cirkel met middelpunt 0 en straal 2.



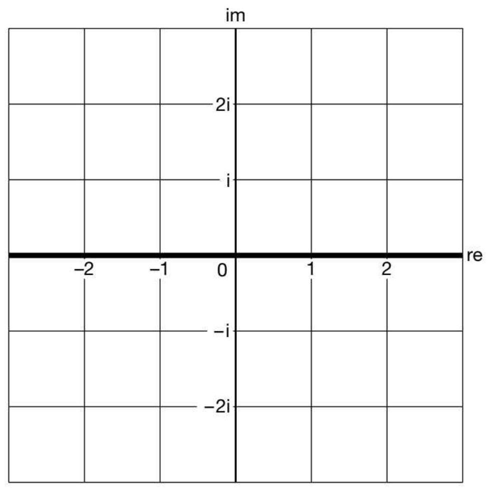
b $|z| = 2{,}5$
Dit is de cirkel met middelpunt 0 en straal 2,5.



c $z \cdot \bar{z} = 2\frac{1}{4}$ geeft $|z| = \sqrt{2\frac{1}{4}} = 1\frac{1}{2}$
Dit is de cirkel met middelpunt 0 en straal $1\frac{1}{2}$.



d $z = \bar{z}$
$a + b\mathrm{i} = a - b\mathrm{i}$
$2b\mathrm{i} = 0$
$b = 0$
$\mathrm{Im}(z) = 0$
Dit is de reële as.



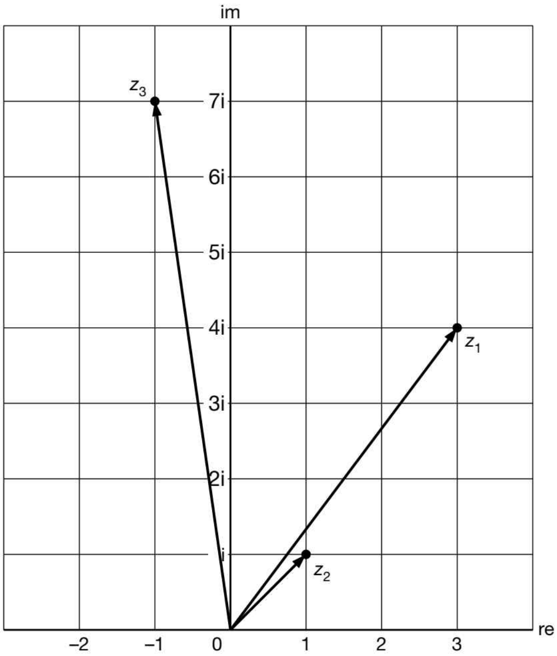
20 $z \cdot \bar{z} = (a + b\mathrm{i}) \cdot (a - b\mathrm{i}) = a^2 - b^2\mathrm{i}^2 = a^2 + b^2$
$|z| = \sqrt{a^2 + b^2}$, dus $|z|^2 = a^2 + b^2$.
Dus $z \cdot \bar{z} = |z|^2$.

21 $|z_1 \cdot z_2| = |(a + bi) \cdot (c + di)| = |ac + adi + bci + bdi^2| = |ac - bd + (bc + ad)i| = \sqrt{(ac - bd)^2 + (bc + ad)^2} =$
$\sqrt{a^2c^2 - 2abcd + b^2d^2 + b^2c^2 + 2abcd + a^2d^2} = \sqrt{a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2}$ ... (1)
$|z_1| \cdot |z_2| = |a + bi| \cdot |c + di| = \sqrt{a^2 + b^2} \cdot \sqrt{c^2 + d^2} = \sqrt{(a^2 + b^2)(c^2 + d^2)} = \sqrt{a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2}$ ... (2)
Uit (1) en (2) volgt $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$.

Bladzijde 52

22 **a** $z_3 = z_1 \cdot z_2 = (3 + 4i)(1 + i) = 3 + 3i + 4i + 4i^2 = 3 + 3i + 4i - 4 = -1 + 7i$

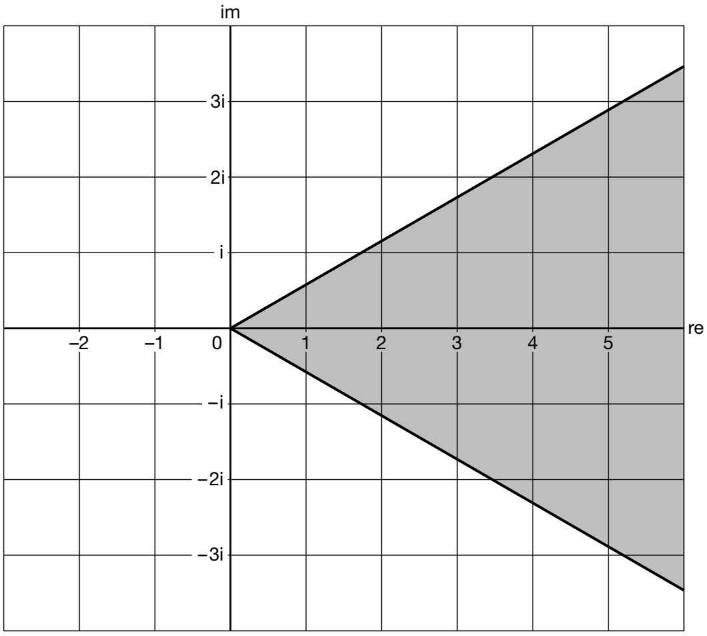
b



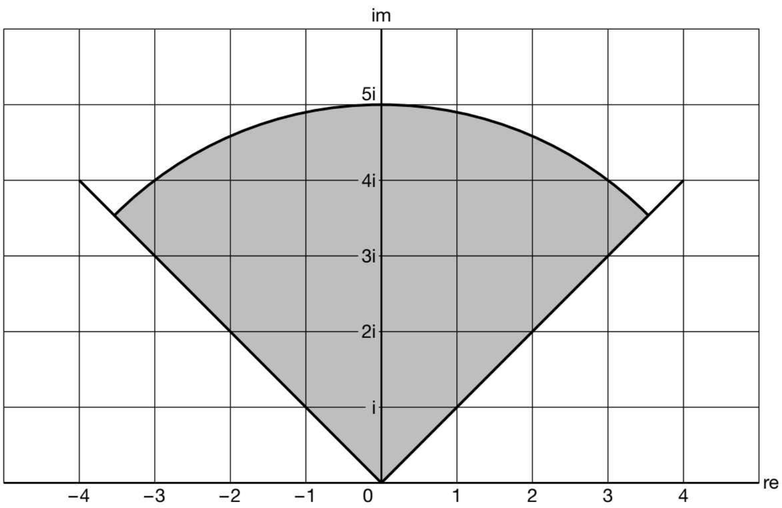
c Voor z_1 geldt: $\tan(\varphi_1) = \frac{4}{3}$ geeft $\varphi_1 \approx 0{,}927$.
Voor z_2 geldt: $\tan(\varphi_2) = \frac{1}{1} = 1$ geeft $\varphi_2 \approx 0{,}785$.
Voor z_3 geldt: $\tan(\varphi_3) = \frac{7}{-1} = -7$ geeft $\varphi_3 \approx 1{,}713$.
Voor de draaiingshoeken geldt $\varphi_1 + \varphi_2 = \varphi_3$.

Bladzijde 54

23 **a**



b



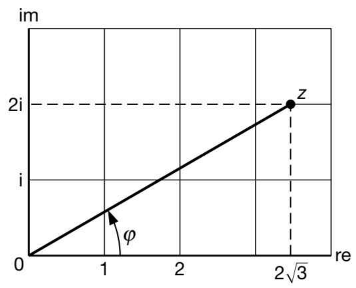
24 **a** $|z| = \sqrt{3^2 + 2^2} = \sqrt{13}$
angle(3 + 2i) (TI) of Arg(3 + 2i) (Casio) geeft $\varphi \approx 0{,}59$

b $|z| = \sqrt{(-2)^2 + 5^2} = \sqrt{29}$
angle(−2 + 5i) (TI) of Arg(−2 + 5i) (Casio) geeft $\varphi \approx 1{,}95$

c $|z| = \sqrt{(-3)^2 + (-4)^2} = \sqrt{25} = 5$
angle(−3 − 4i) (TI) of Arg(−3 − 4i) (Casio) geeft $\varphi \approx -2{,}21$

d $|z| = \sqrt{4^2 + (-1)^2} = \sqrt{17}$
angle(4 − i) (TI) of Arg(4 − i) (Casio) geeft $\varphi \approx -0{,}24$

25 **a**

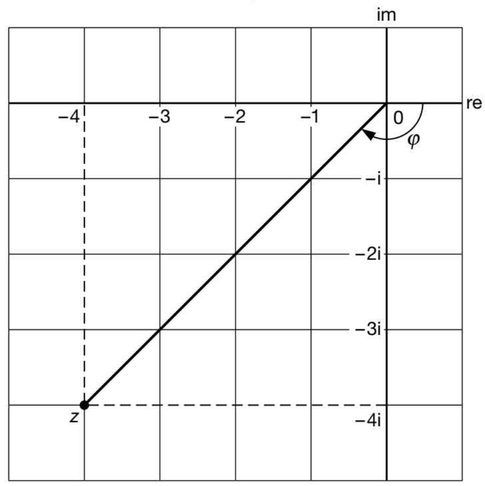


$\tan(\varphi) = \dfrac{2}{2\sqrt{3}} = \frac{1}{3}\sqrt{3}$ geeft $\varphi = \frac{1}{6}\pi$

Dus $\text{Arg}\left(2\sqrt{3} + 2\text{i}\right) = \frac{1}{6}\pi$.

De modulus is $|z| = \sqrt{\left(2\sqrt{3}\right)^2 + 2^2} = \sqrt{16} = 4$.

b

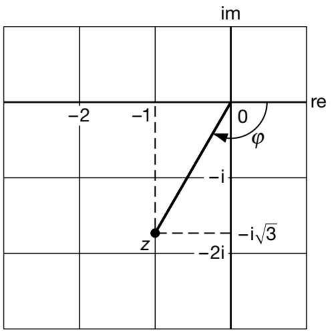


$\tan(\varphi) = \dfrac{-4}{-4} = 1$ geeft $\varphi = -\frac{3}{4}\pi$

Dus $\text{Arg}(-4 - 4\text{i}) = -\frac{3}{4}\pi$.

De modulus is $|z| = \sqrt{(-4)^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$.

c

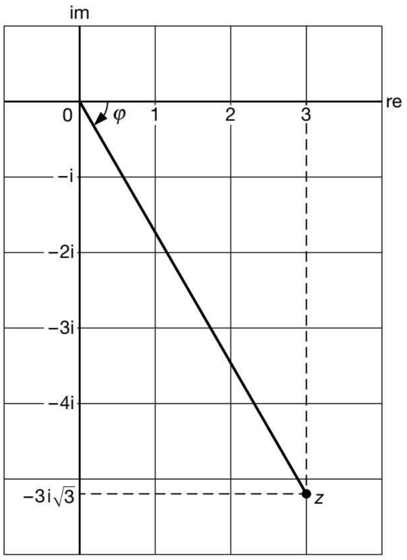


$\tan(\varphi) = \frac{-\sqrt{3}}{-1} = \sqrt{3}$ geeft $\varphi = -\frac{2}{3}\pi$

Dus $\operatorname{Arg}\left(-1 - i\sqrt{3}\right) = -\frac{2}{3}\pi$.

De modulus is $|z| = \sqrt{(-1)^2 + \left(-\sqrt{3}\right)^2} = \sqrt{4} = 2$.

d

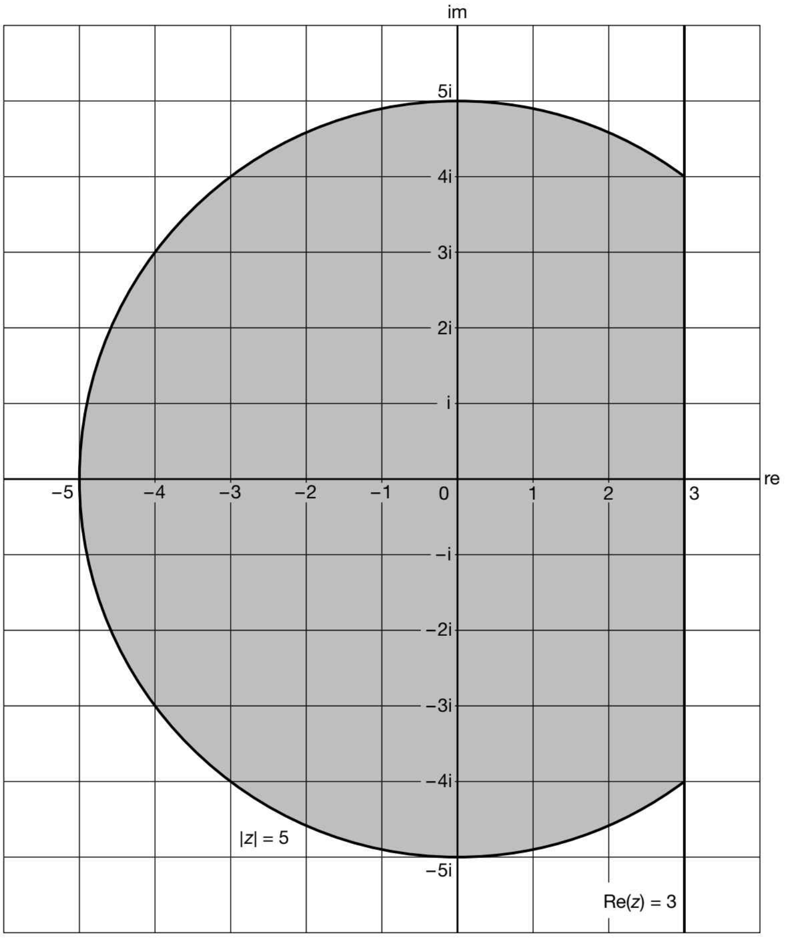


$\tan(\varphi) = \frac{-3\sqrt{3}}{3} = -\sqrt{3}$ geeft $\varphi = -\frac{1}{3}\pi$

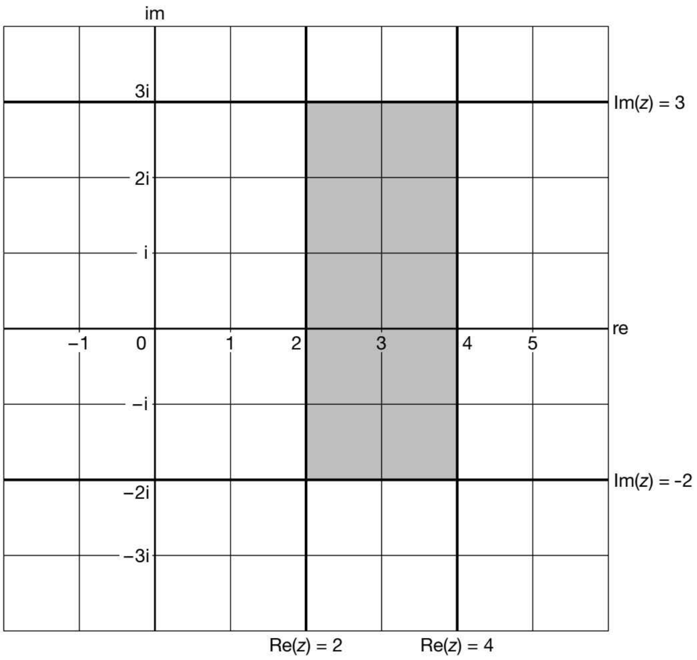
Dus $\operatorname{Arg}\left(3 - 3i\sqrt{3}\right) = -\frac{1}{3}\pi$.

De modulus is $|z| = \sqrt{3^2 + \left(-3\sqrt{3}\right)^2} = \sqrt{36} = 6$.

26 **a**

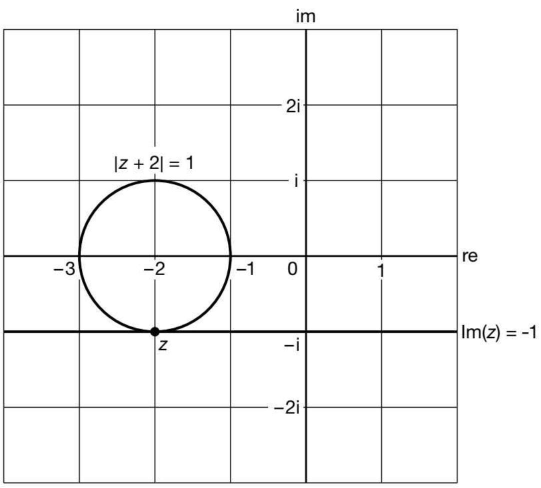


b

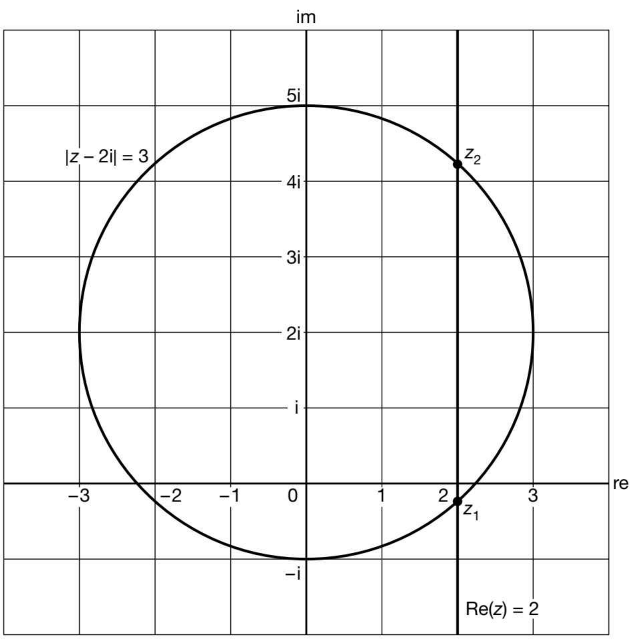


27 **28**

a $|z + 2| = 1$ is de cirkel met middelpunt −2 en straal 1.

