

8 Vectormeetkunde

Voorkennis Vectoren in het platte vlak

Bladzijde 139

1 a $|\vec{a}| = \left| \begin{pmatrix} 4 \\ -3 \end{pmatrix} \right| = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

b $|\vec{b}| = \left| \begin{pmatrix} -\sqrt{5} \\ \sqrt{10} \end{pmatrix} \right| = \sqrt{(-\sqrt{5})^2 + (\sqrt{10})^2} = \sqrt{5 + 10} = \sqrt{15}$

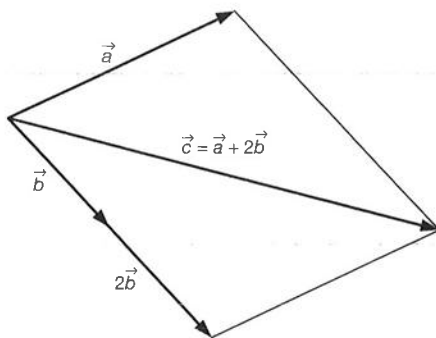
c $|\vec{c}| = 0,8 \cdot \left| \begin{pmatrix} -14 \\ -2 \end{pmatrix} \right| = 0,8 \cdot \sqrt{(-14)^2 + (-2)^2} = 0,8 \cdot \sqrt{196 + 4} = 0,8 \cdot \sqrt{200} = 8\sqrt{2}$

2 a $\vec{c} = \vec{a} - \vec{b} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

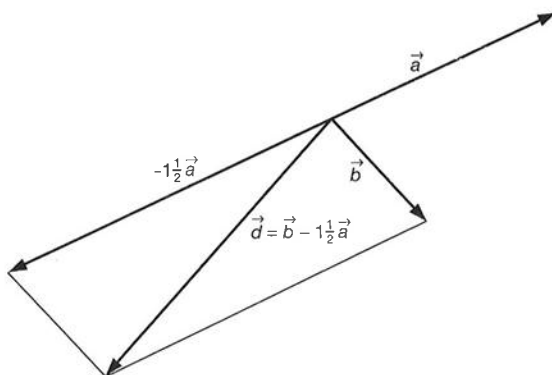
b $\vec{d} = 3\vec{a} - 2\vec{b} = 3 \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix} - 2 \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 13 \end{pmatrix}$

c $\vec{e} = 1\frac{1}{2}\vec{a} + 2\frac{1}{2}\vec{b} = 1\frac{1}{2} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix} + 2\frac{1}{2} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -1\frac{1}{2} \\ 4\frac{1}{2} \end{pmatrix} + \begin{pmatrix} 2\frac{1}{2} \\ -5 \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix}$

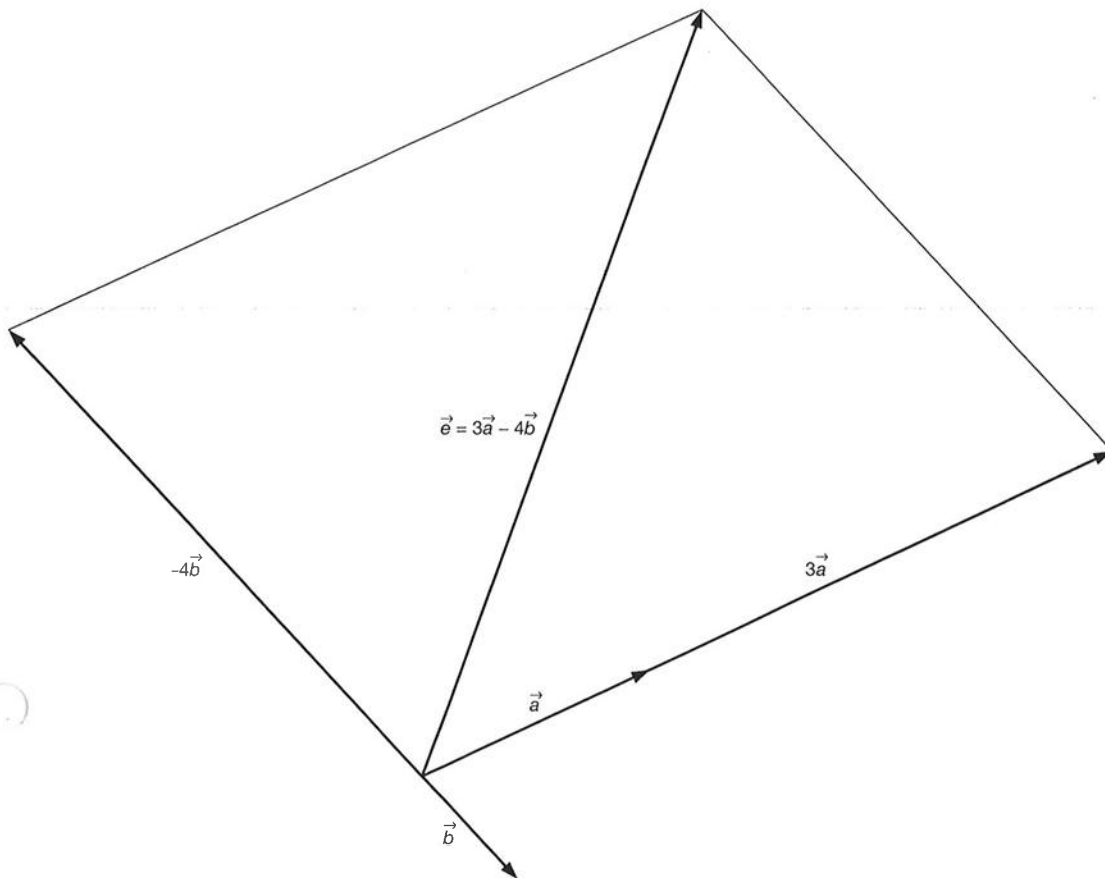
3 a



b



c



$$4 \quad d(A, B) = |\overrightarrow{AB}| = |\vec{b} - \vec{a}| = \left| \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \end{pmatrix} \right| = \left| \begin{pmatrix} -3 \\ 3 \end{pmatrix} \right| = \sqrt{(-3)^2 + 3^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$$

$$d(A, C) = |\overrightarrow{AC}| = |\vec{c} - \vec{a}| = \left| \begin{pmatrix} 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \end{pmatrix} \right| = \left| \begin{pmatrix} -5 \\ 5 \end{pmatrix} \right| = \sqrt{(-5)^2 + 5^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$$

$$d(B, C) = |\overrightarrow{BC}| = |\vec{c} - \vec{b}| = \left| \begin{pmatrix} 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right| = \left| \begin{pmatrix} -2 \\ 2 \end{pmatrix} \right| = \sqrt{(-2)^2 + 2^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{omtrek} = 3\sqrt{2} + 5\sqrt{2} + 2\sqrt{2} = 10\sqrt{2}$$