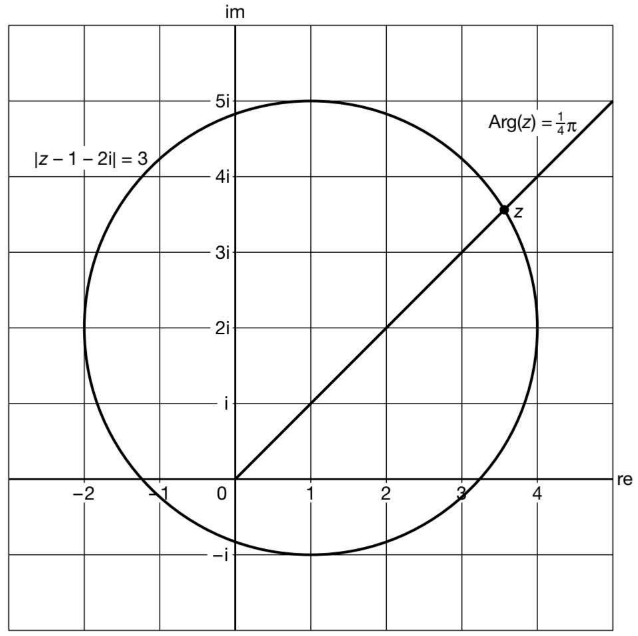


b $|z - 2i| = 3$ is de cirkel met middelpunt 2i en straal 3.

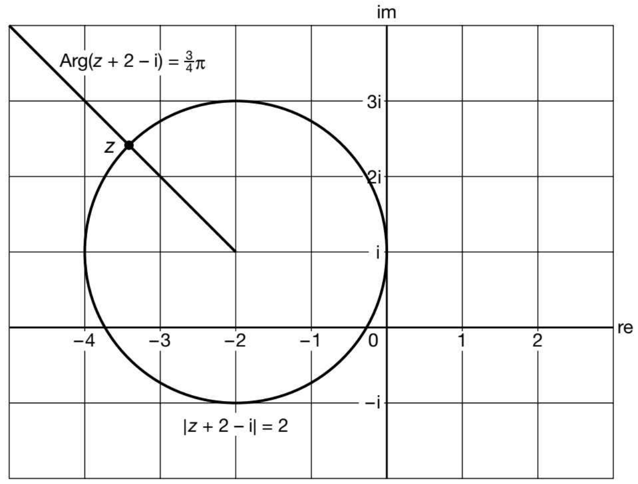


10

c $|z - 1 - 2i| = 3$ is de cirkel met middelpunt $1 + 2i$ en straal 3.

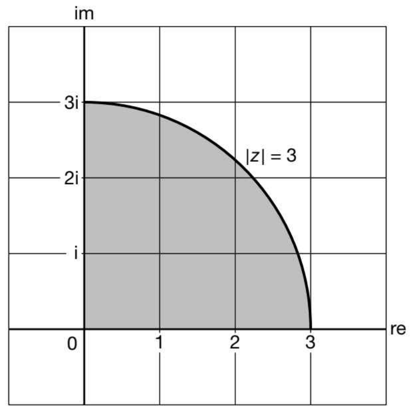


d $|z + 2 - i| = 2$ is de cirkel met middelpunt $-2 + i$ en straal 2.

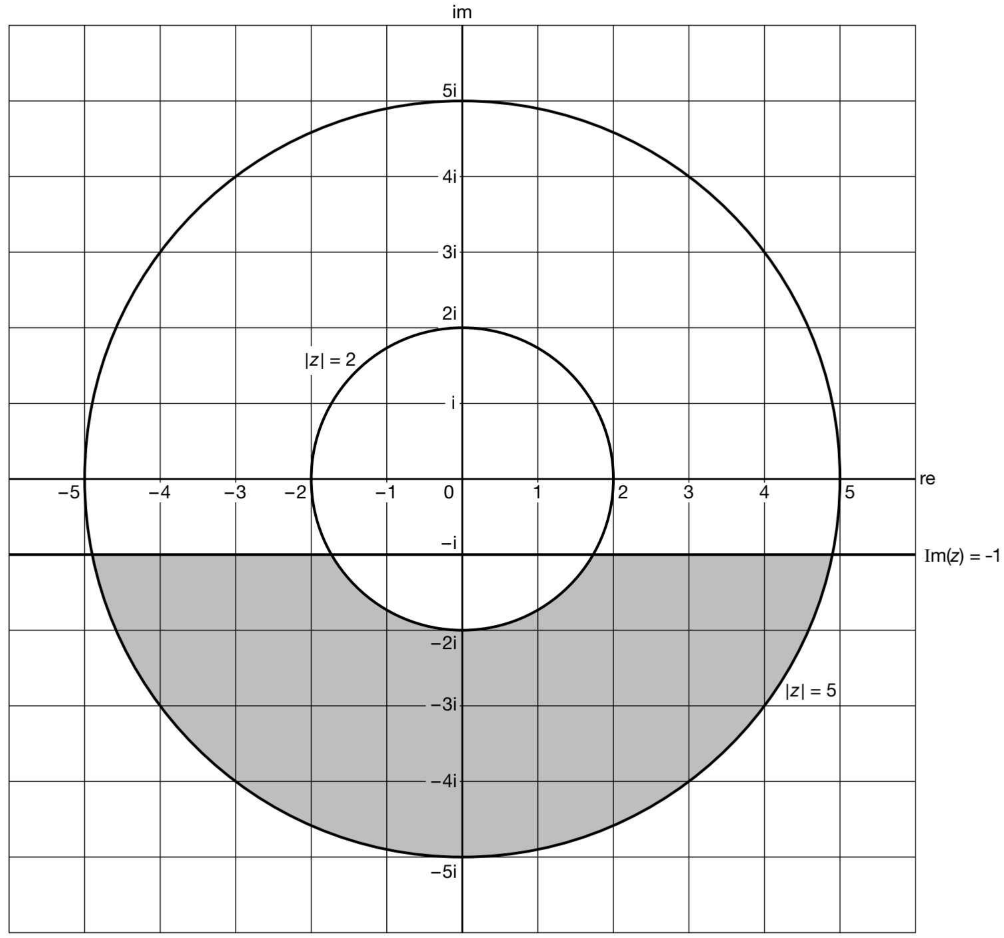


10

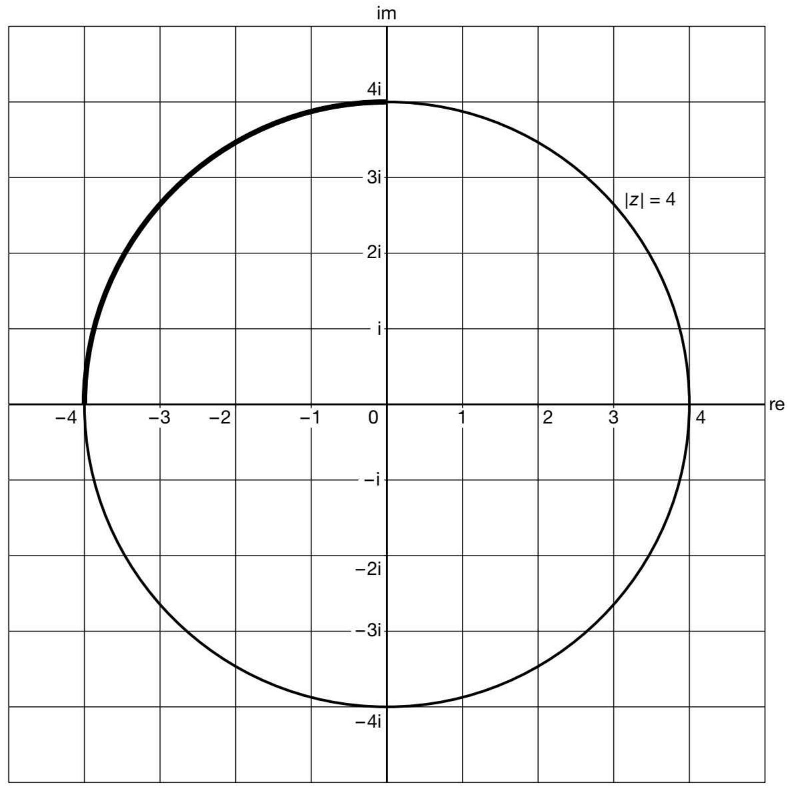
29 **a**



b

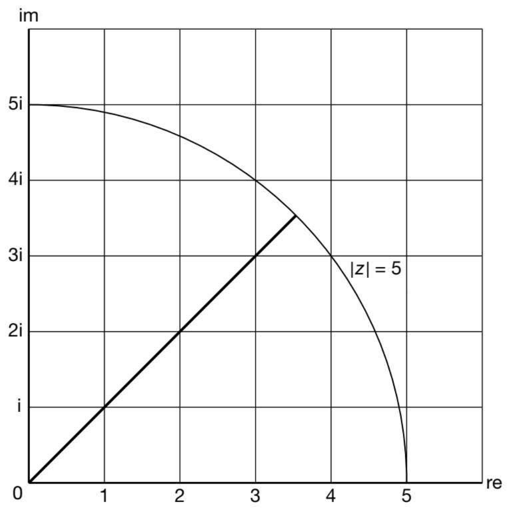

im
5i
4i
3i
2i
|z| = 2
i
re
−5
−4
−3
−2
−1
0
1
2
3
4
5
−i
Im(z) = -1
−2i
|z| = 5
−3i
−4i
−5i

c


im
4i
3i
|z| = 4
2i
i
re
−4
−3
−2
−1
0
1
2
3
4
−i
−2i
−3i
−4i

10

d



10.3 Poolcoördinaten en de formule van Euler

Bladzijde 56

30 $OP = \sqrt{2^2 + 3^2} = \sqrt{13}$

$\tan(\alpha) = \dfrac{3}{2}$ geeft $\alpha \approx 0{,}983$

Bladzijde 57

31 $x_A = r\cos(\theta) = 2\cos\left(\frac{1}{6}\pi\right) = 2 \cdot \frac{1}{2}\sqrt{3} = \sqrt{3}$

$y_A = r\sin(\theta) = 2\sin\left(\frac{1}{6}\pi\right) = 2 \cdot \frac{1}{2} = 1$

Dus $A\left(\sqrt{3}, 1\right)$.

$x_B = r\cos(\theta) = 2\cos\left(\frac{2}{3}\pi\right) = 2 \cdot -\frac{1}{2} = -1$

$y_B = r\sin(\theta) = 2\sin\left(\frac{2}{3}\pi\right) = 2 \cdot \frac{1}{2}\sqrt{3} = \sqrt{3}$

Dus $B\left(-1, \sqrt{3}\right)$.

$x_C = r\cos(\theta) = 4\cos\left(-\frac{3}{4}\pi\right) = 4 \cdot -\frac{1}{2}\sqrt{2} = -2\sqrt{2}$

$y_C = r\sin(\theta) = 4\sin(-\frac{3}{4}\pi) = 4 \cdot -\frac{1}{2}\sqrt{2} = -2\sqrt{2}$

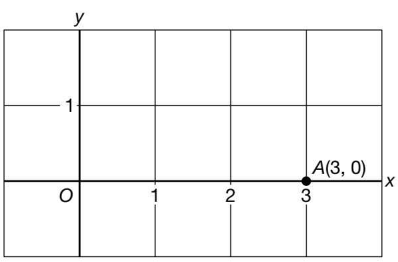
Dus $C\left(-2\sqrt{2}, -2\sqrt{2}\right)$.

$x_D = r\cos(\theta) = 4\cos\left(-\frac{5}{6}\pi\right) = 4 \cdot -\frac{1}{2}\sqrt{3} = -2\sqrt{3}$

$y_D = r\cos(\theta) = 4\sin\left(-\frac{5}{6}\pi\right) = 4 \cdot -\frac{1}{2} = -2$

Dus $D\left(-2\sqrt{3}, -2\right)$.

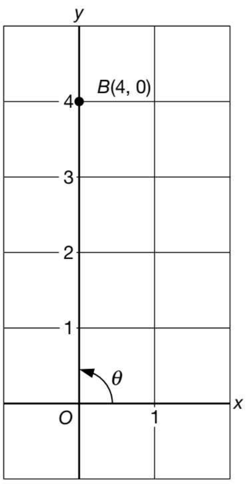
32



$r = 3$

$\tan(\theta) = \dfrac{0}{3} = 0$ geeft $\theta = 0$

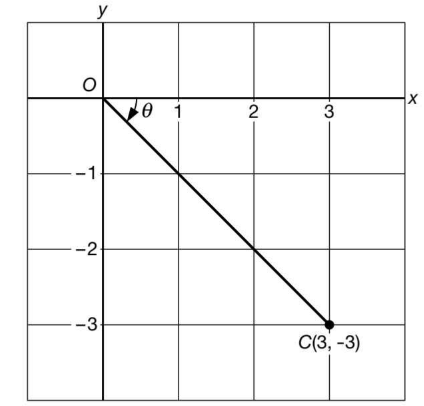
Dus $A(3, 0)$.



$r = 4$

$\theta = \frac{1}{2}\pi$

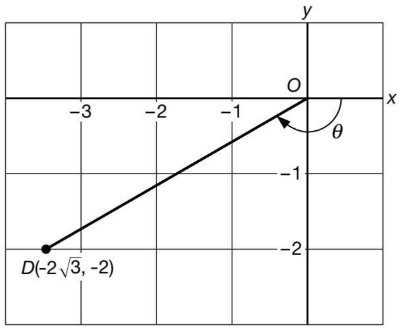
Dus $B\left(4, \frac{1}{2}\pi\right)$.



$r = \sqrt{3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$

$\tan(\theta) = \dfrac{-3}{3} = -1$ geeft $\theta = -\frac{1}{4}\pi$

Dus $C\left(3\sqrt{2}, -\frac{1}{4}\pi\right)$.

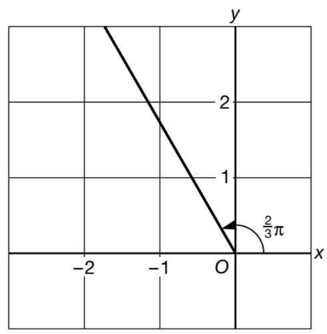


$r = \sqrt{(-2\sqrt{3})^2 + (-2)^2} = \sqrt{16} = 4$

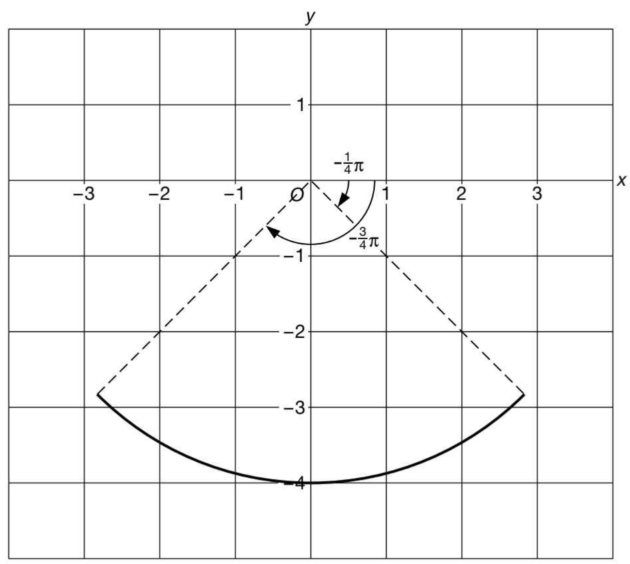
$\tan(\theta) = \dfrac{-2}{-2\sqrt{3}} = \frac{1}{3}\sqrt{3}$ geeft $\theta = -\frac{5}{6}\pi$

Dus $D\left(4, -\frac{5}{6}\pi\right)$.

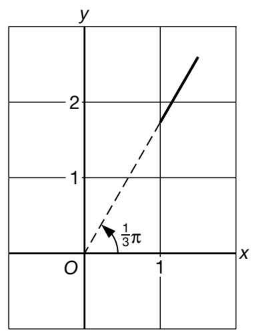
33 **a**



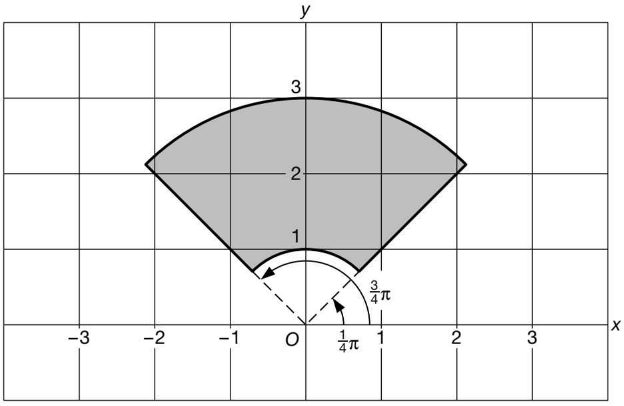
b



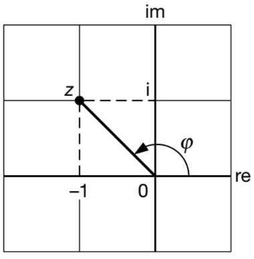
c



d



34 **a**



$\tan(\varphi) = \dfrac{1}{-1} = -1$ geeft $\varphi = \frac{3}{4}\pi$

Dus $\mathrm{Arg}(z) = \mathrm{Arg}(-1 + \mathrm{i}) = \frac{3}{4}\pi$.

De modulus is $|z| = |-1 + \mathrm{i}| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$.

b De poolcoördinaten (r, θ) van het punt P zijn
- de afstand r van P tot O
- de draaiingshoek θ van het lijnstuk OP met $-\pi < \theta \leq \pi$.

De afstand van z tot O is $\sqrt{2}$ en de draaiingshoek van z is $\frac{3}{4}\pi$.
Dus je hebt in vraag a de poolcoördinaten van z berekend.

Bladzijde 58

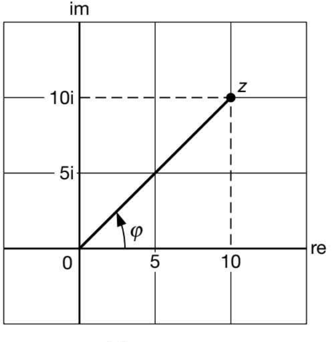
35 **a** $|z| = \sqrt{4^2 + (-3)^2} = 5$ en $\arg(z) \approx -0{,}64$
Dus $z = 5(\cos(-0{,}64) + \mathrm{i}\sin(-0{,}64))$.

b $|z| = \sqrt{12^2 + 5^2} = 13$ en $\arg(z) \approx 0{,}39$
Dus $z = 13(\cos(0{,}39) + \mathrm{i}\sin(0{,}39))$.

c $|z| = \sqrt{\left(-\sqrt{3}\right)^2 + (-2)^2} = \sqrt{7}$ en $\arg(z) \approx -2{,}28$
Dus $z = \sqrt{7}(\cos(-2{,}28) + \mathrm{i}\sin(-2{,}28))$.

d $|z| = \sqrt{2^2 + \left(-\sqrt{5}\right)^2} = 3$ en $\arg(z) \approx -0{,}84$
Dus $z = 3(\cos(-0{,}84) + \mathrm{i}\sin(-0{,}84))$.

36 **a**

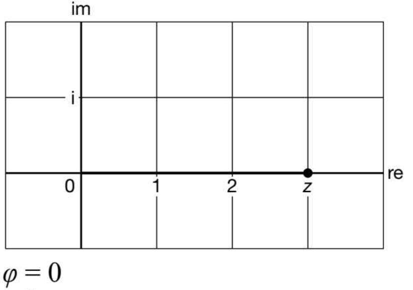


$\tan(\varphi) = \dfrac{10}{10} = 1$ geeft $\varphi = \frac{1}{4}\pi$

$|z| = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$

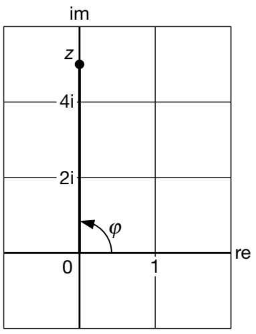
Dus $z = 10\sqrt{2}\left(\cos\left(\frac{1}{4}\pi\right) + \mathrm{i}\sin\left(\frac{1}{4}\pi\right)\right)$.

b



$\varphi = 0$
$|z| = 3$
Dus $z = 3(\cos(0) + \mathrm{i}\sin(0))$.

c

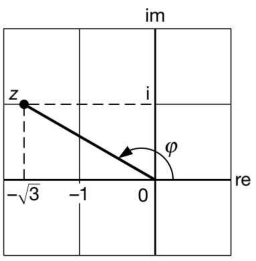


$\varphi = \frac{1}{2}\pi$

$|z| = 5$

Dus $z = 5\left(\cos\left(\frac{1}{2}\pi\right) + i\sin\left(\frac{1}{2}\pi\right)\right)$.

d

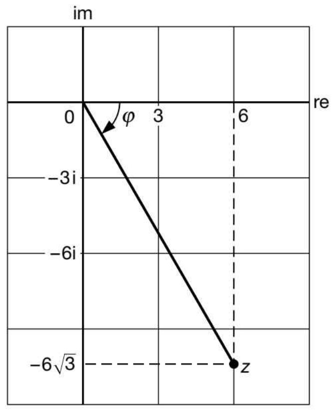


$\tan(\varphi) = \dfrac{1}{-\sqrt{3}} = -\frac{1}{3}\sqrt{3}$ geeft $\varphi = \frac{5}{6}\pi$

$|z| = \sqrt{(-\sqrt{3})^2 + 1^2} = 2$

Dus $z = 2\left(\cos\left(\frac{5}{6}\pi\right) + i\sin\left(\frac{5}{6}\pi\right)\right)$.

e

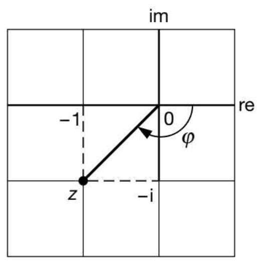


$\tan(\varphi) = \dfrac{-6\sqrt{3}}{6} = -\sqrt{3}$ geeft $\varphi = -\frac{1}{3}\pi$

$|z| = \sqrt{6^2 + (-6\sqrt{3})^2} = 12$

Dus $z = 12\left(\cos\left(-\frac{1}{3}\pi\right) + i\sin\left(-\frac{1}{3}\pi\right)\right)$.

f $z = \dfrac{1-i}{i} = \dfrac{1-i}{i} \cdot \dfrac{i}{i} = \dfrac{i - i^2}{i^2} = \dfrac{i+1}{-1} = -1 - i$



$\tan(\varphi) = \dfrac{-1}{-1} = 1$ geeft $\varphi = -\frac{3}{4}\pi$

$|z| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

Dus $z = \sqrt{2}\left(\cos\left(-\frac{3}{4}\pi\right) + i\sin\left(-\frac{3}{4}\pi\right)\right)$.

Bladzijde 59

37 **a** $\text{Re}(z) = 3\cos\left(\frac{1}{6}\pi\right) = 3 \cdot \frac{1}{2}\sqrt{3} = 1\frac{1}{2}\sqrt{3}$

$\text{Im}(z) = 3\sin\left(\frac{1}{6}\pi\right) = 3 \cdot \frac{1}{2} = 1\frac{1}{2}$

b $a = \text{Re}(z) = 1\frac{1}{2}\sqrt{3}$ en $b = \text{Im}(z) = 1\frac{1}{2}$, dus $z = 1\frac{1}{2}\sqrt{3} + 1\frac{1}{2}i$.

38 **a** $z = 4\left(\cos\left(\frac{1}{3}\pi\right) + i\sin\left(\frac{1}{3}\pi\right)\right) = 4\left(\frac{1}{2} + \frac{1}{2}i\sqrt{3}\right) = 2 + 2i\sqrt{3}$

b $z = 5\left(\cos\left(\frac{3}{4}\pi\right) + i\sin\left(\frac{3}{4}\pi\right)\right) = 5\left(-\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}\right) = -2\frac{1}{2}\sqrt{2} + 2\frac{1}{2}i\sqrt{2}$

c $z = 6\left(\cos\left(-\frac{5}{6}\pi\right) + i\sin\left(-\frac{5}{6}\pi\right)\right) = 6\left(-\frac{1}{2}\sqrt{3} - \frac{1}{2}i\right) = -3\sqrt{3} - 3i$

d $z = 8\left(\cos\left(-\frac{1}{6}\pi\right) + i\sin\left(-\frac{1}{6}\pi\right)\right) = 8\left(\frac{1}{2}\sqrt{3} - \frac{1}{2}i\right) = 4\sqrt{3} - 4i$

39 **a** $|z_1| = \sqrt{5^2 + 3^2} = \sqrt{34}$ en $\arg(z) \approx 0{,}54$

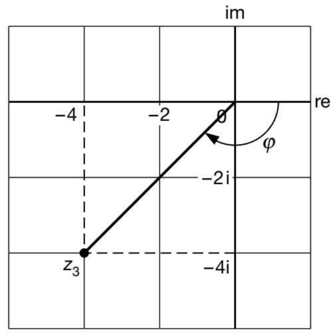
Dus $z_1 = \sqrt{34}(\cos(0{,}54) + i\sin(0{,}54))$.

$|z_2| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$ en $\arg(z) \approx 2{,}16$

Dus $z_2 = \sqrt{13}(\cos(2{,}16) + i\sin(2{,}16))$.

10

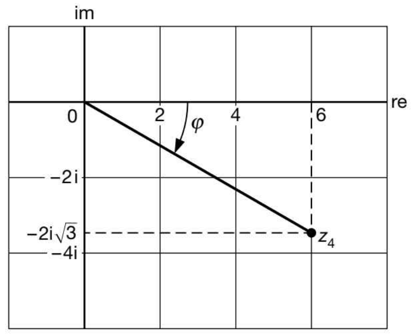
b



$\tan(\varphi) = \dfrac{-4}{-4} = 1$ geeft $\varphi = -\frac{3}{4}\pi$

$|z_3| = \sqrt{(-4)^2 + (-4)^2} = 4\sqrt{2}$

Dus $z_3 = 4\sqrt{2}\left(\cos\left(-\frac{3}{4}\pi\right) + \mathrm{i}\sin\left(-\frac{3}{4}\pi\right)\right)$.



$\tan(\varphi) = \dfrac{-2\sqrt{3}}{6} = -\frac{1}{3}\sqrt{3}$ geeft $\varphi = -\frac{1}{6}\pi$

$|z_4| = \sqrt{6^2 + \left(-2\sqrt{3}\right)^2} = 4\sqrt{3}$

Dus $z_4 = 4\sqrt{3}\left(\cos\left(-\frac{1}{6}\pi\right) + \mathrm{i}\sin\left(-\frac{1}{6}\pi\right)\right)$.

c $z_5 = 3\left(\cos\left(\frac{1}{4}\pi\right) + \mathrm{i}\sin\left(\frac{1}{4}\pi\right)\right) = 3\left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right) = 1\frac{1}{2}\sqrt{2} + 1\frac{1}{2}\mathrm{i}\sqrt{2}$

$z_6 = 4\left(\cos\left(-1\frac{1}{3}\pi\right) + \mathrm{i}\sin\left(-1\frac{1}{3}\pi\right)\right) = 4\left(-\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right) = -2 + 2\mathrm{i}\sqrt{3}$

d $z_7 = 2\left(\cos\left(-\frac{1}{3}\pi\right) + \mathrm{i}\sin\left(-\frac{1}{3}\pi\right)\right) = 2\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right) = 1 - \mathrm{i}\sqrt{3}$

40 **a** $\left.\begin{array}{l} f(0) = \mathrm{e}^0 = 1 \\ f(0) = a_0 + 0 + 0 + 0 + \ldots = a_0 \end{array}\right\} a_0 = 1$

b $\left.\begin{array}{l} f'(0) = \mathrm{e}^0 = 1 \\ f'(0) = a_1 + 0 + 0 + \ldots = a_1 \end{array}\right\} a_1 = 1$

c $f''(x) = \mathrm{e}^x$ en $f''(x) = 2a_2 + 3 \cdot 2 \cdot a_3x + 4 \cdot 3 \cdot a_4x^2 + 5 \cdot 4 \cdot a_5x^3 + \ldots$

$f''(0) = 1$ en $f''(0) = 2a_2$ geeft $2a_2 = 1$, dus $a_2 = \frac{1}{2}$.

$f^3(x) = \mathrm{e}^x$ en $f^3(x) = 3 \cdot 2 \cdot a_3 + 4 \cdot 3 \cdot 2 \cdot a_4x + 5 \cdot 4 \cdot 3 \cdot 2 \cdot a_5x^2 + \ldots$

$f^3(0) = 1$ en $f^3(0) = 6a_3$ geeft $6a_3 = 1$, dus $a_3 = \frac{1}{6}$.

$f^4(x) = \mathrm{e}^x$ en $f^4(x) = 4 \cdot 3 \cdot 2 \cdot a_4 + 5 \cdot 4 \cdot 3 \cdot 2 \cdot a_5x + \ldots$

$f^4(0) = 1$ en $f^4(0) = 24a_4$ geeft $24a_4 = 1$, dus $a_4 = \frac{1}{24}$.

$f^5(x) = \mathrm{e}^x$ en $f^5(x) = 5 \cdot 4 \cdot 3 \cdot 2 \cdot a_5 + 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot a_6x + \ldots$

$f^5(0) = 1$ en $f^5(0) = 120a_5$ geeft $120a_5 = 1$, dus $a_5 = \frac{1}{120}$.

d Er geldt steeds $n! \cdot a_n = 1$ ofwel $a_n = \dfrac{1}{n!}$.

Dus $a_6 = \dfrac{1}{6!}$.

41 **a** $\left.\begin{array}{l} f(0)=\sin(0)=0 \\ f(0)=a_0+0+0+0+\ldots=a_0 \end{array}\right\} a_0=0$

b $\left.\begin{array}{l} f'(0)=\cos(0)=1 \\ f'(0)=a_1+0+0+\ldots=a_1 \end{array}\right\} a_1=1$

c $f''(x)=-\sin(x)$ en $f''(x)=2a_2+3\cdot2\cdot a_3x+4\cdot3\cdot a_4x^2+5\cdot4\cdot a_5x^3+\ldots$
$f''(0)=0$ en $f''(0)=2a_2$ geeft $2a_2=0$, dus $a_2=0$.
$f^3(x)=-\cos(x)$ en $f^3(x)=3\cdot2\cdot a_3+4\cdot3\cdot2\cdot a_4x+5\cdot4\cdot3\cdot2\cdot a_5x^2+\ldots$
$f^3(0)=-1$ en $f^3(0)=6a_3$ geeft $6a_3=-1$, dus $a_3=-\frac{1}{6}$.
$f^4(x)=\sin(x)$ en $f^4(x)=4\cdot3\cdot2\cdot a_4+5\cdot4\cdot3\cdot2\cdot a_5x+\ldots$
$f^4(0)=0$ en $f^4(0)=24a_4$ geeft $24a_4=0$, dus $a_4=0$.
$f^5(0)=f'(0)=1$ geeft $a_5=\dfrac{1}{5!}=\frac{1}{120}$.
$f^6(0)=f''(0)=0$ geeft $a_6=\dfrac{0}{6!}=0$.
$f^7(0)=f^3(0)=-1$ geeft $a_7=\dfrac{-1}{7!}=-\frac{1}{5040}$.

d $f^{10}(0)=f''(0)=0$ geeft $a_{10}=\dfrac{0}{10!}=0$.
$f^{100}(0)=f(0)=0$ geeft $a_{100}=\dfrac{0}{100!}=0$.
$f^{15}(0)=f^3(0)=-1$ geeft $a_{15}=\dfrac{-1}{15!}$.
$f^{27}(0)=f^3(0)=-1$ geeft $a_{27}=\dfrac{-1}{27!}$.

Bladzijde 61

42 **a**
$$\begin{aligned}
e^{ix}&=1+ix+\frac{(ix)^2}{2!}+\frac{(ix)^3}{3!}+\frac{(ix)^4}{4!}+\frac{(ix)^5}{5!}+\frac{(ix)^6}{6!}+\frac{(ix)^7}{7!}+\ldots\\
&=1+ix+\frac{i^2x^2}{2!}+\frac{i^3x^3}{3!}+\frac{i^4x^4}{4!}+\frac{i^5x^5}{5!}+\frac{i^6x^6}{6!}+\frac{i^7x^7}{7!}+\ldots\\
&=1+ix+\frac{i^2x^2}{2!}+\frac{i^2\cdot x^3}{3!}i+\frac{i^2\cdot i^2\cdot x^4}{4!}+\frac{i^2\cdot i^2\cdot x^5}{5!}i+\frac{i^2\cdot i^2\cdot i^2\cdot x^6}{6!}+\frac{i^2\cdot i^2\cdot i^2\cdot x^7}{7!}i+\ldots\\
&=1+ix+\frac{-x^2}{2!}+\frac{-x^3}{3!}i+\frac{x^4}{4!}+\frac{x^5}{5!}i+\frac{-x^6}{6!}+\frac{-x^7}{7!}i+\ldots\\
&=1+ix-\frac{x^2}{2!}-\frac{x^3}{3!}i+\frac{x^4}{4!}+\frac{x^5}{5!}i-\frac{x^6}{6!}-\frac{x^7}{7!}i+\ldots
\end{aligned}$$

b $\cos(x)=1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\dfrac{x^6}{6!}+\ldots$ en $i\sin(x)=i\left(x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\dfrac{x^7}{7!}+\ldots\right)=ix-\dfrac{x^3}{3!}i+\dfrac{x^5}{5!}i-\dfrac{x^7}{7!}i+\ldots$ geeft
$\cos(x)+i\sin(x)=1+ix-\dfrac{x^2}{2!}-\dfrac{x^3}{3!}i+\dfrac{x^4}{4!}+\dfrac{x^5}{5!}i-\dfrac{x^6}{6!}-\dfrac{x^7}{7!}i+\ldots$
Dus $e^{ix}=\cos(x)+i\sin(x)$.