Bladzijde 140

$$5 \quad a \quad \vec{a} \cdot \vec{b} = 2 \cdot 3 + 5 \cdot (-1) = 6 - 5 = 1$$

$$|\vec{a}| = \sqrt{2^2 + 5^2} = \sqrt{29} \text{ en } |\vec{b}| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$$

$$\cos(\angle(\vec{a}, \vec{b})) = \frac{1}{\sqrt{29} \cdot \sqrt{10}} = \frac{1}{\sqrt{290}}$$

$$\angle(\vec{a}, \vec{b}) \approx 86,6^\circ$$

$$b \quad \vec{c} \cdot \vec{d} = -1 \cdot 2 + 4 \cdot (-3) = -2 - 12 = -14$$

$$|\vec{c}| = \sqrt{(-1)^2 + 4^2} = \sqrt{17} \text{ en } |\vec{d}| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$$

$$\cos(\angle(\vec{c}, \vec{d})) = \frac{-14}{\sqrt{17} \cdot \sqrt{13}} = \frac{-14}{\sqrt{221}}$$

$$\angle(\vec{c}, \vec{d}) \approx 160,3^\circ$$

$$6 \quad a \quad \overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ en } \overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 1 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$$

$$b \quad \overrightarrow{AB} \cdot \overrightarrow{AC} = 2 \cdot (-2) + 1 \cdot 6 = -4 + 6 = 2$$

$$|\overrightarrow{AB}| = \sqrt{2^2 + 1^2} = \sqrt{5} \text{ en } |\overrightarrow{AC}| = \sqrt{(-2)^2 + 6^2} = \sqrt{40}$$

$$\cos(\angle BAC) = \frac{2}{\sqrt{5} \cdot \sqrt{40}} = \frac{2}{\sqrt{200}}$$

$$\angle BAC \approx 81,9^\circ$$

Bladzijde 141

- 7 a Uit $\vec{a} \cdot \vec{b} = 0$ volgt dat $\cos(\angle(\vec{a}, \vec{b})) = 0$, dus $\angle(\vec{a}, \vec{b}) = 90^\circ$.

$$\begin{aligned} \text{b } \vec{a} \cdot \vec{b} &= \begin{pmatrix} p \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 6 \end{pmatrix} = 2p - 30 \\ \vec{a} \cdot \vec{b} &= 0 \end{aligned} \quad \left\{ \begin{array}{l} 2p - 30 = 0 \\ 2p = 30 \\ p = 15 \end{array} \right.$$

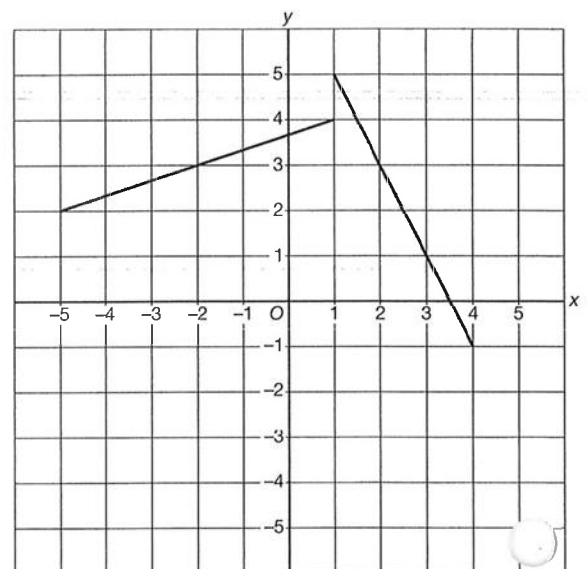
$$\begin{aligned} \text{c } \vec{AB} \cdot \vec{AC} &= \begin{pmatrix} p-3 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 14 \end{pmatrix} = -2p + 6 \\ \vec{AB} \cdot \vec{AC} &= 0 \end{aligned} \quad \left\{ \begin{array}{l} -2p + 6 = 0 \\ -2p = -6 \\ p = 3 \end{array} \right.$$

Bladzijde 142

$$\begin{aligned} \text{8 a } k: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{a} + \lambda(\vec{b} - \vec{a}) \\ \vec{a} &= \begin{pmatrix} 4 \\ -1 \end{pmatrix} \text{ en } \vec{b} - \vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \end{pmatrix} \end{aligned} \quad \left\{ \begin{array}{l} k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 3 \end{pmatrix} \end{array} \right.$$

$$\begin{aligned} \text{b } l: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{c} + \lambda(\vec{d} - \vec{c}) \\ \vec{c} &= \begin{pmatrix} 0 \\ 5 \end{pmatrix} \text{ en } \vec{d} - \vec{c} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \end{aligned} \quad \left\{ \begin{array}{l} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ -5 \end{pmatrix} \end{array} \right.$$

9	λ	-1	2
punt		(4, -1)	(1, 5)
μ	-1	1	
punt		(-5, 2)	(1, 4)



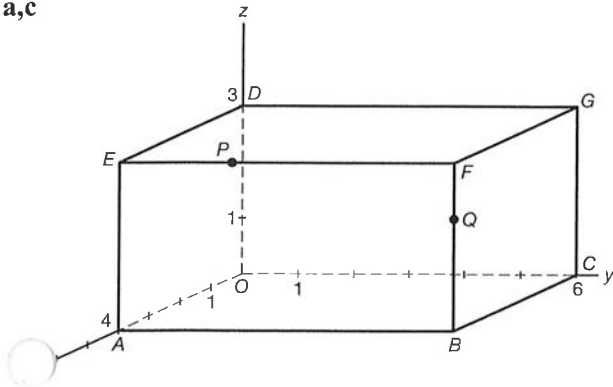
- 10 Punt A(-2, 6).
 $3 - \lambda = -2$
 $-\lambda = -5$
 $\lambda = 5$
 $\lambda = 5$ geeft (-2, 11) dus A ligt niet op k.
- Punt B(0, -7).
 $3 - \lambda = 0$
 $-\lambda = -3$
 $\lambda = 3$
 $\lambda = 3$ geeft (0, 7) dus B ligt niet op k.
- Punt C(13, -19).
 $3 - \lambda = 13$
 $-\lambda = 10$
 $\lambda = -10$
 $\lambda = -10$ geeft (13, -19) dus C ligt op k.

8.1 Coördinaten in de ruimte

Bladzijde 143

- 1 a M 4, $2\frac{1}{2}$ en 3
 b N 2, 5 en 3
 c B 4, 5 en 0
 d G 0, 5 en 3
 e E 4, 0 en 3
 f C 0, 5 en 0

2 a,c



b $O(0, 0, 0)$, $B(4, 6, 0)$, $E(4, 0, 3)$, $F(4, 6, 3)$ en $G(0, 6, 3)$

$$\text{d } \vec{m} = \frac{1}{2}(\vec{p} + \vec{r}) = \frac{1}{2}\left(\begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \\ 2 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 8 \\ 8 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 2\frac{1}{2} \end{pmatrix}$$

Dus $M(4, 4, 2\frac{1}{2})$.

Bladzijde 145

$$3 \text{ a } \vec{m} = \frac{1}{2}(\vec{c} + \vec{f}) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 6 \\ 12 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}$$

Dus $M(3, 6, 3)$.

$$\text{b } \vec{p} = \begin{pmatrix} 6 \\ 3 \\ 6 \end{pmatrix} \text{ en } \vec{q} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}$$

$$\vec{r} = \frac{1}{2}(\vec{p} + \vec{q}) = \frac{1}{2}\left(\begin{pmatrix} 6 \\ 3 \\ 6 \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 12 \\ 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 6 \\ 1\frac{1}{2} \\ 4\frac{1}{2} \end{pmatrix}$$

Dus $R(6, 1\frac{1}{2}, 4\frac{1}{2})$.

$$\text{c } d(C, Q) = |\vec{CQ}| = |\vec{q} - \vec{c}| = \left|\begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}\right| = \left|\begin{pmatrix} 6 \\ -6 \\ 3 \end{pmatrix}\right| = \sqrt{6^2 + (-6)^2 + 3^2} = \sqrt{36 + 36 + 9} = \sqrt{81} = 9$$

$$4 \text{ a } \vec{m} = \frac{1}{2}(\vec{b} + \vec{t}) = \frac{1}{2}\left(\begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \\ 8 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 9 \\ 9 \\ 8 \end{pmatrix} = \begin{pmatrix} 4\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix}$$

Dus $M(4\frac{1}{2}, 4\frac{1}{2}, 4)$.

$$\text{b } \vec{p} = \frac{1}{2}(\vec{c} + \vec{m}) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 4\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 10\frac{1}{2} \\ 10\frac{1}{2} \\ 4 \end{pmatrix} = \begin{pmatrix} 5\frac{1}{4} \\ 5\frac{1}{4} \\ 2 \end{pmatrix}$$

Dus $P(5\frac{1}{4}, 5\frac{1}{4}, 2)$.

$$\text{c } d(M, S) = |\vec{MS}| = |\vec{s} - \vec{m}| = \left|\begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 4\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix}\right| = \left|\begin{pmatrix} -1\frac{1}{2} \\ -1\frac{1}{2} \\ -4 \end{pmatrix}\right| = \sqrt{(-1\frac{1}{2})^2 + (-1\frac{1}{2})^2 + (-4)^2} = \sqrt{20\frac{1}{2}} = \frac{1}{2}\sqrt{82}$$

5 a $D(0, 0, 2)$ en $G(0, 2, 2)$, dus $M\left(\frac{0+0}{2}, \frac{0+2}{2}, \frac{2+2}{2}\right) = M(0, 1, 2)$.