43 **a** $z=4e^{\frac{1}{4}\pi i}=4\left(\cos\left(\frac{1}{4}\pi\right)+i\sin\left(\frac{1}{4}\pi\right)\right)=4\left(\frac{1}{2}\sqrt{2}+\frac{1}{2}i\sqrt{2}\right)=2\sqrt{2}+2i\sqrt{2}$

b $z=\sqrt{3}e^{\frac{1}{6}\pi i}=\sqrt{3}\left(\cos\left(\frac{1}{6}\pi\right)+i\sin\left(\frac{1}{6}\pi\right)\right)=\sqrt{3}\left(\frac{1}{2}\sqrt{3}+\frac{1}{2}i\right)=1\frac{1}{2}+\frac{1}{2}i\sqrt{3}$

c $z=e^{\pi i}=\cos(\pi)+i\sin(\pi)=-1$

44 **a** $|z|=\sqrt{1^2+1^2}=\sqrt{2}$ en $\arg(z)=\frac{1}{4}\pi$ geeft $z=\sqrt{2}e^{\frac{1}{4}\pi i}$

b $|z|=5$ en $\arg(z)=\pi$ geeft $z=5e^{\pi i}$

c $|z|=\sqrt{(\sqrt{3})^2+1^2}=2$ en $\arg(z)=\frac{1}{6}\pi$ geeft $z=2e^{\frac{1}{6}\pi i}$

45 **a** $z=5e^{i}=5(\cos(1)+i\sin(1))=5\cos(1)+5i\sin(1)\approx2{,}70+4{,}21i$
b $z=-4e^{3i}=-4(\cos(3)+i\sin(3))=-4\cos(3)-4i\sin(3)\approx3{,}96-0{,}56i$
c $z=\pi e^{-i}=\pi(\cos(-1)+i\sin(-1))=\pi\cos(-1)+\pi i\sin(-1)\approx1{,}70-2{,}64i$

10

46 **a** $|z| = \sqrt{5^2 + 2^2} = \sqrt{29}$ en $\arg(z) \approx 0{,}38$ geeft $z \approx \sqrt{29}\,\mathrm{e}^{0{,}38\mathrm{i}}$

b $|z| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$ en $\arg(z) \approx 2{,}03$ geeft $z \approx \sqrt{5}\,\mathrm{e}^{2{,}03\mathrm{i}}$

c $|z| = \sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{3}\right)^2} = \sqrt{5}$ en $\arg(z) \approx 0{,}89$ geeft $z \approx \sqrt{5}\,\mathrm{e}^{0{,}89\mathrm{i}}$

Bladzijde 62

47 **a** $\mathrm{e}^{\pi\mathrm{i}} + 1 = \cos(\pi) + \mathrm{i}\sin(\pi) + 1 = -1 + \mathrm{i} \cdot 0 + 1 = 0$ klopt.

b $\mathrm{e}^{\frac{1}{2}\pi\mathrm{i}} = \cos\left(\frac{1}{2}\pi\right) + \mathrm{i}\sin\left(\frac{1}{2}\pi\right) = 0 + \mathrm{i} \cdot 1 = \mathrm{i}$, dus $\mathrm{e}^{\frac{1}{2}\pi\mathrm{i}} - \mathrm{i} = 0$.
$\mathrm{e}^{2\pi\mathrm{i}} = \cos(2\pi) + \mathrm{i}\sin(2\pi) = 1 + \mathrm{i} \cdot 0 = 1$, dus $\mathrm{e}^{2\pi\mathrm{i}} - 1 = 0$.

48 **a** $z = 5\,\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}} = 5\left(\cos\left(\frac{1}{3}\pi\right) + \mathrm{i}\sin\left(\frac{1}{3}\pi\right)\right) = 5\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right) = 2\frac{1}{2} + 2\frac{1}{2}\mathrm{i}\sqrt{3}$

b $|z| = \sqrt{\left(2\sqrt{3}\right)^2 + (-2)^2} = 4$ en $\arg(z) = -\frac{1}{6}\pi$ geeft $z = 4\,\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}}$

c $z = 6\,\mathrm{e}^{4\mathrm{i}} = 6(\cos(4) + \mathrm{i}\sin(4)) = 6\cos(4) + 6\mathrm{i}\sin(4) \approx -3{,}92 - 4{,}54\mathrm{i}$

d $|z| = \sqrt{(-3)^2 + 4^2} = 5$ en $\arg(z) \approx 2{,}21$ geeft $z \approx 5\,\mathrm{e}^{2{,}21\mathrm{i}}$

49 **a** $\mathrm{e}^{\mathrm{i}x} = \cos(x) + \mathrm{i}\sin(x)$ en $\mathrm{e}^{-\mathrm{i}x} = \cos(-x) + \mathrm{i}\sin(-x) = \cos(x) - \mathrm{i}\sin(x)$ geeft
$\mathrm{e}^{\mathrm{i}x} - \mathrm{e}^{-\mathrm{i}x} = \cos(x) + \mathrm{i}\sin(x) - (\cos(x) - \mathrm{i}\sin(x)) = \cos(x) + \mathrm{i}\sin(x) - \cos(x) + \mathrm{i}\sin(x) = 2\mathrm{i}\sin(x)$,
dus $\sin(x) = \dfrac{\mathrm{e}^{\mathrm{i}x} - \mathrm{e}^{-\mathrm{i}x}}{2\mathrm{i}}$.

b $\mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{-\mathrm{i}x} = \cos(x) + \mathrm{i}\sin(x) + \cos(x) - \mathrm{i}\sin(x) = 2\cos(x)$, dus $\cos(x) = \dfrac{\mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{-\mathrm{i}x}}{2}$.

50 **a** $\sin(x) = 2$ geeft

$\dfrac{\mathrm{e}^{\mathrm{i}x} - \mathrm{e}^{-\mathrm{i}x}}{2\mathrm{i}} = 2$

$\mathrm{e}^{\mathrm{i}x} - \mathrm{e}^{-\mathrm{i}x} = 4\mathrm{i}$

Stel $\mathrm{e}^{\mathrm{i}x} = u$.

$u - \dfrac{1}{u} = 4\mathrm{i}$

$u^2 - 1 = 4\mathrm{i}u$

$u^2 - 4\mathrm{i}u - 1 = 0$

$D = (-4\mathrm{i})^2 - 4 \cdot 1 \cdot -1 = 16\mathrm{i}^2 + 4 = -16 + 4 = -12 = 12\mathrm{i}^2$

$u = \dfrac{4\mathrm{i} + \sqrt{12\mathrm{i}^2}}{2} \vee u = \dfrac{4\mathrm{i} - \sqrt{12\mathrm{i}^2}}{2}$

$u = \dfrac{4\mathrm{i} + 2\mathrm{i}\sqrt{3}}{2} \vee u = \dfrac{4\mathrm{i} - 2\mathrm{i}\sqrt{3}}{2}$

$u = 2\mathrm{i} + \mathrm{i}\sqrt{3} \vee u = 2\mathrm{i} - \mathrm{i}\sqrt{3}$

$\mathrm{e}^{\mathrm{i}x} = 2\mathrm{i} + \mathrm{i}\sqrt{3} \vee \mathrm{e}^{\mathrm{i}x} = 2\mathrm{i} - \mathrm{i}\sqrt{3}$

$\mathrm{i}x = \ln\left(2\mathrm{i} + \mathrm{i}\sqrt{3}\right) \vee \mathrm{i}x = \ln\left(2\mathrm{i} - \mathrm{i}\sqrt{3}\right)$

$\mathrm{i}^2x = \mathrm{i}\ln\left(2\mathrm{i} + \mathrm{i}\sqrt{3}\right) \vee \mathrm{i}^2x = \mathrm{i}\ln\left(2\mathrm{i} - \mathrm{i}\sqrt{3}\right)$

$-x = \mathrm{i}\ln\left(2\mathrm{i} + \mathrm{i}\sqrt{3}\right) \vee -x = \mathrm{i}\ln\left(2\mathrm{i} - \mathrm{i}\sqrt{3}\right)$

$x = -\mathrm{i}\ln\left(2\mathrm{i} + \mathrm{i}\sqrt{3}\right) \vee x = -\mathrm{i}\ln\left(2\mathrm{i} - \mathrm{i}\sqrt{3}\right)$

10.4 Complexe e-machten en complexe wortels

Bladzijde 64

51 **a** $(1 + \mathrm{i})^2 = 1 + 2\mathrm{i} + \mathrm{i}^2 = 1 + 2\mathrm{i} - 1 = 2\mathrm{i}$
$(1 + \mathrm{i})^4 = (2\mathrm{i})^2 = 4\mathrm{i}^2 = -4$

b $|1 + \mathrm{i}| = \sqrt{1^2 + 1^2} = \sqrt{2}$ en $\arg(1 + \mathrm{i}) = \frac{1}{4}\pi$, dus $1 + \mathrm{i} = \sqrt{2}\,\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$.

c $\left(\sqrt{2}\,\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}\right)^4 = \left(\sqrt{2}\right)^4 \cdot \left(\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}\right)^4 = 4\,\mathrm{e}^{\pi\mathrm{i}} = 4(\cos(\pi) + \mathrm{i}\sin(\pi)) = 4\cos(\pi) + 4\mathrm{i}\sin(\pi) = 4 \cdot -1 + 4\mathrm{i} \cdot 0 = -4$

10

Bladzijde 65

52 a $1 - i = \sqrt{2}\,e^{-\frac{1}{4}\pi i}$, dus $z = (1 - i)^6 = \left(\sqrt{2}\,e^{-\frac{1}{4}\pi i}\right)^6 = 8\,e^{-1\frac{1}{2}\pi i}$

b $1 + i = \sqrt{2}\,e^{\frac{1}{4}\pi i}$ en $3 - 3i\sqrt{3} = 6\,e^{-\frac{1}{3}\pi i}$, dus $z = (1 + i)(3 - 3i\sqrt{3}) = \sqrt{2}\,e^{\frac{1}{4}\pi i} \cdot 6\,e^{-\frac{1}{3}\pi i} = 6\sqrt{2}\,e^{-\frac{1}{12}\pi i}$

c $4 + 4i\sqrt{3} = 8\,e^{\frac{1}{3}\pi i}$ en $\sqrt{3} - i = 2\,e^{-\frac{1}{6}\pi i}$, dus $\dfrac{4 + 4i\sqrt{3}}{\sqrt{3} - i} = \dfrac{8\,e^{\frac{1}{3}\pi i}}{2\,e^{-\frac{1}{6}\pi i}} = 4\,e^{\frac{1}{2}\pi i}$

d $5 + 5i = 5\sqrt{2}\,e^{\frac{1}{4}\pi i}$, dus $(5 + 5i)^{-3} = \left(5\sqrt{2}\,e^{\frac{1}{4}\pi i}\right)^{-3} = \dfrac{1}{\left(5\sqrt{2}\,e^{\frac{1}{4}\pi i}\right)^3} = \dfrac{1}{250\sqrt{2}\,e^{\frac{3}{4}\pi i}} = \dfrac{1}{250\sqrt{2}}\,e^{-\frac{3}{4}\pi i}$

e $2\sqrt{3} + 2i = 4\,e^{\frac{1}{6}\pi i}$ en $2 - 2i\sqrt{3} = 4\,e^{-\frac{1}{3}\pi i}$, dus $(2\sqrt{3} + 2i)(2 - 2i\sqrt{3})^{-2} = 4\,e^{\frac{1}{6}\pi i} \cdot (4\,e^{-\frac{1}{3}\pi i})^{-2} = 4\,e^{\frac{1}{6}\pi i} \cdot \frac{1}{16}\,e^{\frac{2}{3}\pi i} = \frac{1}{4}\,e^{\frac{5}{6}\pi i}$

f $3\sqrt{3} - 3i = 6\,e^{-\frac{1}{6}\pi i}$ en $2\sqrt{3} + 2i = 4\,e^{\frac{1}{6}\pi i}$, dus $\dfrac{(3\sqrt{3} - 3i)^2}{2\sqrt{3} + 2i} = \dfrac{\left(6\,e^{-\frac{1}{6}\pi i}\right)^2}{4\,e^{\frac{1}{6}\pi i}} = \dfrac{36\,e^{-\frac{1}{3}\pi i}}{4\,e^{\frac{1}{6}\pi i}} = 9\,e^{-\frac{1}{2}\pi i}$

53 a $|2 + 3i| = \sqrt{13}$, dus $|(2 + 3i)^5| = (\sqrt{13})^5 = 169\sqrt{13}$

Met de GR krijg je $z = (2 + 3i)^5 \approx 169\sqrt{13}\,e^{-1{,}37i}$.

b De GR geeft $z = (1 + 4i)(4 - i) \approx 17\,e^{1{,}08i}$.

c De GR geeft $z = \dfrac{2 - 4i}{2 + i} \approx 2\,e^{-1{,}57i}$.

d $|3 - i\sqrt{3}| = 2\sqrt{3}$, dus $|(3 - i\sqrt{3})^{-4}| = (2\sqrt{3})^{-4} = \dfrac{1}{(2\sqrt{3})^4} = \frac{1}{144}$

Met de GR krijg je $z = (3 - i\sqrt{3})^{-4} \approx \frac{1}{144}\,e^{2{,}09i}$.

e $|2 - i| = \sqrt{5}$ en $|3 + i| = \sqrt{10}$, dus

$|(2 - i)^3 \cdot (3 + i)^2| = |(2 - i)^3| \cdot |(3 + i)^2| = (\sqrt{5})^3 \cdot (\sqrt{10})^2 = 50\sqrt{5}$

Met de GR krijg je $z \approx (2 - i)^3 \cdot (3 + i)^2 = 50\sqrt{5}\,e^{-0{,}75i}$.

f De GR geeft $z = \dfrac{(1 + 5i)^4}{(1 - 5i)^2} \approx 135\frac{1}{5}\,e^{1{,}42i}$.

54 a $z_1 = \sqrt{3} + i = 2\,e^{\frac{1}{6}\pi i}$

$z_2 = -2 + 2i\sqrt{3} = 4\,e^{\frac{2}{3}\pi i}$

$z_1 \cdot z_2 = 2\,e^{\frac{1}{6}\pi i} \cdot 4\,e^{\frac{2}{3}\pi i} = 8\,e^{\frac{5}{6}\pi i}$

Dus $|z_1 \cdot z_2| = 8$ en $\arg(z_1 \cdot z_2) = \frac{5}{6}\pi$.

b $z_3 = 3\cos\left(-\frac{3}{4}\pi i\right) + i\sin\left(-\frac{3}{4}\pi i\right)$, dus $|z_3| = 3$ en $\arg(z_3) = -\frac{3}{4}\pi$, dus $z_3 = 3\,e^{-\frac{3}{4}\pi i}$.

$z_1 \cdot z_3 = 2\,e^{\frac{1}{6}\pi i} \cdot 3\,e^{-\frac{3}{4}\pi i} = 6\,e^{-\frac{7}{12}\pi i}$

Dus $|z_1 \cdot z_3| = 6$ en $\arg(z_1 \cdot z_3) = -\frac{7}{12}\pi$.

c $\dfrac{z_2}{z_3} = \dfrac{4\,e^{\frac{2}{3}\pi i}}{3\,e^{-\frac{3}{4}\pi i}} = 1\frac{1}{3}\,e^{1\frac{5}{12}\pi i}$

Dus $\left|\dfrac{z_2}{z_3}\right| = 1\frac{1}{3}$ en $\arg\left(\dfrac{z_2}{z_3}\right) = 1\frac{5}{12}\pi$.

55 a $\sqrt{2} + i\sqrt{2} = 2\,e^{\frac{1}{4}\pi i}$, dus $z = e^{\frac{1}{4}\pi i} + \sqrt{2} + i\sqrt{2} = e^{\frac{1}{4}\pi i} + 2\,e^{\frac{1}{4}\pi i} = 3\,e^{\frac{1}{4}\pi i}$

b $1 + i\sqrt{3} = 2\,e^{\frac{1}{3}\pi i}$, dus $z = 1 + i\sqrt{3} - e^{\frac{1}{3}\pi i} = 2\,e^{\frac{1}{3}\pi i} - e^{\frac{1}{3}\pi i} = e^{\frac{1}{3}\pi i}$

c $2\sqrt{3} - 2i = 4\,e^{-\frac{1}{6}\pi i}$, dus $z = (2\sqrt{3} - 2i)^2 - 10\,e^{1\frac{2}{3}\pi i} = \left(4\,e^{-\frac{1}{6}\pi i}\right)^2 - 10\,e^{1\frac{2}{3}\pi i} = 16\,e^{-\frac{1}{3}\pi i} - 10\,e^{-\frac{1}{3}\pi i} = 6\,e^{-\frac{1}{3}\pi i}$

d $1 + i = \sqrt{2}\,e^{\frac{1}{4}\pi i}$, dus $z = \dfrac{(1 + i)^6}{4\,e^{\pi i}} = \dfrac{\left(\sqrt{2}\,e^{\frac{1}{4}\pi i}\right)^6}{4\,e^{\pi i}} = \dfrac{8\,e^{1\frac{1}{2}\pi i}}{4\,e^{\pi i}} = 2\,e^{\frac{1}{2}\pi i}$

Bladzijde 66

56 a Er bestaat een lineair verband tussen u_n en de twee voorafgaande termen u_{n-1} en $_{n-2}$, dus je hebt met een lineaire differentievergelijking van de tweede orde te maken.

b Stel $u_n = g^n$.
Substitutie van $u_n = g^n$ in $u_n = 2u_{n-1} - 4u_{n-2}$ geeft $g^n = 2^{n-1} - 4g^{n-2}$ ofwel $g^n - 2g^{n-1} + 4g^{n-2} = 0$.
Delen door g^{n-2} geeft vervolgens de karakteristieke vergelijking $g^2 - 2g + 4 = 0$.

c $g^2 - 2g + 4 = 0$
$(g-1)^2 - 1 + 4 = 0$
$(g-1)^2 = -3$
$(g-1)^2 = 3i^2$
$g - 1 = i\sqrt{3} \vee g - 1 = -i\sqrt{3}$
$g = 1 + i\sqrt{3} \vee g = 1 - i\sqrt{3}$

d $\left|1 + i\sqrt{3}\right| = 2$ en $\arg\left(1 + i\sqrt{3}\right) = \frac{1}{3}\pi$
$\left|1 - i\sqrt{3}\right| = 2$ en $\arg\left(1 - i\sqrt{3}\right) = -\frac{1}{3}\pi$
Dus $g = 2\left(\cos\left(\frac{1}{3}\pi\right) + i\sin\left(\frac{1}{3}\pi\right)\right) \vee g = 2\left(\cos\left(-\frac{1}{3}\pi\right) + i\sin\left(-\frac{1}{3}\pi\right)\right)$.