$B(2, 2, 0)$ en $C(0, 2, 0)$, dus $N\left(\frac{2+0}{2}, \frac{2+2}{2}, \frac{0+0}{2}\right) = N(1, 2, 0)$.

$$\overrightarrow{MA} = \vec{a} - \vec{m} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$

$$\overrightarrow{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

b $|\overrightarrow{MA}| = \sqrt{(2)^2 + (-1)^2 + (-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$

$|\overrightarrow{MN}| = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$

Bladzijde 147

6 De vectoren $\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$ en $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ hebben dezelfde richting én de vectoren $\begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$ en $\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$ hebben dezelfde

richting, dus $\angle\left(\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}\right) = \angle\left(\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}\right)$.

De vectoren $\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$ en $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ hebben dezelfde richting maar de vectoren $\begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$ en $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ hebben niet

dezelfde richting, dus $\angle\left(\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}\right) \neq \angle\left(\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}\right)$.

7 $d(A, B) = |\overrightarrow{AB}| = |\vec{b} - \vec{a}| = \left| \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{pmatrix} \right| = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2} =$

$$\sqrt{a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 - 2a_1b_1 - 2a_2b_2 - 2a_3b_3}$$

$$d(O, A) = |\vec{a}| = \left| \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \right| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$d(O, B) = |\vec{b}| = \left| \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \right| = \sqrt{b_1^2 + b_2^2 + b_3^2}$$

De cosinusregel in driehoek OAB geeft

$$d(A, B)^2 = d(O, A)^2 + d(O, B)^2 - 2 \cdot d(O, A) \cdot d(O, B) \cdot \cos(\varphi)$$

$$a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 - 2a_1b_1 - 2a_2b_2 - 2a_3b_3 = a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 - 2 \cdot |\vec{a}| \cdot |\vec{b}| \cdot \cos(\varphi)$$

$$2 \cdot |\vec{a}| \cdot |\vec{b}| \cdot \cos(\varphi) = 2a_1b_1 + 2a_2b_2 + 2a_3b_3$$

$$\cos(\varphi) = \frac{2a_1b_1 + 2a_2b_2 + 2a_3b_3}{2 \cdot |\vec{a}| \cdot |\vec{b}|} = \frac{a_1b_1 + a_2b_2 + a_3b_3}{|\vec{a}| \cdot |\vec{b}|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

8 $\angle ENG = \angle(\overrightarrow{NE}, \overrightarrow{NG})$

$$\overrightarrow{NE} = \vec{e} - \vec{n} = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}$$

$$\overrightarrow{NG} = \vec{g} - \vec{n} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{NE}, \overrightarrow{NG})) = \frac{\begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix} \right|} = \frac{-4 + 0 + 16}{6 \cdot \sqrt{20}} = \frac{12}{6\sqrt{20}} = \frac{2}{\sqrt{20}}$$

$$\angle ENG = \angle(\overrightarrow{NE}, \overrightarrow{NG}) \approx 63,4^\circ$$

9 a $\overrightarrow{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 3 \end{pmatrix}$

$$\overrightarrow{CE} = \vec{e} - \vec{c} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{BD}, \overrightarrow{CE})) = \frac{\begin{pmatrix} -6 \\ -4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}}{\left| \begin{pmatrix} -6 \\ -4 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix} \right|} = \frac{-36 + 16 + 9}{\sqrt{61} \cdot \sqrt{61}} = \frac{-11}{61}$$

$$\angle(\overrightarrow{BD}, \overrightarrow{CE}) \approx 100,4$$

b $\angle BGD = \angle(\overrightarrow{GB}, \overrightarrow{GD})$

$$\overrightarrow{GB} = \vec{b} - \vec{g} = \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ -2 \end{pmatrix}$$

$$\overrightarrow{GD} = \vec{d} - \vec{g} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{GB}, \overrightarrow{GD})) = \frac{\begin{pmatrix} 6 \\ 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} 6 \\ 0 \\ -2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix} \right|} = \frac{0 + 0 - 2}{\sqrt{40} \cdot \sqrt{17}} = \frac{-2}{\sqrt{680}}$$

$$\angle BGD = \angle(\overrightarrow{GB}, \overrightarrow{GD}) \approx 94,4^\circ$$

Bladzijde 148

- 10 a $A(3, 3, 0)$, $B(-3, 3, 0)$, $C(-3, -3, 0)$, $D(3, -3, 0)$, $E(3, 3, 6)$, $F(-3, 3, 6)$, $G(-3, -3, 6)$, $H(3, -3, 6)$ en $T(0, 0, 10)$.

b $\overrightarrow{TH} = \vec{t} - \vec{h} = \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}$ dus een richtingsvector van de lijn HT is $\begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}$.

$T(0, 0, 10)$ ligt op de lijn HT dus een steunvector is $\begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$.

- c Het Oxy -vlak bestaat uit alle punten waarvan de z -coördinaat gelijk is aan 0.
 P ligt in dit vlak dus $z_P = 0$.

d $\left. \begin{array}{l} z_P = 0 \\ z_P = 10 - 4\lambda \end{array} \right\} \begin{array}{l} 10 - 4\lambda = 0 \\ -4\lambda = -10 \\ \lambda = 2\frac{1}{2} \end{array}$

e $\lambda = 2\frac{1}{2}$ geeft $P(7\frac{1}{2}, -7\frac{1}{2}, 0)$

Bladzijde 149

- 11 a Het Oxz -vlak bestaat uit alle punten waarvan de y -coördinaat gelijk is aan 0.
 Q ligt in dit vlak dus $y_Q = 0$.
 b Het Oyz -vlak bestaat uit alle punten waarvan de x -coördinaat gelijk is aan 0.
 De x -coördinaat van deze punten is dus gelijk aan 0.

12 a $\vec{m} = \begin{pmatrix} 4 \\ 0 \\ 1\frac{1}{2} \end{pmatrix}$ en $\vec{p} - \vec{m} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 1\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \\ 1\frac{1}{2} \end{pmatrix}$, dus $MP: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 1 \\ 1\frac{1}{2} \end{pmatrix}$.

$z_T = 0$ geeft $3 + 1\frac{1}{2}\lambda = 0$, dus $\lambda = -2$.

$\lambda = -2$ geeft $\vec{t} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} - 2 \cdot \begin{pmatrix} -4 \\ 1 \\ 1\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \\ 0 \end{pmatrix}$

Dus $T(8, -1, 0)$.

b $\vec{p} - \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$ en $\vec{q} - \vec{p} = \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix}$, dus $PQ: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix}$.

$z_U = 0$ geeft $3 - \lambda = 0$, dus $\lambda = 3$.

$\lambda = 3$ geeft $\vec{u} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + 3 \cdot \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 12 \\ 13 \\ 0 \end{pmatrix}$

Dus $U(12, 13, 0)$.

Bladzijde 150

13 a $\vec{m} = \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix}$ en $\vec{p} - \vec{m} = \begin{pmatrix} 5\frac{1}{4} \\ 5\frac{1}{4} \\ 2 \end{pmatrix} - \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix} = \begin{pmatrix} 3\frac{3}{4} \\ \frac{3}{4} \\ -2 \end{pmatrix}$, dus $MP: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3\frac{3}{4} \\ \frac{3}{4} \\ -2 \end{pmatrix}$.

$z_D = 0$ geeft $4 - 2\lambda = 0$, dus $\lambda = 2$.

$\lambda = 2$ geeft $\vec{d} = \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix} + 2 \cdot \begin{pmatrix} 3\frac{3}{4} \\ \frac{3}{4} \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ 0 \end{pmatrix}$

Dus $D(9, 6, 0)$.

$$\text{b } \vec{q} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} \text{ en } \vec{p} - \vec{q} = \begin{pmatrix} 5\frac{1}{4} \\ 5\frac{1}{4} \\ 2 \end{pmatrix} - \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} = \begin{pmatrix} 1\frac{1}{2} \\ 3 \\ -4 \end{pmatrix}, \text{ dus } QP: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1\frac{1}{2} \\ 3 \\ -4 \end{pmatrix}.$$

$$x_E = 0 \text{ geeft } 3\frac{3}{4} + 1\frac{1}{2}\lambda = 0, \text{ dus } \lambda = -2\frac{1}{2}.$$

$$\lambda = -2\frac{1}{2} \text{ geeft } \vec{e} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} - 2\frac{1}{2} \cdot \begin{pmatrix} 1\frac{1}{2} \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ -5\frac{1}{4} \\ 16 \end{pmatrix}$$

$$\text{Dus } E(0, -5\frac{1}{4}, 16).$$

$$\text{c } \vec{q} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} \text{ en } \vec{m} - \vec{q} = \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix} - \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} = \begin{pmatrix} -2\frac{1}{4} \\ 2\frac{1}{4} \\ -2 \end{pmatrix}, \text{ dus } QM: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -2\frac{1}{4} \\ 2\frac{1}{4} \\ -2 \end{pmatrix}.$$

$$y_F = 0 \text{ geeft } 2\frac{1}{4} + 2\frac{1}{4}\lambda = 0, \text{ dus } \lambda = -1.$$

$$\lambda = -1 \text{ geeft } \vec{f} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} - \begin{pmatrix} -2\frac{1}{4} \\ 2\frac{1}{4} \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 8 \end{pmatrix}$$

$$\text{Dus } F(6, 0, 8).$$

8.2 Vergelijkingen van vlakken

Bladzijde 152

- 14 a $A(3, 0, 0)$ invullen geeft $20 \cdot 3 + 15 \cdot 0 + 12 \cdot 0 = 60$ klopt.
 $B(0, 4, 0)$ invullen geeft $20 \cdot 0 + 15 \cdot 4 + 12 \cdot 0 = 60$ klopt.
 $C(0, 0, 5)$ invullen geeft $20 \cdot 0 + 15 \cdot 0 + 12 \cdot 5 = 60$ klopt.

b Het linker- en rechterlid delen door 60 geeft $\frac{20x}{60} + \frac{15y}{60} + \frac{12z}{60} = \frac{60}{60}$ ofwel $\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 1$.

- c Je kunt de coördinaten van de snijpunten met de assen herkennen aan de getallen in de noemers.

Bladzijde 154

- 15 a Voor het snijpunt met de x -as geldt $y = 0$ en $z = 0$.

$$\text{Dit geeft } ax = d$$

$$x = \frac{d}{a}$$

$$\text{Dus het snijpunt is } A\left(\frac{d}{a}, 0, 0\right).$$

$$\text{Voor het snijpunt met de } y\text{-as geldt } x = 0 \text{ en } z = 0.$$

$$\text{Dit geeft } by = d$$

$$y = \frac{d}{b}$$

$$\text{Dus het snijpunt is } B\left(0, \frac{d}{b}, 0\right).$$

$$\text{Voor het snijpunt met de } z\text{-as geldt } x = 0 \text{ en } y = 0.$$

$$\text{Dit geeft } cz = d$$

$$z = \frac{d}{c}$$

$$\text{Dus het snijpunt is } C\left(0, 0, \frac{d}{c}\right).$$

$$\text{b } \overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 0 \\ \frac{d}{b} \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{d}{a} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{d}{a} \\ \frac{d}{b} \\ 0 \end{pmatrix}$$

$$\overrightarrow{AB} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -\frac{d}{a} \\ \frac{d}{b} \\ 0 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = -d + d + 0 = 0, \text{ dus } \overrightarrow{AB} \perp \begin{pmatrix} a \\ b \\ c \end{pmatrix}.$$

$$\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 0 \\ 0 \\ \frac{d}{c} \end{pmatrix} - \begin{pmatrix} \frac{d}{a} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{d}{a} \\ 0 \\ \frac{d}{c} \end{pmatrix}$$

$$\overrightarrow{AC} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -\frac{d}{a} \\ 0 \\ \frac{d}{c} \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = -d + 0 + d = 0, \text{ dus } \overrightarrow{AC} \perp \begin{pmatrix} a \\ b \\ c \end{pmatrix}.$$

c $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ staat loodrecht op $\overrightarrow{AB}$ en op $\overrightarrow{AC}$ dus ook loodrecht op de snijdende lijnen AB en AC .

Dan moet $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ loodrecht op V staan omdat AB en AC in V liggen, en is $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ dus een normaalvector van V .

Bladzijde 155

- 16 a ACD snijdt de x -as in $A(4, 0, 0)$, de y -as in $C(0, 4, 0)$ en de z -as in $D(0, 0, 4)$,

$$\text{dus } ACD: \frac{x}{4} + \frac{y}{4} + \frac{z}{4} = 1 \text{ ofwel } x + y + z = 4.$$

$$\text{Dus } \vec{n}_{ACD} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

- b Van C naar M ga je 4 in de x -richting en -2 in de y -richting.

Ga je vanuit $M(4, 2, 0)$ opnieuw 4 in de x -richting en -2 in de y -richting, dan kom je in het punt $(8, 0, 0)$.

De lijn CM en dus ook het vlak CMD snijdt de x -as in het punt $(8, 0, 0)$.

Dus CMD snijdt de x -as in $(8, 0, 0)$, de y -as in $C(0, 4, 0)$ en de z -as in $D(0, 0, 4)$,

$$\text{dus } CMD: \frac{x}{8} + \frac{y}{4} + \frac{z}{4} = 1 \text{ ofwel } CMD: x + 2y + 2z = 8.$$

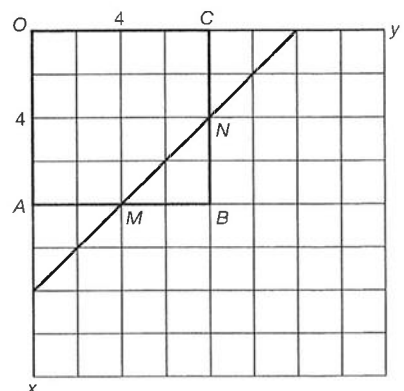
$$\text{Dus } \vec{n}_{CMD} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}.$$

- c Zie de figuur: MN snijdt de x -as in $(6, 0, 0)$ en de y -as in $(0, 6, 0)$.