Bladzijde 67

57 **a** Substitutie van $u_n = g^n$ in $u_n = 2\sqrt{3} \cdot u_{n-1} - 4u_{n-2}$ geeft
$g^2 - 2\sqrt{3} \cdot g + 4 = 0$
$\left(g - \sqrt{3}\right)^2 - 3 + 4 = 0$
$\left(g - \sqrt{3}\right)^2 = -1$
$\left(g - \sqrt{3}\right)^2 = i^2$
$g - \sqrt{3} = i \vee g - \sqrt{3} = -i$
$g = \sqrt{3} + i \vee g = \sqrt{3} - i$
$\left|\sqrt{3} + i\right| = 2$ en $\arg\left(\sqrt{3} + i\right) = \frac{1}{6}\pi$
$u_n = \left(A\cos\left(\frac{1}{6}\pi n\right) + B\sin\left(\frac{1}{6}\pi n\right)\right) \cdot 2^n$
$$\left.\begin{array}{l} u_0 = 4 \text{ geeft } A = 4 \\ u_1 = \sqrt{3} \text{ geeft } \left(A \cdot \frac{1}{2}\sqrt{3} + B \cdot \frac{1}{2}\right) \cdot 2 = \sqrt{3} \text{ ofwel } \sqrt{3} \cdot A + B = \sqrt{3} \end{array}\right\} A = 4 \text{ en } B = -3\sqrt{3}$$
Dus $u_n = \left(4\cos\left(\frac{1}{6}\pi n\right) - 3\sqrt{3}\sin\left(\frac{1}{6}\pi n\right)\right) \cdot 2^n$.

b Substitutie van $v_n = g^n$ in $v_n = -4v_{n-1} - 16v_{n-2}$ geeft
$g^2 + 4g + 16 = 0$
$(g+2)^2 - 4 + 16 = 0$
$(g+2)^2 = -12$
$(g+2)^2 = 12i^2$
$g + 2 = 2i\sqrt{3} \vee g + 2 = -2i\sqrt{3}$
$g = -2 + 2i\sqrt{3} \vee g = -2 - 2i\sqrt{3}$
$\left|-2 + 2i\sqrt{3}\right| = 4$ en $\arg\left(-2 + 2i\sqrt{3}\right) = \frac{2}{3}\pi$
$v_n = \left(A\cos\left(\frac{2}{3}\pi n\right) + B\sin\left(\frac{2}{3}\pi n\right)\right) \cdot 4^n$
$$\left.\begin{array}{l} v_0 = 1 \text{ geeft } A = 1 \\ v_1 = 2 \text{ geeft } \left(A \cdot -\frac{1}{2} + B \cdot \frac{1}{2}\sqrt{3}\right) \cdot 4 = 2 \text{ ofwel } -2A + 2\sqrt{3} \cdot B = 2 \end{array}\right\} A = 1 \text{ en } B = \tfrac{2}{3}\sqrt{3}$$
Dus $v_n = \left(\cos\left(\frac{2}{3}\pi n\right) + \frac{2}{3}\sqrt{3}\sin\left(\frac{2}{3}\pi n\right)\right) \cdot 4^n$.

Bladzijde 68

58 **a** Substitutie van $u_n = g^n$ in $u_n = -6u_{n-1} - 12u_{n-2}$ geeft
$g^2 + 6g + 12 = 0$
$(g+3)^2 - 9 + 12 = 0$
$(g+3)^2 = -3$
$(g+3)^2 = 3i^2$
$g + 3 = i\sqrt{3} \vee g + 3 = -i\sqrt{3}$
$g = -3 + i\sqrt{3} \vee g = -3 - i\sqrt{3}$
$\left|-3 + i\sqrt{3}\right| = 2\sqrt{3}$ en $\arg\left(-3 + i\sqrt{3}\right) = \frac{5}{6}\pi$
$u_n = \left(A\cos\left(\frac{5}{6}\pi n\right) + B\sin\left(\frac{5}{6}\pi n\right)\right) \cdot \left(2\sqrt{3}\right)^n$
$$\left.\begin{array}{l} u_0 = 1 \text{ geeft } A = 1 \\ u_1 = 1 \text{ geeft } \left(A \cdot -\frac{1}{2}\sqrt{3} + B \cdot \frac{1}{2}\right) \cdot 2\sqrt{3} = 1 \text{ ofwel } -3A + \sqrt{3} \cdot B = 1 \end{array}\right\} A = 1 \text{ en } B = 1\tfrac{1}{3}\sqrt{3}$$
Dus $u_n = \left(\cos\left(\frac{5}{6}\pi n\right) + 1\frac{1}{3}\sqrt{3}\sin\left(\frac{5}{6}\pi n\right)\right) \cdot \left(2\sqrt{3}\right)^n$.

b Substitutie van $x_n = g^n$ in $x_n = -\frac{1}{4}x_{n-2}$ geeft
$g^2 = -\frac{1}{4}$
$g^2 = \frac{1}{4}\mathrm{i}^2$
$g = \frac{1}{2}\mathrm{i} \vee g = -\frac{1}{2}\mathrm{i}$
$\left|\frac{1}{2}\mathrm{i}\right| = \frac{1}{2}$ en $\arg\left(\frac{1}{2}\mathrm{i}\right) = \frac{1}{2}\pi$
$x_n = \left(A\cos\left(\frac{1}{2}\pi n\right) + B\sin\left(\frac{1}{2}\pi n\right)\right) \cdot \left(\frac{1}{2}\right)^n$
$x_0 = 16$ geeft $A = 16$
$x_1 = 12$ geeft $(A \cdot 0 + B \cdot 1) \cdot \frac{1}{2} = 12$ ofwel $B = 24$
Dus $x_n = \left(16\cos\left(\frac{1}{2}\pi n\right) + 24\sin\left(\frac{1}{2}\pi n\right)\right) \cdot \left(\frac{1}{2}\right)^n$.

59 **a** $\left.\begin{array}{l} x_n = -2x_{n-1} + 4y_{n-1} \text{ geeft } x_{n+1} = -2x_n + 4y_n \\ y_n = -x_{n-1} - 2y_{n-1} \end{array}\right\}$
$x_{n+1} = -2x_n + 4(-x_{n-1} - 2y_{n-1})$
$\left.\begin{array}{l} x_{n+1} = -2x_n - 4x_{n-1} - 2 \cdot 4y_{n-1} \\ x_n = -2x_{n-1} + 4y_{n-1} \text{ geeft } 4y_{n-1} = x_n + 2x_{n-1} \end{array}\right\}$
$x_{n+1} = -2x_n - 4x_{n-1} - 2 \cdot (x_n + 2x_{n-1})$
$x_{n+1} = -2x_n - 4x_{n-1} - 2x_n - 4x_{n-1}$
$x_{n+1} = -4x_n - 8x_{n-1}$
$x_n = -4x_{n-1} - 8x_{n-2}$

b Substitutie van $x_n = g^n$ in $x_n = -4x_{n-1} - 8x_{n-2}$ geeft
$g^2 + 4g + 8 = 0$
$(g + 2)^2 - 4 + 8 = 0$
$(g + 2)^2 = -4$
$(g + 2)^2 = 4\mathrm{i}^2$
$g + 2 = 2\mathrm{i} \vee g + 2 = -2\mathrm{i}$
$g = -2 + 2\mathrm{i} \vee g = -2 - 2\mathrm{i}$
$|-2 + 2\mathrm{i}| = 2\sqrt{2}$ en $\arg(-2 + 2\mathrm{i}) = \frac{3}{4}\pi$
$x_n = \left(A\cos\left(\frac{3}{4}\pi n\right) + B\sin\left(\frac{3}{4}\pi n\right)\right) \cdot \left(2\sqrt{2}\right)^n$
$x_0 = 2$ en $y_0 = 3$ geeft $x_1 = -2 \cdot 2 + 4 \cdot 3 = 8$
$\left.\begin{array}{l} x_0 = 2 \text{ geeft } A = 2 \\ x_1 = 8 \text{ geeft } \left(A \cdot -\frac{1}{2}\sqrt{2} + B \cdot \frac{1}{2}\sqrt{2}\right) \cdot 2\sqrt{2} = 8 \text{ ofwel } -2A + 2B = 8 \end{array}\right\} A = 2 \text{ en } B = 6$
Dus $x_n = \left(2\cos\left(\frac{3}{4}\pi n\right) + 6\sin\left(\frac{3}{4}\pi n\right)\right) \cdot \left(2\sqrt{2}\right)^n$.

$\left.\begin{array}{l} y_n = -x_{n-1} - 2y_{n-1} \text{ geeft } y_{n+1} = -x_n - 2y_n \\ x_n = -2x_{n-1} + 4y_{n-1} \end{array}\right\}$
$y_{n+1} = -x_n - 2y_n = -(-2x_{n-1} + 4y_{n-1}) - 2y_n = 2x_{n-1} - 4y_{n-1} - 2y_n$
$\left.\begin{array}{l} y_{n+1} = 2x_{n-1} - 4y_{n-1} - 2y_n \\ y_n = -x_{n-1} - 2y_{n-1} \text{ geeft } x_{n-1} = -y_n - 2y_{n-1} \end{array}\right\}$
$y_{n+1} = 2(-y_n - 2y_{n-1}) - 4y_{n-1} - 2y_n$
$y_{n+1} = -2y_n - 4y_{n-1} - 4y_{n-1} - 2y_n$
$y_{n+1} = -4y_n - 8y_{n-1}$
$y_n = -4y_{n-1} - 8y_{n-2}$
Dus $y_n = \left(A\cos\left(\frac{3}{4}\pi n\right) + B\sin\left(\frac{3}{4}\pi n\right)\right) \cdot \left(2\sqrt{2}\right)^n$.
$x_0 = 2$ en $y_0 = 3$ geeft $y_1 = -2 - 2 \cdot 3 = -8$
$\left.\begin{array}{l} y_0 = 3 \text{ geeft } A = 3 \\ y_1 = -8 \text{ geeft } \left(A \cdot -\frac{1}{2}\sqrt{2} + B \cdot \frac{1}{2}\sqrt{2}\right) \cdot 2\sqrt{2} = -8 \text{ ofwel } -2A + 2B = -8 \end{array}\right\} A = 3 \text{ en } B = -1$
Dus $y_n = \left(3\cos\left(\frac{3}{4}\pi n\right) - \sin\left(\frac{3}{4}\pi n\right)\right) \cdot \left(2\sqrt{2}\right)^n$.

60 **a** $\left|\sqrt{3} + \mathrm{i}\right| = 2$ en $\arg\left(\sqrt{3} + \mathrm{i}\right) = \frac{1}{6}\pi$, dus
$\left(\sqrt{3} + \mathrm{i}\right)^3 = \left(2\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}\right)^3 = 8\mathrm{e}^{\frac{1}{2}\pi\mathrm{i}} = 8\left(\cos\left(\frac{1}{2}\pi\right) + \mathrm{i}\sin\left(\frac{1}{2}\pi\right)\right) = 8\cos\left(\frac{1}{2}\pi\right) + 8\mathrm{i}\sin\left(\frac{1}{2}\pi\right) = 8\mathrm{i}$
$\left|-\sqrt{3} + \mathrm{i}\right| = 2$ en $\arg\left(-\sqrt{3} + \mathrm{i}\right) = \frac{5}{6}\pi$, dus
$\left(-\sqrt{3} + \mathrm{i}\right)^3 = \left(2\mathrm{e}^{\frac{5}{6}\pi\mathrm{i}}\right)^3 = 8\mathrm{e}^{2\frac{1}{2}\pi i} = 8\left(\cos\left(2\frac{1}{2}\pi\right) + \mathrm{i}\sin\left(2\frac{1}{2}\pi\right)\right) = 8\cos\left(2\frac{1}{2}\pi\right) + 8\mathrm{i}\sin\left(2\frac{1}{2}\pi\right) = 8\mathrm{i}$
$(-2\mathrm{i})^3 = -8\mathrm{i}^3 = -8\mathrm{i}^2 \cdot \mathrm{i} = 8\mathrm{i}$

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b $z^3 = 8\text{i}$ geeft $z = \sqrt{3} + \text{i} \vee z = -\sqrt{3} + \text{i} \vee z = -2\text{i}$

c $2\,\text{e}^{\frac{1}{6}\pi\text{i}} = 2\left(\cos\left(\tfrac{1}{6}\pi\right) + \text{i}\sin\left(\tfrac{1}{6}\pi\right)\right) = 2\cos\left(\tfrac{1}{6}\pi\right) + 2\text{i}\sin\left(\tfrac{1}{6}\pi\right) = 2\cdot\tfrac{1}{2}\sqrt{3} + 2\text{i}\cdot\tfrac{1}{2} = \sqrt{3} + \text{i}$

$2\,\text{e}^{2\frac{1}{6}\pi\text{i}} = 2\left(\cos\left(2\tfrac{1}{6}\pi\right) + \text{i}\sin\left(2\tfrac{1}{6}\pi\right)\right) = 2\cos\left(2\tfrac{1}{6}\pi\right) + 2\text{i}\sin\left(2\tfrac{1}{6}\pi\right) = 2\cdot\tfrac{1}{2}\sqrt{3} + 2\text{i}\cdot\tfrac{1}{2} = \sqrt{3} + \text{i}$

d $2\,\text{e}^{\frac{5}{6}\pi\text{i}} = 2\left(\cos\left(\tfrac{5}{6}\pi\right) + \text{i}\sin\left(\tfrac{5}{6}\pi\right)\right) = 2\cos\left(\tfrac{5}{6}\pi\right) + 2\text{i}\sin\left(\tfrac{5}{6}\pi\right) = 2\cdot-\tfrac{1}{2}\sqrt{3} + 2\text{i}\cdot\tfrac{1}{2} = -\sqrt{3} + \text{i}$

$2\,\text{e}^{2\frac{5}{6}\pi\text{i}} = 2\left(\cos\left(2\tfrac{5}{6}\pi\right) + \text{i}\sin\left(2\tfrac{5}{6}\pi\right)\right) = 2\cos\left(2\tfrac{5}{6}\pi\right) + 2\text{i}\sin\left(2\tfrac{5}{6}\pi\right) = 2\cdot-\tfrac{1}{2}\sqrt{3} + 2\text{i}\cdot\tfrac{1}{2} = -\sqrt{3} + \text{i}$

Dus $2\,\text{e}^{\frac{5}{6}\pi\text{i}} = 2\,\text{e}^{2\frac{5}{6}\pi\text{i}}$.

e $|-2\text{i}| = 2$ en $\arg(-2\text{i}) = -\tfrac{1}{2}\pi$

$-2\text{i} = 2\,\text{e}^{-\frac{1}{2}\pi\text{i}} = 2\,\text{e}^{1\frac{1}{2}\pi\text{i}} = 2\,\text{e}^{3\frac{1}{2}\pi\text{i}}$

Bladzijde 70

61 **a** $(z-1)^2 = \tfrac{1}{2} + \tfrac{1}{2}\text{i}\sqrt{3}$

$(z-1)^2 = \text{e}^{\frac{1}{3}\pi\text{i} + k\cdot 2\pi\text{i}}$

$z - 1 = \text{e}^{\frac{1}{6}\pi\text{i} + k\cdot\pi\text{i}}$

$z - 1 = \text{e}^{-\frac{5}{6}\pi\text{i}} \vee z - 1 = \text{e}^{\frac{1}{6}\pi\text{i}}$

$z - 1 = -\tfrac{1}{2}\sqrt{3} - \tfrac{1}{2}\text{i} \vee z - 1 = \tfrac{1}{2}\sqrt{3} + \tfrac{1}{2}\text{i}$

$z = 1 - \tfrac{1}{2}\sqrt{3} - \tfrac{1}{2}\text{i} \vee z = 1 + \tfrac{1}{2}\sqrt{3} + \tfrac{1}{2}\text{i}$

b $(z - 1 - \text{i})^2 = -\text{i}$

$(z - 1 - \text{i})^2 = \text{e}^{-\frac{1}{2}\pi\text{i} + k\cdot 2\pi\text{i}}$

$z - 1 - \text{i} = \text{e}^{-\frac{1}{4}\pi\text{i} + k\cdot\pi\text{i}}$

$z - 1 - \text{i} = \tfrac{1}{2}\sqrt{2} - \tfrac{1}{2}\text{i}\sqrt{2} \vee z - 1 - \text{i} = -\tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}\text{i}\sqrt{2}$

$z = 1 + \tfrac{1}{2}\sqrt{2} + \text{i} - \tfrac{1}{2}\text{i}\sqrt{2} \vee z = 1 - \tfrac{1}{2}\sqrt{2} + \text{i} + \tfrac{1}{2}\text{i}\sqrt{2}$

$z = 1 + \tfrac{1}{2}\sqrt{2} + \left(1 - \tfrac{1}{2}\sqrt{2}\right)\text{i} \vee z = 1 - \tfrac{1}{2}\sqrt{2} + \left(1 + \tfrac{1}{2}\sqrt{2}\right)\text{i}$

c $\arg(z^3) = \arg(27) = 0$, dus één van de oplossingen heeft als argument 0.

$|z^3| = |27| = 27$ geeft $|z| = 3$, dus de straal van de cirkel is 3.

De argumenten verschillen steeds $\dfrac{2\pi}{3} = \tfrac{2}{3}\pi$ van elkaar.

De oplossing is $z = 3 \vee z = -1\tfrac{1}{2} + 1\tfrac{1}{2}\text{i}\sqrt{3} \vee z = -1\tfrac{1}{2} - 1\tfrac{1}{2}\text{i}\sqrt{3}$.

d $\arg(z^4) = \arg(-81) = \pi$, dus één van de oplossingen heeft als argument $\tfrac{1}{4}\pi$.

$|z^4| = |-81| = 81$ geeft $|z| = 3$, dus de straal van de cirkel is 3.

De argumenten verschillen steeds $\dfrac{2\pi}{4} = \tfrac{1}{2}\pi$ van elkaar.

De oplossing is $z = 1\tfrac{1}{2}\sqrt{2} + 1\tfrac{1}{2}\text{i}\sqrt{2} \vee z = -1\tfrac{1}{2}\sqrt{2} + 1\tfrac{1}{2}\text{i}\sqrt{2} \vee z = -1\tfrac{1}{2}\sqrt{2} - 1\tfrac{1}{2}\text{i}\sqrt{2} \vee z = 1\tfrac{1}{2}\sqrt{2} - 1\tfrac{1}{2}\text{i}\sqrt{2}$.