MND snijdt de x -as in $(6, 0, 0)$, de y -as in $(0, 6, 0)$ en de z -as in $D(0, 0, 4)$,

$$\text{dus } MND: \frac{x}{6} + \frac{y}{6} + \frac{z}{4} = 1 \text{ ofwel } MND: 2x + 2y + 3z = 12.$$

$$\text{Dus } \vec{n}_{MND} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}.$$



d MNF snijdt de x -as in $(6, 0, 0)$ en de y -as in $(0, 6, 0)$, dus MNF: $\frac{x}{6} + \frac{y}{6} + \frac{z}{r} = 1$.

$$\begin{aligned} F(4, 4, 4) \text{ invullen in } \frac{x}{6} + \frac{y}{6} + \frac{z}{r} = 1 \text{ geeft } \frac{4}{6} + \frac{4}{6} + \frac{4}{r} &= 1 \\ \frac{2}{3} + \frac{2}{3} + \frac{4}{r} &= 1 \\ \frac{4}{r} &= -\frac{1}{3} \\ r &= -12 \end{aligned}$$

$$\text{MNF: } \frac{x}{6} + \frac{y}{6} - \frac{z}{12} = 1 \text{ ofwel MNF: } 2x + 2y - z = 12$$

$$\text{Dus } \vec{n}_{\text{MNF}} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}.$$

$$\text{e } \vec{d} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \text{ en } \vec{b} - \vec{d} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -4 \end{pmatrix}, \text{ dus BD: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}.$$

Substitutie van $x = \lambda, y = \lambda$ en $z = 4 - \lambda$ in $2x + 2y - z = 12$ geeft

$$\lambda + 2\lambda - (4 - \lambda) = 12$$

$$2\lambda + 2\lambda - 4 + \lambda = 12$$

$$5\lambda = 16$$

$$\lambda = 3\frac{1}{5}$$

Dus het snijpunt is $(3\frac{1}{5}, 3\frac{1}{5}, \frac{4}{5})$.

17 a ACD snijdt de x -as in $A(5, 0, 0)$, de y -as in $C(0, 6, 0)$ en de z -as in $D(0, 0, 4)$,

$$\text{dus ACD: } \frac{x}{5} + \frac{y}{6} + \frac{z}{4} = 1 \text{ ofwel } 12x + 10y + 15z = 60.$$

b Van C naar M ga je 5 in de x -richting en -3 in de y -richting.

Ga je vanuit $M(5, 3, 0)$ opnieuw 5 in de x -richting en -3 in de y -richting, dan kom je in het punt $(10, 0, 0)$.

De lijn CM en dus ook het vlak CMD snijdt de x -as in het punt $(10, 0, 0)$.

Dus CMD snijdt de x -as in $(10, 0, 0)$, de y -as in $C(0, 6, 0)$ en de z -as in $D(0, 0, 4)$,

$$\text{dus CMD: } \frac{x}{10} + \frac{y}{6} + \frac{z}{4} = 1 \text{ ofwel CMD: } 6x + 10y + 15z = 60.$$

$$\text{Dus } \vec{n}_{\text{CMD}} = \begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix}.$$

$$\text{c } \vec{d} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \text{ en } \vec{b} - \vec{d} = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ -4 \end{pmatrix}, \text{ dus BD: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 6 \\ -4 \end{pmatrix}.$$

ACF snijdt de x -as in $(5, 0, 0)$, de y -as in $(0, 6, 0)$ en de z -as in $(0, 0, -4)$,

$$\text{dus ACF: } \frac{x}{5} + \frac{y}{6} - \frac{z}{4} = 1 \text{ ofwel ACF: } 12x + 10y - 15z = 60.$$

Substitutie van $x = 5\lambda, y = 6\lambda$ en $z = 4 - 4\lambda$ in $12x + 10y - 15z = 60$ geeft

$$12 \cdot 5\lambda + 10 \cdot 6\lambda - 15(4 - 4\lambda) = 60$$

$$60\lambda + 60\lambda - 60 + 60\lambda = 60$$

$$180\lambda = 120$$

$$\lambda = \frac{2}{3}$$

Dus het snijpunt is $(3\frac{1}{3}, 4, 1\frac{1}{3})$.

- 18 a** ACP snijdt de x -as in $A(4, 0, 0)$ en de y -as in $C(0, 6, 0)$, dus ACP : $\frac{x}{4} + \frac{y}{6} + \frac{z}{r} = 1$.

$$P(1, 1\frac{1}{2}, 2\frac{1}{2}) \text{ invullen in } \frac{x}{4} + \frac{y}{6} + \frac{z}{r} = 1 \text{ geeft } \frac{1}{4} + \frac{1\frac{1}{2}}{6} + \frac{2\frac{1}{2}}{r} = 1$$

$$\frac{1}{4} + \frac{1}{4} + \frac{2\frac{1}{2}}{r} = 1$$

$$\frac{2\frac{1}{2}}{r} = \frac{1}{2}$$

$$r = 5$$

$$ACP: \frac{x}{4} + \frac{y}{6} + \frac{z}{5} = 1 \text{ ofwel } ACP: 15x + 10y + 12z = 60.$$

- b** MNP snijdt de x -as in $(6, 0, 0)$ en de y -as in $(0, 9, 0)$, dus MNP : $\frac{x}{6} + \frac{y}{9} + \frac{z}{r} = 1$.

$$P(1, 1\frac{1}{2}, 2\frac{1}{2}) \text{ invullen in } \frac{x}{6} + \frac{y}{9} + \frac{z}{r} = 1 \text{ geeft } \frac{1}{6} + \frac{1\frac{1}{2}}{9} + \frac{2\frac{1}{2}}{r} = 1$$

$$\frac{1}{6} + \frac{1}{6} + \frac{2\frac{1}{2}}{r} = 1$$

$$\frac{2\frac{1}{2}}{r} = \frac{2}{3}$$

$$2r = 7\frac{1}{2}$$

$$r = 3\frac{3}{4}$$

$$MNP: \frac{x}{6} + \frac{y}{9} + \frac{z}{3\frac{3}{4}} = 1 \text{ ofwel } MNP: 15x + 10y + 24z = 90.$$

$$\text{Dus } \vec{n}_{MNP} = \begin{pmatrix} 15 \\ 10 \\ 24 \end{pmatrix}.$$

- c** ANT snijdt de x -as in $(4, 0, 0)$ en de y -as in $(0, 12, 0)$, dus ANT : $\frac{x}{4} + \frac{y}{12} + \frac{z}{r} = 1$.

$$T(2, 3, 5) \text{ invullen in } \frac{x}{4} + \frac{y}{12} + \frac{z}{r} = 1 \text{ geeft } \frac{2}{4} + \frac{3}{12} + \frac{5}{r} = 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{5}{r} = 1$$

$$\frac{5}{r} = \frac{1}{4}$$

$$r = 20$$

$$ANT: \frac{x}{4} + \frac{y}{12} + \frac{z}{20} = 1 \text{ ofwel } ANT: 15x + 5y + 3z = 60.$$

$$\text{Dus } \vec{r}_l = \vec{n}_{ANT} = \begin{pmatrix} 15 \\ 5 \\ 3 \end{pmatrix} \text{ en } l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1\frac{1}{2} \\ 2\frac{1}{2} \end{pmatrix} + \lambda \begin{pmatrix} 15 \\ 5 \\ 3 \end{pmatrix}.$$

$$l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1\frac{1}{2} \\ 2\frac{1}{2} \end{pmatrix} + \lambda \begin{pmatrix} 15 \\ 5 \\ 3 \end{pmatrix} \text{ snijden met } ANT: 15x + 5y + 3z = 60 \text{ geeft}$$

$$15(1 + 15\lambda) + 5(1\frac{1}{2} + 5\lambda) + 3(2\frac{1}{2} + 3\lambda) = 60$$

$$15 + 225\lambda + 7\frac{1}{2} + 25\lambda + 7\frac{1}{2} + 9\lambda = 60$$

$$259\lambda = 30$$

$$\lambda = \frac{30}{259}$$

$$\text{Het snijpunt is } (2\frac{191}{259}, 2\frac{41}{518}, 2\frac{439}{518}).$$