62 **a** $\sqrt{2 + \text{i}}$, dus los op $z^2 = 2 + \text{i}$

$z^2 \approx \sqrt{5}\,\text{e}^{0{,}46\text{i} + k\cdot 2\pi\text{i}}$

$z \approx 1{,}50\,\text{e}^{0{,}23\text{i} + k\cdot\pi\text{i}}$

$z \approx 1{,}50\,\text{e}^{-2{,}91\text{i}} \vee z \approx 1{,}50\,\text{e}^{0{,}23\text{i}}$

Dus $\sqrt{2 + \text{i}} \approx 1{,}50\,\text{e}^{-2{,}91\text{i}} \vee \sqrt{2 + \text{i}} \approx 1{,}50\,\text{e}^{0{,}23\text{i}}$.

b $\sqrt{-4 + 3\text{i}}$, dus los op $z^2 = -4 + 3\text{i}$

$z^2 \approx 5\,\text{e}^{2{,}50\text{i} + k\cdot 2\pi\text{i}}$

$z \approx 2{,}24\,\text{e}^{1{,}25\text{i} + k\cdot\pi\text{i}}$

$z \approx 2{,}24\,\text{e}^{-1{,}89\text{i}} \vee z \approx 2{,}24\,\text{e}^{1{,}25\text{i}}$

Dus $\sqrt{-4 + 3\text{i}} \approx 2{,}24\,\text{e}^{-1{,}89\text{i}} \vee \sqrt{-4 + 3\text{i}} \approx 2{,}24\,\text{e}^{1{,}25\text{i}}$.

c $\sqrt[3]{-6 + 3\text{i}}$, dus los op $z^3 = -6 + 3\text{i}$

$z^3 \approx \sqrt{45}\,\text{e}^{2{,}68\text{i} + k\cdot 2\pi\text{i}}$

$z \approx 1{,}89\,\text{e}^{0{,}89\text{i} + k\cdot\frac{2}{3}\pi\text{i}}$

$z \approx 1{,}89\,\text{e}^{-1{,}20\text{i}} \vee z \approx 1{,}89\,\text{e}^{0{,}89\text{i}} \vee z \approx 1{,}89\,\text{e}^{2{,}99\text{i}}$

Dus $\sqrt[3]{-6 + 3\text{i}} \approx 1{,}89\,\text{e}^{-1{,}20\text{i}} \vee \sqrt[3]{-6 + 3\text{i}} \approx 1{,}89\,\text{e}^{0{,}89\text{i}} \vee \sqrt[3]{-6 + 3\text{i}} \approx 1{,}89\,\text{e}^{2{,}99\text{i}}$.

d $\sqrt[4]{10}$, dus los op $z^4 = 10$

$z^4 = 10\,\text{e}^{k\cdot 2\pi\text{i}}$

$z \approx 1{,}78\,\text{e}^{k\cdot\frac{1}{2}\pi\text{i}}$

$z \approx 1{,}78\,\text{e}^{-1{,}57\text{i}} \vee z \approx 1{,}78 \vee z \approx 1{,}78\,\text{e}^{1{,}57\text{i}} \vee z \approx 1{,}78\,\text{e}^{3{,}14\text{i}}$

Dus $\sqrt[4]{10} \approx 1{,}78\,\text{e}^{-1{,}57\text{i}} \vee \sqrt[4]{10} \approx 1{,}78 \vee \sqrt[4]{10} \approx 1{,}78\,\text{e}^{1{,}57\text{i}} \vee \sqrt[4]{10} \approx 1{,}78\,\text{e}^{3{,}14\text{i}}$.

63 **a** $\sqrt{-5\mathrm{i}}$, dus los op $z^2 = -5\mathrm{i}$.
$\arg(z^2) = \arg(-5\mathrm{i}) = -\frac{1}{2}\pi$, dus één van de oplossingen heeft als argument $-\frac{1}{4}\pi$.
$|z^2| = |-5\mathrm{i}| = 5$ geeft $|z| = \sqrt{5}$, dus de straal van de cirkel is $\sqrt{5}$.
De argumenten verschillen steeds $\frac{2\pi}{2} = \pi$.
De oplossing is $z = \sqrt{5}\left(\frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}\right) = \frac{1}{2}\sqrt{10} - \frac{1}{2}\mathrm{i}\sqrt{10} \vee z = \sqrt{5}\left(-\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right) = -\frac{1}{2}\sqrt{10} + \frac{1}{2}\mathrm{i}\sqrt{10}$.
Dus $\sqrt{-5\mathrm{i}} = \frac{1}{2}\sqrt{10} - \frac{1}{2}\mathrm{i}\sqrt{10} \vee \sqrt{-5\mathrm{i}} = -\frac{1}{2}\sqrt{10} + \frac{1}{2}\mathrm{i}\sqrt{10}$.

b $\frac{1}{1+\mathrm{i}\sqrt{3}} = \left(1+\mathrm{i}\sqrt{3}\right)^{-1} = \left(2\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}\right)^{-1} = \frac{1}{2}\mathrm{e}^{-\frac{1}{3}\pi\mathrm{i}}$

$\sqrt{\frac{1}{1+\mathrm{i}\sqrt{3}}}$, dus los op $z^2 = \frac{1}{1+\mathrm{i}\sqrt{3}}$

$$z^2 = \tfrac{1}{2}\mathrm{e}^{-\frac{1}{3}\pi\mathrm{i} + k\cdot 2\pi\mathrm{i}}$$
$$z = \tfrac{1}{2}\sqrt{2}\,\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i} + k\cdot\pi\mathrm{i}}$$
$$z = \tfrac{1}{2}\sqrt{2}\,\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}} \vee z = \tfrac{1}{2}\sqrt{2}\,\mathrm{e}^{\frac{5}{6}\pi\mathrm{i}}$$
$$z = \tfrac{1}{2}\sqrt{2}\left(\tfrac{1}{2}\sqrt{3} - \tfrac{1}{2}\mathrm{i}\right) \vee z = \tfrac{1}{2}\sqrt{2}\left(-\tfrac{1}{2}\sqrt{3} + \tfrac{1}{2}\mathrm{i}\right)$$
$$z = \tfrac{1}{4}\sqrt{6} - \tfrac{1}{4}\mathrm{i}\sqrt{2} \vee z = -\tfrac{1}{4}\sqrt{6} + \tfrac{1}{4}\mathrm{i}\sqrt{2}$$

Dus $\sqrt{\frac{1}{1+\mathrm{i}\sqrt{3}}} = \frac{1}{4}\sqrt{6} - \frac{1}{4}\mathrm{i}\sqrt{2} \vee \sqrt{\frac{1}{1+\mathrm{i}\sqrt{3}}} = -\frac{1}{4}\sqrt{6} + \frac{1}{4}\mathrm{i}\sqrt{2}$.

c $\sqrt[3]{-2+2\mathrm{i}}$, dus los op $z^3 = -2 + 2\mathrm{i}$

$$z^3 = 2\sqrt{2}\,\mathrm{e}^{\frac{3}{4}\pi\mathrm{i} + k\cdot 2\pi\mathrm{i}}$$
$$z = \sqrt{2}\,\mathrm{e}^{\frac{1}{4}\pi\mathrm{i} + k\cdot\frac{2}{3}\pi\mathrm{i}}$$
$$z = \sqrt{2}\,\mathrm{e}^{-\frac{5}{12}\pi\mathrm{i}} \vee z = \sqrt{2}\,\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}} \vee z = \sqrt{2}\,\mathrm{e}^{\frac{11}{12}\pi\mathrm{i}}$$
$$z = \sqrt{2}\left(\cos\left(-\tfrac{5}{12}\pi\right) + \mathrm{i}\sin\left(-\tfrac{5}{12}\pi\right)\right) \vee z = \sqrt{2}\left(\tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}\mathrm{i}\sqrt{2}\right) \vee z = \sqrt{2}\left(\cos\left(\tfrac{11}{12}\pi\right) + \mathrm{i}\sin\left(\tfrac{11}{12}\pi\right)\right)$$
$$z = \sqrt{2}\cos\left(-\tfrac{5}{12}\pi\right) + \mathrm{i}\sqrt{2}\sin\left(-\tfrac{5}{12}\pi\right) \vee z = 1 + \mathrm{i} \vee z = \sqrt{2}\cos\left(\tfrac{11}{12}\pi\right) + \mathrm{i}\sqrt{2}\sin\left(\tfrac{11}{12}\pi\right)$$

Dus $\sqrt[3]{-2+2\mathrm{i}} = \sqrt{2}\cos\left(-\frac{5}{12}\pi\right) + \mathrm{i}\sqrt{2}\sin\left(-\frac{5}{12}\pi\right) \vee \sqrt[3]{-2+2\mathrm{i}} = 1 + \mathrm{i} \vee \sqrt[3]{-2+2\mathrm{i}} = \sqrt{2}\cos\left(\frac{11}{12}\pi\right) + \mathrm{i}\sqrt{2}\sin\left(\frac{11}{12}\pi\right)$.

d $\sqrt[4]{-8+8\mathrm{i}\sqrt{3}}$, dus los op $z^4 = -8 + 8\mathrm{i}\sqrt{3}$

$$z^4 = 16\mathrm{e}^{\frac{2}{3}\pi\mathrm{i} + k\cdot 2\pi\mathrm{i}}$$
$$z = 2\mathrm{e}^{\frac{1}{6}\pi\mathrm{i} + k\cdot\frac{1}{2}\pi\mathrm{i}}$$
$$z = 2\mathrm{e}^{-\frac{5}{6}\pi\mathrm{i}} \vee z = 2\mathrm{e}^{-\frac{1}{3}\pi\mathrm{i}} \vee z = 2\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}} \vee z = 2\mathrm{e}^{\frac{2}{3}\pi\mathrm{i}}$$
$$z = 2\left(-\tfrac{1}{2}\sqrt{3} - \tfrac{1}{2}\mathrm{i}\right) \vee z = 2\left(\tfrac{1}{2} - \tfrac{1}{2}\mathrm{i}\sqrt{3}\right) \vee z = 2\left(\tfrac{1}{2}\sqrt{3} + \tfrac{1}{2}\mathrm{i}\right) \vee z = 2\left(-\tfrac{1}{2} + \tfrac{1}{2}\mathrm{i}\sqrt{3}\right)$$
$$z = -\sqrt{3} - \mathrm{i} \vee z = 1 - \mathrm{i}\sqrt{3} \vee z = \sqrt{3} + \mathrm{i} \vee z = -1 + \mathrm{i}\sqrt{3}$$

Dus $\sqrt[4]{-8+8\mathrm{i}\sqrt{3}} = -\sqrt{3} - \mathrm{i} \vee \sqrt[4]{-8+8\mathrm{i}\sqrt{3}} = 1 - \mathrm{i}\sqrt{3} \vee \sqrt[4]{-8+8\mathrm{i}\sqrt{3}} = \sqrt{3} + \mathrm{i} \vee \sqrt[4]{-8+8\mathrm{i}\sqrt{3}} = -1 + \mathrm{i}\sqrt{3}$.

64 **a** $\arg(z^2) = \arg(-4\mathrm{i}) = -\frac{1}{2}\pi$, dus één van de oplossingen heeft als argument $-\frac{1}{4}\pi$.
$|z^2| = |-4\mathrm{i}| = 4$ geeft $|z| = 2$, dus de straal van de cirkel is 2.
De argumenten verschillen steeds $\frac{2\pi}{2} = \pi$.
De oplossing is $z = 2\left(\frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}\right) = \sqrt{2} - \mathrm{i}\sqrt{2} \vee z = 2\left(-\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right) = -\sqrt{2} + \mathrm{i}\sqrt{2}$.

b $\arg(z^6) = \arg(-64) = \pi$, dus één van de oplossingen heeft als argument $\frac{1}{6}\pi$.
$|z^6| = |-64| = 64$ geeft $|z| = 2$, dus de straal van de cirkel is 2.
De argumenten verschillen steeds $\frac{2\pi}{6} = \frac{1}{3}\pi$.
De oplossing is $z = \sqrt{3} + \mathrm{i} \vee z = 2\mathrm{i} \vee z = -\sqrt{3} + \mathrm{i} \vee z = -\sqrt{3} - \mathrm{i} \vee z = -2\mathrm{i} \vee z = \sqrt{3} - \mathrm{i}$.

c $z^2 - 4z + 4 = \frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}$

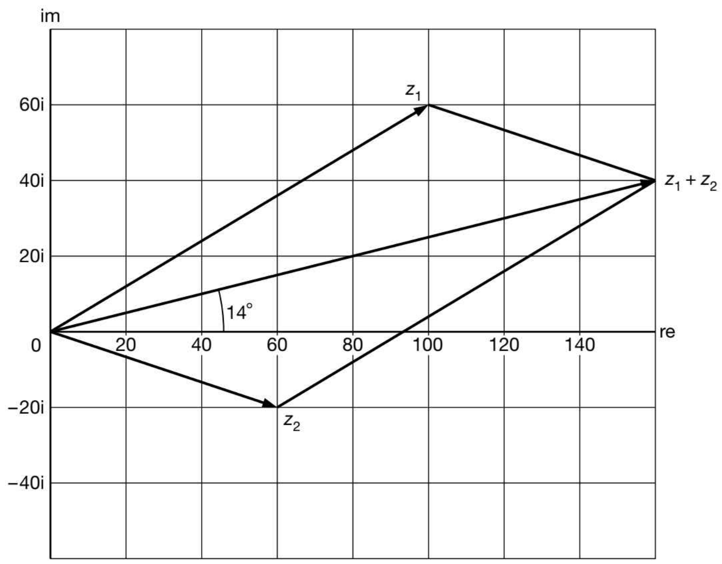
$$(z-2)^2 = \mathrm{e}^{-\frac{1}{3}\pi\mathrm{i} + k\cdot 2\pi\mathrm{i}}$$
$$z - 2 = \mathrm{e}^{-\frac{1}{6}\pi\mathrm{i} + k\cdot\pi\mathrm{i}}$$
$$z - 2 = \mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}} \vee z - 2 = \mathrm{e}^{\frac{5}{6}\pi\mathrm{i}}$$
$$z - 2 = \tfrac{1}{2}\sqrt{3} - \tfrac{1}{2}\mathrm{i} \vee z - 2 = -\tfrac{1}{2}\sqrt{3} + \tfrac{1}{2}\mathrm{i}$$
$$z = 2 + \tfrac{1}{2}\sqrt{3} - \tfrac{1}{2}\mathrm{i} \vee z = 2 - \tfrac{1}{2}\sqrt{3} + \tfrac{1}{2}\mathrm{i}$$

d $z^2 - 6z + 10 = \mathrm{i}\sqrt{3}$
$(z-3)^2 - 9 + 10 = \mathrm{i}\sqrt{3}$
$(z-3)^2 = -1 + \mathrm{i}\sqrt{3}$
$(z-3)^2 = 2\mathrm{e}^{\frac{2}{3}\pi\mathrm{i} + k\cdot 2\pi\mathrm{i}}$
$z - 3 = \sqrt{2}\mathrm{e}^{\frac{1}{3}\pi\mathrm{i} + k\cdot\pi\mathrm{i}}$
$z - 3 = \sqrt{2}\mathrm{e}^{-\frac{2}{3}\pi\mathrm{i}} \vee z - 3 = \sqrt{2}\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$
$z - 3 = \sqrt{2}\left(-\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right) \vee z - 3 = \sqrt{2}\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)$
$z - 3 = -\frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{6} \vee z - 3 = \frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{6}$
$z = 3 - \frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{6} \vee z = 3 + \frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{6}$

10.5 Krachten en snelheden met complexe getallen

Bladzijde 72

65 **a**



b lengte ≈ 8,25 cm dus $|z_1 + z_2| \approx 165$.
hoek ≈ 14°
c $|z_1 + z_2| = |160 + 40\mathrm{i}| \approx 165$
$\arg(z_1 + z_2) = \arg(160 + 40\mathrm{i}) \approx 14°$

10

Bladzijde 73

66 Bij de krachten $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$ en $\vec{F}_4$ horen de complexe getallen
$z_1 = 120$
$z_2 = 250(\cos(35°) + \mathrm{i}\sin(35°))$
$z_3 = 200(\cos(100°) + \mathrm{i}\sin(100°))$
$z_4 = 180(\cos(170°) + \mathrm{i}\sin(170°))$
De GR geeft $|z_1 + z_2 + z_3 + z_4| \approx 388$ en $\arg(z_1 + z_2 + z_3 + z_4) \approx 73°$.
De resultante heeft een grootte van ongeveer 388 N en maakt een hoek van 73° met de horizontale as.

67 Bij de krachten $\vec{F}_z$, $\vec{F}_1$ en $\vec{F}_r$ horen de complexe getallen
$z_z = -200\mathrm{i}$
$z_1 = 300(\cos(25°) + \mathrm{i}\sin(25°))$
$z_r = 300(\cos(70°) + \mathrm{i}\sin(70°))$
en z_2.
$z_r = z_1 + z_2 + z_z$ dus $z_2 = z_r - z_1 - z_z$
De GR geeft $|z_r - z_1 - z_z| \approx 393$ en $\arg(z_r - z_1 - z_z) \approx 115°$.
Dus de grootte van $\vec{F}_2$ is 393 N en de hoek $\alpha \approx 180° - 115° = 65°$.

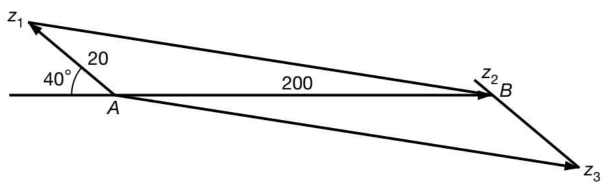
68 Bij de krachten $\vec{F}_z$, $\vec{F}_1$, $\vec{F}_2$ en $\vec{F}_t$ horen de complexe getallen
$z_z = -500\text{i}$
$z_1 = 300(\cos(200°) + \text{i}\sin(200°))$
$z_2 = 150(\cos(-40°) + \text{i}\sin(-40°))$
en z_t.
$z_z + z_1 + z_2 + z_t = 0$ geeft $z_t = -z_z - z_1 - z_2$
De GR geeft $|-z_z - z_1 - z_2| \approx 719$ en $\arg(-z_z - z_1 - z_2) \approx 77°$.
Dus de grootte van $\vec{F}_t \approx 719$ N en $\alpha \approx 90° - 77° = 13°$.

Bladzijde 74

69 Bij de snelheden horen de complexe getallen
$z_1 = 4(\cos(-155°) + \text{i}\sin(-155°))$ en $z_2 = 3\text{i}$.
De GR geeft $|z_1 + z_2| \approx 3{,}9$ en $\arg(z_1 + z_2) \approx 160°$.
De resulterende snelheid is 3,9 km/uur en de koers is 290°.

70 Bij de snelheden horen de complexe getallen
$z_w = 50(\cos(45°) + \text{i}\sin(45°))$ en
$z_v = 250(\cos(-80°) + \text{i}\sin(-80°))$.
De GR geeft $|z_w + z_v| \approx 225{,}08$.
Het vliegtuig doet er $\dfrac{1000}{225{,}08} \approx 4{,}443$ uur $\approx$ 4 uur en 27 minuten over.

71



Bij de snelheden horen de complexe getallen
$z_1 = 20(\cos(140°) + \text{i}\sin(140°))$
$z_2 = 200$ en z_3.
$z_1 + z_3 = z_2$ dus $z_3 = z_2 - z_1$
De GR geeft $|z_2 - z_1| \approx 216$.
De snelheid van het vliegtuig is 216 km/uur ten opzichte van de omringende lucht.

Diagnostische toets

Bladzijde 76

1
a $2x - 1 + 2\text{i} = 4x - 2 + 4\text{i}$
$-2x = -1 + 2\text{i}$
$x = \frac{1}{2} - \text{i}$

b $(2x + 1)^2 + 9 = 0$
$(2x + 1)^2 = -9$
$(2x + 1)^2 = 9\text{i}^2$
$2x + 1 = 3\text{i} \vee 2x + 1 = -3\text{i}$
$2x = -1 + 3\text{i} \vee 2x = -1 - 3\text{i}$
$x = -\frac{1}{2} + 1\frac{1}{2}\text{i} \vee x = -\frac{1}{2} - 1\frac{1}{2}\text{i}$

c $x^2 - 4x + 10 = 0$
$(x - 2)^2 - 4 + 10 = 0$
$(x - 2)^2 = -6$
$(x - 2)^2 = 6\text{i}^2$
$x - 2 = \text{i}\sqrt{6} \vee x - 2 = -\text{i}\sqrt{6}$
$x = 2 + \text{i}\sqrt{6} \vee x = 2 - \text{i}\sqrt{6}$

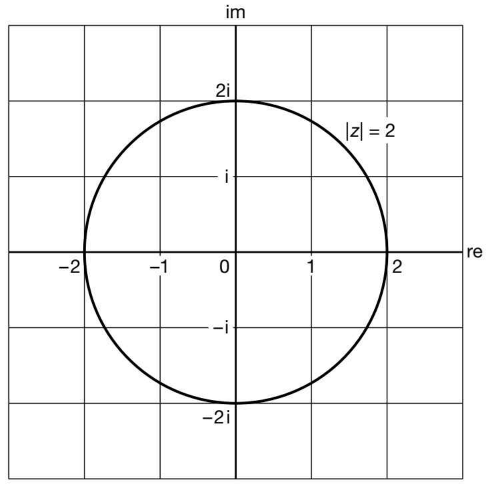
d $4x^2 + 16x + 17 = 0$
$(2x + 4)^2 - 16 + 17 = 0$
$(2x + 4)^2 = -1$
$(2x + 4)^2 = \text{i}^2$
$2x + 4 = \text{i} \vee 2x + 4 = -\text{i}$
$2x = -4 + \text{i} \vee 2x = -4 - \text{i}$
$x = -2 + \frac{1}{2}\text{i} \vee x = -2 - \frac{1}{2}\text{i}$

2
a $z = (2 - \text{i}) - (5 - 8\text{i}) = 2 - \text{i} - 5 + 8\text{i} = -3 + 7\text{i}$
b $z = (6 - 2\text{i})(3 + 2\text{i}) = 18 + 12\text{i} - 6\text{i} - 4\text{i}^2 = 18 + 12\text{i} - 6\text{i} + 4 = 22 + 6\text{i}$
c $z = \dfrac{3 - \text{i}}{4 + \text{i}} = \dfrac{3 - \text{i}}{4 + \text{i}} \cdot \dfrac{4 - \text{i}}{4 - \text{i}} = \dfrac{12 - 3\text{i} - 4\text{i} + \text{i}^2}{16 - \text{i}^2} = \dfrac{12 - 3\text{i} - 4\text{i} - 1}{16 + 1} = \dfrac{11 - 7\text{i}}{17} = \frac{11}{17} - \frac{7}{17}\text{i}$
d $z = \dfrac{2 - 5\text{i}}{6\text{i}} = \dfrac{2 - 5\text{i}}{6\text{i}} \cdot \dfrac{\text{i}}{\text{i}} = \dfrac{2\text{i} - 5\text{i}^2}{6\text{i}^2} = \dfrac{2\text{i} + 5}{-6} = -\frac{5}{6} - \frac{1}{3}\text{i}$

e $z = \overline{(2-3\mathrm{i})^2} = \overline{4 - 12\mathrm{i} + 9\mathrm{i}^2} = \overline{4 - 12\mathrm{i} - 9} = \overline{-5 - 12\mathrm{i}} = -5 + 12\mathrm{i}$

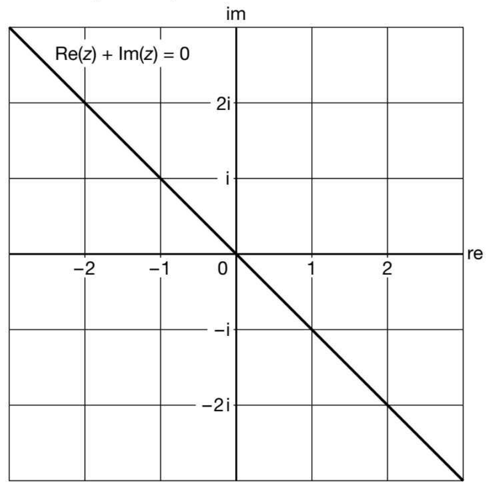
f $z = \dfrac{\overline{2-\mathrm{i}}}{2+3\mathrm{i}} = \dfrac{2+\mathrm{i}}{2+3\mathrm{i}} \cdot \dfrac{2-3\mathrm{i}}{2-3\mathrm{i}} = \dfrac{4 - 6\mathrm{i} + 2\mathrm{i} - 3\mathrm{i}^2}{4 - 9\mathrm{i}^2} = \dfrac{4 - 6\mathrm{i} + 2\mathrm{i} + 3}{4+9} = \dfrac{7-4\mathrm{i}}{13} = \frac{7}{13} - \frac{4}{13}\mathrm{i}$

3 **a**



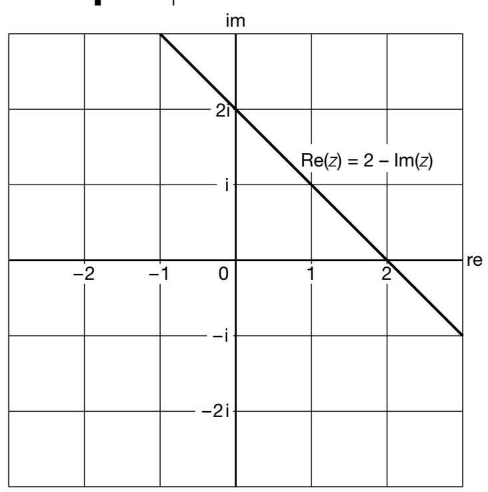
b

Re(z)	0	1
Im(z)	0	-1

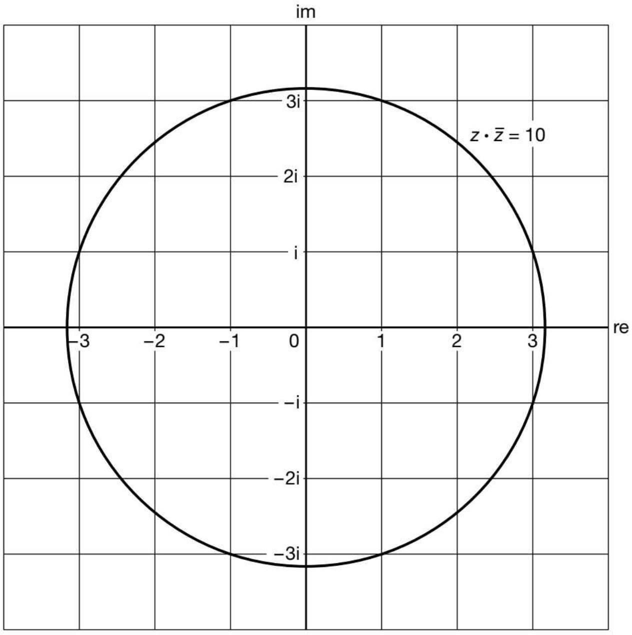


c

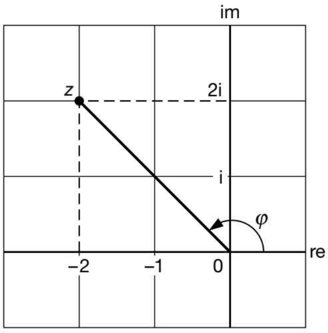
Re(z)	0	2
Im(z)	2	0



d $z \cdot \bar{z} = 10$ geeft $|z| = \sqrt{10}$
Dit is de cirkel met middelpunt 0 en straal $\sqrt{10}$.



4 **a**

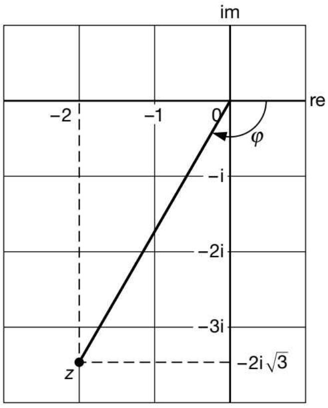


$\tan(\varphi) = \dfrac{2}{-2} = -1$ geeft $\varphi = \frac{3}{4}\pi$

Dus $\text{Arg}(-2 + 2\text{i}) = \frac{3}{4}\pi$.

De modulus is $|z| = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$.

b

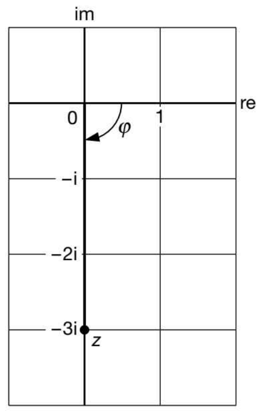


$\tan(\varphi) = \dfrac{-2\sqrt{3}}{-2} = \sqrt{3}$ geeft $\varphi = -\frac{2}{3}\pi$

Dus $\text{Arg}\left(-2 - 2\text{i}\sqrt{3}\right) = -\frac{2}{3}\pi$.

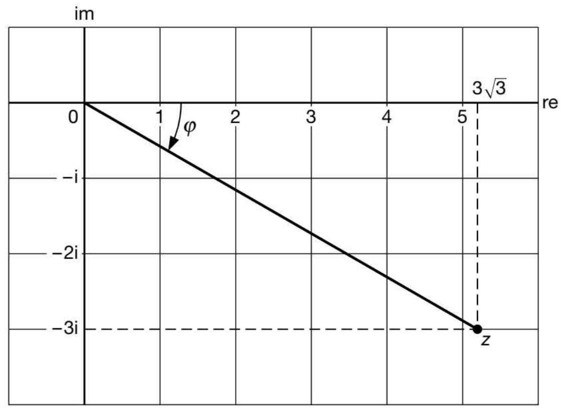
De modulus is $|z| = \sqrt{(-2)^2 + \left(-2\sqrt{3}\right)^2} = \sqrt{16} = 4$.

c



$\varphi = -\frac{1}{2}\pi$, dus $\text{Arg}(-3\text{i}) = -\frac{1}{2}\pi$.
De modulus is $|z| = 3$.

d

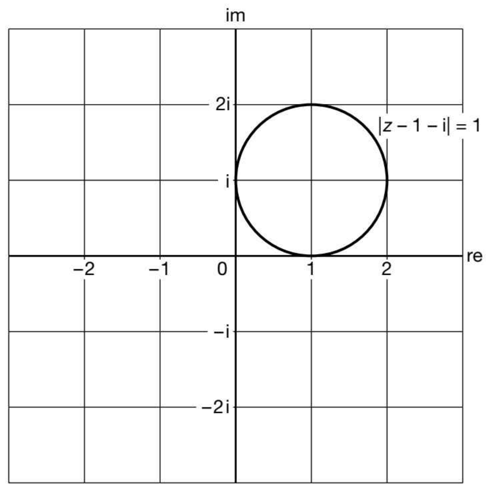


$\tan(\varphi) = \dfrac{3}{-3\sqrt{3}} = -\frac{1}{3}\sqrt{3}$ geeft $\varphi = -\frac{1}{6}\pi$

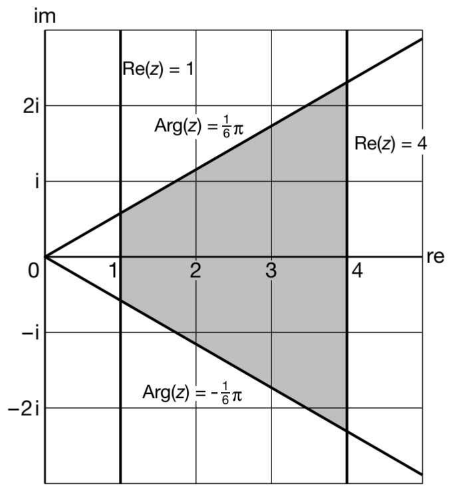
Dus $\text{Arg}\left(2\sqrt{3} - 3\text{i}\right) = -\frac{1}{6}\pi$.

De modulus is $|z| = \sqrt{\left(3\sqrt{3}\right)^2 + (-3)^2} = \sqrt{36} = 6$.

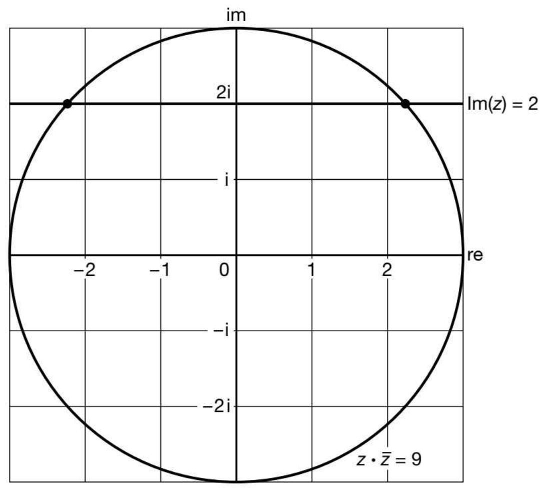
5 **a** $|z - 1 - \text{i}| = 1$ is de cirkel met middelpunt $1 + \text{i}$ en straal 1.



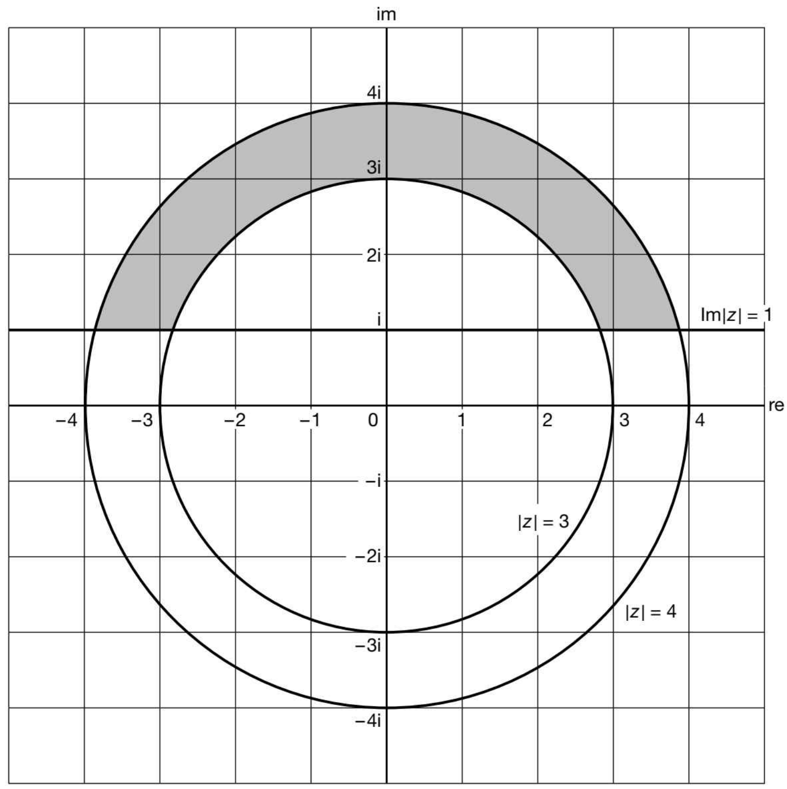
b



c $z \cdot \bar{z} = 9 \ \wedge \ \text{Im}(z) = 2$ geeft $|z| = 3 \ \wedge \ \text{Im}(z) = 2$



d



6 **a** $|z| = \sqrt{5^2 + (-12)^2} = 13$ en $\arg(z) \approx -1{,}18$

Dus $z = 13(\cos(-1{,}18) + \mathrm{i}\sin(-1{,}18))$.

b $|z| = \sqrt{(\sqrt{5})^2 + (-2)^2} = 3$ en $\arg(z) \approx -0{,}73$

Dus $z = 3(\cos(-0{,}73) + \mathrm{i}\sin(-0{,}73))$.

c $|z| = \sqrt{2^2 + 4^2} = 2\sqrt{5}$ en $\arg(z) \approx 1{,}11$

Dus $z = 2\sqrt{5}(\cos(1{,}11) + \mathrm{i}\sin(1{,}11))$.

d $|z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\sqrt{2}\right)^2} = 1\frac{1}{2}$ en $\arg(z) \approx -1{,}91$

Dus $z = 1\frac{1}{2}(\cos(-1{,}91) + \mathrm{i}\sin(-1{,}91))$.