19 a $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} = a_1(a_2b_3 - a_3b_2) + a_2(a_3b_1 - a_1b_3) + a_3(a_1b_2 - a_2b_1) =$

$$a_1a_2b_3 - a_1a_3b_2 + a_2a_3b_1 - a_1a_2b_3 + a_1a_3b_2 - a_2a_3b_1 = 0$$

b $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \cdot \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} = b_1(a_2b_3 - a_3b_2) + b_2(a_3b_1 - a_1b_3) + b_3(a_1b_2 - a_2b_1) =$

$$a_2b_1b_3 - a_3b_1b_2 + a_3b_1b_2 - a_1b_2b_3 + a_1b_2b_3 - a_2b_1b_3 = 0$$

Bladzijde 157

20 a $\begin{matrix} 3 & 1 \\ 1 & \times & 2 \\ 2 & \times & 3 \\ 3 & \times & 1 \\ 1 & \times & 2 \\ 2 & & 3 \end{matrix}$

$$1 \cdot 3 - 2 \cdot 2 = -1$$

$$2 \cdot 1 - 3 \cdot 3 = -7$$

$$3 \cdot 2 - 1 \cdot 1 = 5$$

$$\text{Dus } \vec{a} \times \vec{b} = \begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix}.$$

b $\begin{matrix} 3 & 12 \\ 1 & \times & 4 \\ 2 & \times & 8 \\ 3 & \times & 12 \\ 1 & \times & 4 \\ 2 & & 8 \end{matrix}$

$$1 \cdot 8 - 2 \cdot 4 = 0$$

$$2 \cdot 12 - 3 \cdot 8 = 0$$

$$3 \cdot 4 - 1 \cdot 12 = 0$$

$$\text{Dus } \vec{a} \times \vec{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

c $\begin{matrix} 1 & -3 \\ 2 & \times & -1 \\ 3 & \times & -2 \\ 1 & \times & -3 \\ 2 & \times & -1 \\ 3 & & -2 \end{matrix}$

$$2 \cdot -2 - 3 \cdot -1 = -1$$

$$3 \cdot -3 - 1 \cdot -2 = -7$$

$$1 \cdot -1 - 2 \cdot -3 = 5$$

$$\text{Dus } \vec{b} \times \vec{d} = \begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix}.$$

d $\begin{matrix} 1 & 2 \\ 2 & \times & 4 \\ 3 & \times & 6 \\ 1 & \times & 2 \\ 2 & \times & 4 \\ 3 & & 6 \end{matrix}$

$$2 \cdot 6 - 3 \cdot 4 = 0$$

$$3 \cdot 2 - 1 \cdot 6 = 0$$

$$1 \cdot 4 - 2 \cdot 2 = 0$$

$$\text{Dus } \vec{b} \times \vec{e} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

e $\begin{matrix} 1 & 3 \\ 2 & \times & 1 \\ 3 & \times & 2 \\ 1 & \times & 3 \\ 2 & \times & 1 \\ 3 & & 2 \end{matrix}$

$$2 \cdot 2 - 3 \cdot 1 = 1$$

$$3 \cdot 3 - 1 \cdot 2 = 7$$

$$1 \cdot 1 - 2 \cdot 3 = -5$$

$$\text{Dus } \vec{b} \times \vec{d} = \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix}.$$

f $\begin{matrix} -3 & 1 \\ -1 & \times & 2 \\ -2 & \times & 3 \\ -3 & \times & 1 \\ -1 & \times & 2 \\ -2 & & 3 \end{matrix}$

$$-1 \cdot 3 - -2 \cdot 2 = 1$$

$$-2 \cdot 1 - -3 \cdot 3 = 7$$

$$-3 \cdot 2 - -1 \cdot 1 = -5$$

$$\text{Dus } \vec{d} \times \vec{b} = \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix}.$$

21 a $\vec{a} \cdot \vec{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3$

$$\vec{b} \cdot \vec{a} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = b_1a_1 + b_2a_2 + b_3a_3 = a_1b_1 + a_2b_2 + a_3b_3$$

Dus $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ is waar voor elk tweetal vectoren.

b Bij 20c vond je $\vec{b} \times \vec{d} = \begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix}$.

Bij 20f vond je $\vec{d} \times \vec{b} = \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix}$.

Dus $\vec{d} \times \vec{b} = \vec{b} \times \vec{d}$ is niet waar voor elk tweetal vectoren.

Bladzijde 158

$$22 \text{ a } \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \cdot 1 - 2 \cdot -2 \\ 2 \cdot 4 - 3 \cdot 1 \\ 3 \cdot -2 - -1 \cdot 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$$

$$\text{b } \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \cdot 1 - 1 \cdot 0 \\ 1 \cdot 2 - 0 \cdot 1 \\ 0 \cdot 0 - 5 \cdot 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ -10 \end{pmatrix}$$

$$\text{c } \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \cdot 0 - 1 \cdot 1 \\ 1 \cdot 0 - 1 \cdot 0 \\ 1 \cdot 1 - 1 \cdot 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\text{d } \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \cdot 0 - 0 \cdot 1 \\ 0 \cdot 0 - 1 \cdot 0 \\ 1 \cdot 1 - 0 \cdot 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$23 \quad \vec{r}_k \times \vec{r}_m = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \cdot 1 - 2 \cdot 1 \\ 2 \cdot 1 - 2 \cdot 1 \\ 2 \cdot 1 - -1 \cdot 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}, \text{ dus } \vec{r}_l = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$

$$l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + v \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$24 \text{ a } C(0, 2, 0) \text{ en } G(0, 2, 2), \text{ dus } M(0, 2, 1).$$

$$\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

$$\overrightarrow{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$

$$\overrightarrow{AB} \times \overrightarrow{BM} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 - 0 \cdot 0 \\ 0 \cdot -2 - 0 \cdot 1 \\ 0 \cdot 0 - 2 \cdot -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$$

De vector $\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ staat loodrecht op $\overrightarrow{AB}$ en $\overrightarrow{BM}$, dus de vector $\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ staat loodrecht op het vlak waar

$\overrightarrow{AB}$ en $\overrightarrow{BM}$ richtingsvectoren van zijn. Dat is het vlak ABM .

Dus de vector $\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ staat loodrecht op het vlak ABM en dus $\vec{n}_{ABM} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$.

$$\text{b } \vec{n}_{ABM} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}, \text{ dus } ABM: 2x + 4z = d.$$

$A(2, 0, 0)$ invullen geeft $d = 4$.

Dus $ABM: 2x + 4z = 4$ ofwel $x + 2z = 2$.

Bladzijde 160

25 a $\overrightarrow{EG} = \vec{g} - \vec{e} = \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$$\overrightarrow{EP} = \vec{p} - \vec{e} = \begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$$

$$\vec{n}_{EGP} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ -5 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}$$

$$\left. \begin{array}{l} EGP: 2x + 2y + 5z = d \\ \text{door } E(5, 0, 5) \end{array} \right\} d = 10 + 0 + 25 = 35$$

$$\text{Dus } EGP: 2x + 2y + 5z = 35.$$

b $AD \perp OEF$, dus $\overrightarrow{AD} = \vec{n}_{OEF}$.

$$\overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 5 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\left. \begin{array}{l} OEF: x - z = d \\ \text{door } O(0, 0, 0) \end{array} \right\} d = 0$$

$$\text{Dus } OEF: x - z = 0.$$

c $\overrightarrow{DE} = \vec{e} - \vec{d} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\overrightarrow{EP} = \vec{p} - \vec{e} = \begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$$

$$\vec{n}_{DEP} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix}$$

$$\left. \begin{array}{l} DEP: 2y + 5z = d \\ \text{door } D(0, 0, 5) \end{array} \right\} d = 25$$

$$\text{Dus } DEP: 2y + 5z = 25.$$

26 a $BD \perp ACT$, dus $\overrightarrow{BD} = \vec{n}_{ACT}$.

$$\overrightarrow{BD} = \vec{d} - \vec{b} = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\left. \begin{array}{l} ACT: x + y = d \\ \text{door } A(2, -2, 0) \end{array} \right\} d = 2 - 2 = 0$$

$$\text{Dus } ACT: x + y = 0.$$

b $C(-2, 2, 0)$ en $T(0, 0, 6)$, dus $M(-1, 1, 3)$.

$$\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\overrightarrow{AM} = \vec{m} - \vec{a} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{n}_{ABM} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$ABM: x + z = d$
 door $B(2, 2, 0)$ } $d = 2$
 Dus $ABM: x + z = 2$.

c $\overrightarrow{BD} = \vec{d} - \vec{b} = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$$\overrightarrow{BM} = \vec{m} - \vec{b} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix}$$

$$\vec{n}_{BDM} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$$

$BDM: 3x - 3y + 2z = d$
 door $B(2, 2, 0)$ } $d = 6 - 6 + 0 = 0$
 Dus $BDM: 3x - 3y + 2z = 0$.