7 **a** $z = 3\left(\cos\left(\frac{2}{3}\pi\right) + \mathrm{i}\sin\left(\frac{2}{3}\pi\right)\right) = 3\left(-\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right) = -1\frac{1}{2} + 1\frac{1}{2}\mathrm{i}\sqrt{3}$

b $z = 2\sqrt{2}\left(\cos\left(\frac{1}{4}\pi\right) + \mathrm{i}\sin\left(\frac{1}{4}\pi\right)\right) = 2\sqrt{2}\left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right) = 2 + 2\mathrm{i}$

8 **a** $z = 10\,\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}} = 10\left(\cos\left(\frac{1}{3}\pi\right) + \mathrm{i}\sin\left(\frac{1}{3}\pi\right)\right) = 10\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right) = 5 + 5\mathrm{i}\sqrt{3}$

b $z = 6\,\mathrm{e}^{\frac{1}{2}\pi\mathrm{i}} = 6\left(\cos\left(\frac{1}{2}\pi\right) + \mathrm{i}\sin\left(\frac{1}{2}\pi\right)\right) = 6(0 + \mathrm{i}) = 6\mathrm{i}$

c $z = \dfrac{3}{\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}}} = 3\,\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}} = 3\left(\cos\left(\frac{1}{6}\pi\right) + \mathrm{i}\sin\left(\frac{1}{6}\pi\right)\right) = 3\left(\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}\right) = 1\frac{1}{2}\sqrt{3} + 1\frac{1}{2}\mathrm{i}$

9 **a** $|z| = \sqrt{3^2 + 3^2} = 3\sqrt{2}$ en $\arg(z) = \frac{1}{4}\pi$ geeft $z = 3\sqrt{2}\,\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$

b $|z| = 2$ en $\arg(z) = \pi$ geeft $z = 2\,\mathrm{e}^{\pi\mathrm{i}}$

c $|z| = \sqrt{\left(\frac{1}{2}\sqrt{2}\right)^2 + \left(-\frac{1}{2}\sqrt{2}\right)^2} = 1$ en $\arg(z) = -\frac{1}{4}\pi$ geeft $z = \mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}}$

Bladzijde 77

10 **a** $3 - 3\mathrm{i}\sqrt{3} = 6\,\mathrm{e}^{-\frac{1}{3}\pi\mathrm{i}}$ en $1 + \mathrm{i} = \sqrt{2}\,\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$, dus $z = (3 - 3\mathrm{i}\sqrt{3})(1 + \mathrm{i}) = 6\,\mathrm{e}^{-\frac{1}{3}\pi\mathrm{i}} \cdot \sqrt{2}\,\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}} = 6\sqrt{2}\,\mathrm{e}^{-\frac{1}{12}\pi\mathrm{i}}$

b $1 + \mathrm{i} = \sqrt{2}\,\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$ en $1 - \mathrm{i} = \sqrt{2}\,\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}}$, dus $z = \dfrac{(1+\mathrm{i})^3}{(1-\mathrm{i})^4} = \dfrac{\left(\sqrt{2}\,\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}\right)^3}{\left(\sqrt{2}\,\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}}\right)^4} = \dfrac{2\sqrt{2}\,\mathrm{e}^{\frac{3}{4}\pi\mathrm{i}}}{4\,\mathrm{e}^{-\pi\mathrm{i}}} = \frac{1}{2}\sqrt{2}\,\mathrm{e}^{1\frac{3}{4}\pi\mathrm{i}} = \frac{1}{2}\sqrt{2}\,\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}}$

c $3 - \mathrm{i}\sqrt{3} = 2\sqrt{3}\,\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}}$, dus $z = \left(3 - \mathrm{i}\sqrt{3}\right)^{-4} = \left(2\sqrt{3}\,\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}}\right)^{-4} = \dfrac{1}{\left(2\sqrt{3}\,\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}}\right)^4} = \dfrac{1}{144\,\mathrm{e}^{-\frac{2}{3}\pi\mathrm{i}}} = 144\,\mathrm{e}^{\frac{2}{3}\pi\mathrm{i}}$

d $\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}} = \cos\left(-\frac{1}{4}\pi\right) + \mathrm{i}\sin\left(-\frac{1}{4}\pi\right) = \frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}$, dus $z = \mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}} - \frac{1}{2}\sqrt{2} + \mathrm{i}\sqrt{2} = \frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2} - \frac{1}{2}\sqrt{2} + \mathrm{i}\sqrt{2} = \frac{1}{2}\mathrm{i}\sqrt{2} = \frac{1}{2}\sqrt{2}\,\mathrm{e}^{\frac{1}{2}\pi\mathrm{i}}$

e $\sqrt{3} + \mathrm{i} = 2\,\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}$, dus $z = \left(\sqrt{3} + \mathrm{i}\right)^2 - 6\,\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}} = \left(2\,\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}\right)^2 - 6\,\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}} = 4\,\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}} - 6\,\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}} = -2\,\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$

f $2 + 2\mathrm{i} = 2\sqrt{2}\,\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$ en $1 - \mathrm{i} = \sqrt{2}\,\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}}$, dus $z = \dfrac{(2+2\mathrm{i})^3}{1-\mathrm{i}} = \dfrac{\left(2\sqrt{2}\,\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}\right)^3}{\sqrt{2}\,\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}}} = \dfrac{16\sqrt{2}\,\mathrm{e}^{\frac{3}{4}\pi\mathrm{i}}}{\sqrt{2}\,\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}}} = 16\,\mathrm{e}^{\pi\mathrm{i}}$

11 Substitutie van $u_n = g^n$ in $u_n = -2u_{n-1} - 2u_{n-2}$ geeft
$g^2 + 2g + 2 = 0$
$(g + 1)^2 - 1 + 2 = 0$
$(g + 1)^2 = -1$
$(g + 1)^2 = \mathrm{i}^2$
$g + 1 = \mathrm{i} \vee g + 1 = -\mathrm{i}$
$g = -1 + \mathrm{i} \vee g = -1 - \mathrm{i}$
$|-1 + \mathrm{i}| = \sqrt{2}$ en $\arg(-1 + \mathrm{i}) = \frac{3}{4}\pi$

$u_n = \left(A\cos\left(\frac{3}{4}\pi n\right) + B\sin\left(\frac{3}{4}\pi n\right)\right) \cdot \left(\sqrt{2}\right)^n$

$$\left.\begin{array}{l} u_0 = 2\sqrt{2} \text{ geeft } A = 2\sqrt{2} \\ u_1 = 1 \text{ geeft } \left(A \cdot -\frac{1}{2}\sqrt{2} + B \cdot \frac{1}{2}\sqrt{2}\right) \cdot \sqrt{2} = 1 \text{ ofwel } -A + B = 1 \end{array}\right\} A = 2\sqrt{2} \text{ en } B = 1 + 2\sqrt{2}$$

Dus $u_n = \left(2\sqrt{2}\cos\left(\frac{3}{4}\pi n\right) + \left(1 + 2\sqrt{2}\right)\sin\left(\frac{3}{4}\pi n\right)\right) \cdot \left(\sqrt{2}\right)^n$.

12 **a** $\arg(z^2) = \arg(-16\mathrm{i}) = -\frac{1}{2}\pi$, dus één van de oplossingen heeft als argument $-\frac{1}{4}\pi$.
$|z^2| = |-16\mathrm{i}| = 16$ geeft $|z| = 4$, dus de straal van de cirkel is 4.

De argumenten verschillen steeds $\dfrac{2\pi}{2} = \pi$ van elkaar.

De oplossing is $z = 2\sqrt{2} - 2\mathrm{i}\sqrt{2} \vee z = -2\sqrt{2} + 2\mathrm{i}\sqrt{2}$.

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b $\arg(z^3) = \arg(27\mathrm{i}) = \frac{1}{2}\pi$, dus één van de oplossingen heeft als argument $\frac{1}{6}\pi$.
$|z^3| = |27\mathrm{i}| = 27$ geeft $|z| = 3$, dus de straal van de cirkel is 3.
De argumenten verschillen steeds $\frac{2\pi}{3} = \frac{2}{3}\pi$ van elkaar.
De oplossing is $z = 3\left(\frac{1}{2}\sqrt{3} + \frac{1}{2}\right) = 1\frac{1}{2}\sqrt{3} + 1\frac{1}{2}\mathrm{i} \vee z = 3\left(-\frac{1}{2}\sqrt{3} + \frac{1}{2}\right) = -1\frac{1}{2}\sqrt{3} + 1\frac{1}{2}\mathrm{i} \vee z = 3(0 - \mathrm{i}) = -3\mathrm{i}$.

c $(z - 2\mathrm{i})^2 = -4\mathrm{i}$
$(z - 2\mathrm{i})^2 = 4\mathrm{e}^{-\frac{1}{2}\pi\mathrm{i} + k \cdot 2\pi\mathrm{i}}$
$z - 2\mathrm{i} = 2\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i} + k \cdot \pi\mathrm{i}}$
$z - 2\mathrm{i} = 2\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}} \vee z - 2\mathrm{i} = 2\mathrm{e}^{\frac{3}{4}\pi\mathrm{i}}$
$z - 2\mathrm{i} = 2\left(\frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}\right) \vee z - 2\mathrm{i} = 2\left(-\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right)$
$z - 2\mathrm{i} = \sqrt{2} - \mathrm{i}\sqrt{2} \vee z - 2\mathrm{i} = -\sqrt{2} + \mathrm{i}\sqrt{2}$
$z = \sqrt{2} + 2\mathrm{i} - \mathrm{i}\sqrt{2} \vee z = -\sqrt{2} + 2\mathrm{i} + \mathrm{i}\sqrt{2}$
$z = \sqrt{2} + \left(2 - \sqrt{2}\right)\mathrm{i} \vee z = -\sqrt{2} + \left(2 + \sqrt{2}\right)\mathrm{i}$

d $z^2 + 4z + 2 = 2\mathrm{i}\sqrt{3}$
$(z + 2)^2 - 4 + 2 = 2\mathrm{i}\sqrt{3}$
$(z + 2)^2 = 2 + 2\mathrm{i}\sqrt{3}$
$(z + 2)^2 = 4\mathrm{e}^{\frac{1}{3}\pi\mathrm{i} + k \cdot 2\pi\mathrm{i}}$
$z + 2 = 2\mathrm{e}^{\frac{1}{6}\pi\mathrm{i} + k \cdot \pi\mathrm{i}}$
$z + 2 = 2\mathrm{e}^{-\frac{5}{6}\pi\mathrm{i}} \vee z + 2 = 2\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}$
$z + 2 = 2\left(-\frac{1}{2}\sqrt{3} - \frac{1}{2}\mathrm{i}\right) \vee z + 2 = 2\left(\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}\right)$
$z + 2 = -\sqrt{3} - \mathrm{i} \vee z + 2 = \sqrt{3} + \mathrm{i}$
$z = -2 - \sqrt{3} - \mathrm{i} \vee z = -2 + \sqrt{3} + \mathrm{i}$

13 **a** $\sqrt{\mathrm{i}}$, dus los op $z^2 = \mathrm{i}$
$z^2 = \mathrm{e}^{\frac{1}{2}\pi\mathrm{i} + k \cdot 2\pi\mathrm{i}}$
$z = \mathrm{e}^{\frac{1}{4}\pi\mathrm{i} + k \cdot \pi\mathrm{i}}$
$z = \mathrm{e}^{-\frac{3}{4}\pi\mathrm{i}} \vee z = \mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$
Dus $\sqrt{\mathrm{i}} = \mathrm{e}^{-\frac{3}{4}\pi\mathrm{i}} \vee \sqrt{\mathrm{i}} = \mathrm{e}^{\frac{1}{4}\pi}$.

b $\sqrt[4]{-16}$, dus los op $z^4 = -16$
$z^4 = 16\mathrm{e}^{\pi\mathrm{i} + k \cdot 2\pi\mathrm{i}}$
$z = 2\mathrm{e}^{\frac{1}{4}\pi\mathrm{i} + k \cdot \frac{1}{2}\pi\mathrm{i}}$
$z = 2\mathrm{e}^{-\frac{3}{4}\pi\mathrm{i}} \vee z = 2\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}} \vee z = 2\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}} \vee z = 2\mathrm{e}^{\frac{3}{4}\pi\mathrm{i}}$
Dus $\sqrt[4]{-16} = 2\mathrm{e}^{-\frac{3}{4}\pi\mathrm{i}} \vee \sqrt[4]{-16} = 2\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}} \vee \sqrt[4]{-16} = 2\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}} \vee \sqrt[4]{-16} = 2\mathrm{e}^{\frac{3}{4}\pi\mathrm{i}}$.

c $\sqrt[4]{-2 - 2\mathrm{i}\sqrt{3}}$, dus los op $z^4 = -2 - 2\mathrm{i}\sqrt{3}$
$z^4 = 4\mathrm{e}^{-\frac{2}{3}\pi\mathrm{i} + k \cdot 2\pi\mathrm{i}}$
$z = \sqrt{2}\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i} + k \cdot \frac{1}{2}\pi\mathrm{i}}$
$z = \sqrt{2}\mathrm{e}^{-\frac{2}{3}\pi\mathrm{i}} \vee z = \sqrt{2}\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}} \vee z = \sqrt{2}\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}} \vee z = \sqrt{2}\mathrm{e}^{\frac{5}{6}\pi\mathrm{i}}$
Dus $\sqrt[4]{-2 - 2\mathrm{i}\sqrt{3}} = \sqrt{2}\mathrm{e}^{-\frac{2}{3}\pi\mathrm{i}} \vee \sqrt[4]{-2 - 2\mathrm{i}\sqrt{3}} = \sqrt{2}\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}} \vee \sqrt[4]{-2 - 2\mathrm{i}\sqrt{3}} = \sqrt{2}\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}} \vee \sqrt[4]{-2 - 2\mathrm{i}\sqrt{3}} = \sqrt{2}\mathrm{e}^{\frac{5}{6}\pi\mathrm{i}}$.

d $\sqrt[3]{-\frac{1}{\mathrm{i}}} = \sqrt[3]{\frac{-1}{\mathrm{i}}} = \sqrt[3]{\frac{\mathrm{i}^2}{\mathrm{i}}} = \sqrt[3]{\mathrm{i}}$, dus los op $z^3 = \mathrm{i}$
$z^3 = \mathrm{e}^{\frac{1}{2}\pi\mathrm{i} + k \cdot 2\pi\mathrm{i}}$
$z = \mathrm{e}^{\frac{1}{6}\pi\mathrm{i} + k \cdot \frac{2}{3}\pi\mathrm{i}}$
$z = \mathrm{e}^{-\frac{1}{2}\pi\mathrm{i}} \vee z = \mathrm{e}^{\frac{1}{6}\pi\mathrm{i}} \vee z = \mathrm{e}^{\frac{5}{6}\pi\mathrm{i}}$
Dus $\sqrt[3]{-\frac{1}{\mathrm{i}}} = \mathrm{e}^{-\frac{1}{2}\pi\mathrm{i}} \vee \sqrt[3]{-\frac{1}{\mathrm{i}}} = \mathrm{e}^{\frac{1}{6}\pi\mathrm{i}} \vee \sqrt[3]{-\frac{1}{\mathrm{i}}} = \mathrm{e}^{\frac{5}{6}\pi\mathrm{i}}$.

14 Bij de krachten $\vec{F_1}$, $\vec{F_2}$ en $\vec{F_3}$ horen de complexe getallen
$z_1 = 1200(\cos(20°) + \mathrm{i}\sin(20°))$
$z_2 = 400(\cos(-65°) + \mathrm{i}\sin(-65°))$ en
$z_3 = 1500(\cos(145°) + \mathrm{i}\sin(145°))$.
De GR geeft $|z_1 + z_2 + z_3| \approx 911$ en $\arg(z_1 + z_2 + z_3) \approx 86°$.
De resultante is 911 N groot en maakt een hoek van 86° met de positieve reële as.

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15 **a** Bij de snelheid van het schip en de stroomsnelheid van het water horen de complexe getallen
$z_1 = 8(\cos(50°) + \text{i}\sin(50°))$ en
$z_2 = 3(\cos(135°) + \text{i}\sin(135°))$.
De GR geeft $|z_1 + z_2| \approx 8{,}8$ en $\arg(z_1 + z_2) \approx 70°$.
De werkelijke snelheid is ongeveer 8,8 knopen.
De koers is 20°.

b Stel de nieuwe snelheid ten opzichte van het water is v knopen.
$z_1 = v(\cos(50°) + \text{i}\sin(50°))$
$z_2 = 3(\cos(135°) + \text{i}\sin(135°))$
$\text{Re}(z_1 + z_2) = v \cdot \cos(50°) + 3\cos(135°) = 0$
$v \cdot \cos(50°) = -3\cos(135°)$
$v = \dfrac{-3\cos(135°)}{\cos(50°)} \approx 3{,}3$
Hij moet zijn snelheid met $8 - 3{,}3 = 4{,}7$ knopen verlagen.

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