27 a $\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 6 \\ 0 \end{pmatrix}$

$$\overrightarrow{CG} = \vec{g} - \vec{c} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{n}_{ACG} = \begin{pmatrix} -5 \\ 6 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \\ 0 \end{pmatrix}$$

$ACG: 6x + 5y = d$
 door $A(5, 0, 0)$ } $d = 30$
 Dus $ACG: 6x + 5y = 30$.

b $\overrightarrow{BE} = \vec{e} - \vec{b} = \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$

$$\overrightarrow{BG} = \vec{g} - \vec{b} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 4 \end{pmatrix}$$

$$\vec{n}_{BEG} = \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} \times \begin{pmatrix} -5 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -12 \\ -10 \\ 15 \end{pmatrix} \triangleq \begin{pmatrix} 12 \\ 10 \\ 15 \end{pmatrix}$$

8.3 Hoeken in de ruimte

Bladzijde 162

28 a $\overrightarrow{MA} = \vec{a} - \vec{m} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$

$$\overrightarrow{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{MA}, \overrightarrow{MN})) = \frac{\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \right|} = \frac{2 - 1 + 4}{\sqrt{9} \cdot \sqrt{6}} = \frac{5}{3\sqrt{6}}$$

$$\angle(\overrightarrow{MA}, \overrightarrow{MN}) \approx 47,1^\circ$$

b De hoek tussen de vectoren is een scherpe hoek waarvan de benen langs de lijnen AM en MN liggen.

Dit is juist de hoek tussen de lijnen AM en MN .

Bladzijde 163

29 a $\vec{r}_{CE} = \vec{e} - \vec{c} = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$$\vec{r}_{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$\cos(\angle(CE, MN)) = \frac{\left| \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \right|} = \frac{|1 - 1 - 2|}{\sqrt{3} \cdot \sqrt{6}} = \frac{2}{\sqrt{18}}$$

$$\text{Dus } \angle(CE, MN) \approx 61,9^\circ.$$

b $\vec{r}_{AG} = \vec{g} - \vec{a} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$$\vec{r}_{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$$

$$\cos(\angle(AG, BM)) = \frac{\left| \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} \right|}{\left| \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} \right|} = \frac{|2 - 1 + 2|}{\sqrt{3} \cdot 3} = \frac{1}{\sqrt{3}}$$

$$\text{Dus } \angle(AG, CE) \approx 54,7^\circ.$$

c $\angle AMC = \angle(\overrightarrow{MA}, \overrightarrow{MC})$

$$\overrightarrow{MA} = \vec{a} - \vec{m} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$

$$\overrightarrow{MC} = \vec{c} - \vec{m} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{MA}, \overrightarrow{MC})) = \frac{\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \right|} = \frac{0 - 1 + 4}{3 \cdot \sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\angle AMC = \angle(\overrightarrow{MA}, \overrightarrow{MC}) \approx 63,4^\circ$$

Bladzijde 164

- 30 a $D(0, 0, 3)$ en $G(0, 4, 2)$, dus $M(0, 2, 2\frac{1}{2})$.

$$\overrightarrow{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 0 \\ 2 \\ 2\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \\ 2\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} -12 \\ 4 \\ 5 \end{pmatrix}$$

$$\overrightarrow{CE} = \vec{e} - \vec{c} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}$$

$$\cos(\angle(AM, CE)) = \frac{\begin{vmatrix} -12 \\ 4 \\ 5 \end{vmatrix} \cdot \begin{vmatrix} 6 \\ -4 \\ 3 \end{vmatrix}}{\left| \begin{pmatrix} -12 \\ 4 \\ 5 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix} \right|} = \frac{-72 - 16 + 15}{\sqrt{185} \cdot \sqrt{61}} = \frac{73}{\sqrt{11285}}$$

Dus $\angle(AM, CE) \approx 46,6^\circ$.

b $\overrightarrow{AF} = \vec{f} - \vec{a} = \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$

$$\overrightarrow{EG} = \vec{g} - \vec{e} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}$$

$$\cos(\angle(AF, EG)) = \frac{\begin{vmatrix} 0 \\ 2 \\ 1 \end{vmatrix} \cdot \begin{vmatrix} -6 \\ 4 \\ -1 \end{vmatrix}}{\left| \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} \right|} = \frac{0 + 8 - 1}{\sqrt{5} \cdot \sqrt{53}} = \frac{7}{\sqrt{265}}$$

Dus $\angle(AF, EG) \approx 64,5^\circ$.

$$\text{c } \angle EMC = \angle(\overrightarrow{ME}, \overrightarrow{MC})$$

$$\overrightarrow{ME} = \vec{e} - \vec{m} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 2\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ \frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}$$

$$\overrightarrow{MC} = \vec{c} - \vec{m} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 2\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -2\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 4 \\ -5 \end{pmatrix}$$

$$\cos(\angle(AM, CE)) = \frac{\begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 4 \\ -5 \end{pmatrix}}{\left| \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 0 \\ 4 \\ -5 \end{pmatrix} \right|} = \frac{0 - 16 - 5}{\sqrt{161} \cdot \sqrt{41}} = \frac{-21}{\sqrt{6601}}$$

$$\text{Dus } \angle EMC = \angle(\overrightarrow{ME}, \overrightarrow{MC}) \approx 105,0^\circ.$$

31 a $C(0, 4, 0)$ en $T(2, 2, 6)$, dus $M(1, 3, 3)$.

$$\overrightarrow{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\overrightarrow{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix}$$

$$\cos(\angle(AM, BM)) = \frac{\left| \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix} \right|}{\left| \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix} \right|} = \frac{|3 - 1 + 3|}{\sqrt{3} \cdot \sqrt{19}} = \frac{5}{\sqrt{57}}$$

$$\text{Dus } \angle(AM, BM) \approx 48,5^\circ.$$

$$\text{b } \vec{r}_{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{r}_{BT} = \vec{t} - \vec{b} = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$$

$$\cos(\angle(AM, BT)) = \frac{\left| \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix} \right|}{\left| \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix} \right|} = \frac{|1 - 1 + 3|}{\sqrt{3} \cdot \sqrt{11}} = \frac{3}{\sqrt{33}}$$

$$\text{Dus } \angle(AM, BT) \approx 58,5^\circ.$$

c $\angle OMB = \angle(\overrightarrow{MO}, \overrightarrow{MB})$

$$\overrightarrow{MO} = \vec{o} - \vec{m} = \begin{pmatrix} -1 \\ -3 \\ -3 \end{pmatrix}$$

$$\overrightarrow{MB} = \vec{b} - \vec{m} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -3 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{MO}, \overrightarrow{MB})) = \frac{\begin{pmatrix} -1 \\ -3 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ -3 \end{pmatrix}}{\left| \begin{pmatrix} -1 \\ -3 \\ -3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ 1 \\ -3 \end{pmatrix} \right|} = \frac{-3 - 3 + 9}{\sqrt{19} \cdot \sqrt{19}} = \frac{3}{19}$$

$$\angle OMB = \angle(\overrightarrow{MO}, \overrightarrow{MB}) \approx 80,9^\circ$$

32 a $\vec{r}_{OT} = \vec{r} = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ dus $OT: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$.

$\lambda = 0$ geeft $O(0, 0, 0)$ en $\lambda = 2$ geeft $T(2, 2, 6)$.

Dus $P(\lambda, \lambda, 3\lambda)$ met $0 \leq \lambda \leq 2$ is voor elke λ een punt op het lijnstuk OT .

b $\vec{r}_{BP} = \vec{p} - \vec{b} = \begin{pmatrix} \lambda \\ \lambda \\ 3\lambda \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda - 4 \\ \lambda - 4 \\ 3\lambda \end{pmatrix}$ en $\vec{r}_{OT} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$.

$BP \perp OT$, dus $\vec{r}_{BP} \cdot \vec{r}_{OT} = 0$

$$\begin{pmatrix} \lambda - 4 \\ \lambda - 4 \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = 0$$

$$\lambda - 4 + \lambda - 4 + 9\lambda = 0$$

$$11\lambda = 8$$

$$\lambda = \frac{8}{11}$$

Dus $P(\frac{8}{11}, \frac{8}{11}, 2\frac{2}{11})$.

c De hoek tussen BP en OT is maximaal 90° voor $P(\frac{8}{11}, \frac{8}{11}, 2\frac{2}{11})$.

De hoek tussen BP en OT is minimaal als $P = O$ of als $P = T$.

Bereken dus $\angle BOT$ en $\angle BTO$.

$$\tan(\angle BOT) = \frac{TS}{OS} = \frac{6}{\sqrt{8}}, \text{ dus } \angle BOT \approx 64,8^\circ.$$

$$\angle BTO = 180^\circ - 2\angle BOT \approx 50,5^\circ$$

Dus $\angle(BP, OT)$ kan alle waarden aannemen van $50,5^\circ$ tot en met 90° .

d $\angle(BP, OT) = 55^\circ$

$$\cos(55^\circ) = \frac{\left| \begin{pmatrix} \lambda - 4 \\ \lambda - 4 \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \right|}{\left| \begin{pmatrix} \lambda - 4 \\ \lambda - 4 \\ 3\lambda \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \right|}$$

$$\cos(55^\circ) = \frac{|\lambda - 4 + \lambda - 4 + 9\lambda|}{\sqrt{(\lambda - 4)^2 + (\lambda - 4)^2 + 9\lambda^2} \cdot \sqrt{11}}$$

$$\cos(55^\circ) = \frac{|11\lambda - 8|}{\sqrt{22(\lambda - 4)^2 + 99\lambda^2}}$$

Voer in $y_1 = \frac{\text{abs}(11x - 8)}{\sqrt{22(x - 4)^2 + 99x^2}}$ en $y_2 = \cos(55^\circ)$.

Intersect geeft $x = 1,8075\dots$

Dus $P(1,81; 1,81; 5,42)$.

Bladzijde 166

- 33 a Lijn n is een normaal van ACD .

$$\vec{r}_{OF} = \vec{f} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \text{ en } \vec{r}_n = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}.$$

$$\cos(\angle(OF, n)) = \frac{\left| \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix} \right|}{\left| \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix} \right|} = \frac{|12 + 12 + 12|}{\sqrt{29} \cdot \sqrt{61}} = \frac{36}{\sqrt{1769}}, \text{ dus } \angle(OF, n) \approx 31,1^\circ.$$

Dus $\angle(OF, ACD) \approx 90^\circ - 31,1^\circ = 58,9^\circ$.

b $\vec{r}_{CE} = \vec{e} - \vec{c} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix}$

Lijn n is een normaal van OBG .

$$\vec{r}_{OB} = \vec{b} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} \text{ en } \vec{r}_{OG} = \vec{g} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{OBG} = \vec{r}_{OB} \times \vec{r}_{OG} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \\ 12 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}$$

$$\cos(\angle(CE, n)) = \frac{\left| \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix} \right|}{\left| \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix} \right|} = \frac{|12 + 12 + 12|}{\sqrt{29} \cdot \sqrt{61}} = \frac{36}{\sqrt{1769}}, \text{ dus } \angle(CE, n) \approx 31,1^\circ.$$

Dus $\angle(CE, OBG) \approx 90^\circ - 31,1^\circ = 58,9^\circ$.

- 34 a Kies het assenstelsel zo, dat je krijgt $A(4, 0, 0)$, $C(0, 4, 0)$ en $D(0, 0, 4)$.

$$\vec{r}_{AG} = \vec{g} - \vec{a} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

Lijn n is een normaal van EBG .

$$OF \perp EBG, \text{ dus } \vec{r}_n = \vec{OF} = \vec{f} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

$$\cos(\angle(AG, n)) = \frac{\left| \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right|}{\left| \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right|} = \frac{|-1 + 1 + 1|}{\sqrt{3} \cdot \sqrt{3}} = \frac{1}{3}, \text{ dus } \angle(AG, n) \approx 70,5^\circ.$$

Dus $\angle(AG, EBG) \approx 90^\circ - 70,5^\circ = 19,5^\circ$.

$$\text{b } \vec{r}_{DM} = \vec{m} - \vec{d} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

Lijn n is een normaal van ACD .

$$OF \perp ACD, \text{ dus } \vec{r}_n = \overrightarrow{OF} = \vec{f} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

$$\cos(\angle(DM, n)) = \frac{\left| \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right|} = \frac{|2 + 1 - 2|}{3 \cdot \sqrt{3}} = \frac{1}{3\sqrt{3}}, \text{ dus } \angle(DM, n) \approx 78,9^\circ.$$

Dus $\angle(DM, ACD) \approx 90^\circ - 78,9^\circ = 11,1^\circ$.

$$\text{c } \vec{r}_{GM} = \vec{m} - \vec{g} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$

Lijn n is een normaal van CDM .

$$\vec{r}_{CD} = \vec{d} - \vec{c} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$\vec{r}_{CM} = \vec{m} - \vec{c} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{CDM} = \vec{r}_{CD} \times \vec{r}_{CM} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$\cos(\angle(GM, n)) = \frac{\left| \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right|} = \frac{|2 - 2 - 4|}{3 \cdot 3} = \frac{4}{9}, \text{ dus } \angle(GM, n) \approx 63,6^\circ.$$

Dus $\angle(GM, CDM) \approx 90^\circ - 63,6^\circ = 26,4^\circ$.

- 35 Voor de hellingshoek φ geldt $\cos(\varphi) = \frac{3,0}{3,8}$, dus $\varphi \approx 37,9^\circ$.

Bladzijde 167

- 36 Het vlak MND snijdt de assen in $(6, 0, 0)$, $(0, 6, 0)$ en $(0, 0, 4)$.

$$MND: \frac{x}{6} + \frac{y}{6} + \frac{z}{4} = 1 \text{ ofwel } MND: 2x + 2y + 3z = 12$$

$$\text{Dus } \vec{r}_m = \vec{n}_{MND} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}.$$

- 37 a Lijn m is een normaal van MNF en lijn n is een normaal van OND .

$$\vec{r}_m = \vec{n}_{MNF} = \overrightarrow{MN} \times \overrightarrow{MF} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{OND} = \overrightarrow{ON} \times \overrightarrow{OD} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 16 \\ -8 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \right|} = \frac{|4 - 2 + 0|}{3 \cdot \sqrt{5}} = \frac{2}{3\sqrt{5}}$$

Dus $\angle(MNF, OND) = \angle(m, n) \approx 72,7^\circ$.

- b Lijn m is een normaal van MND en lijn n is een normaal van OMD .

$$\vec{r}_m = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \text{ en } \vec{r}_n = \vec{n}_{OMD} = \overrightarrow{OM} \times \overrightarrow{OD} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ -16 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \right|} = \frac{|2 - 4 + 0|}{\sqrt{17} \cdot \sqrt{5}} = \frac{2}{\sqrt{85}}$$

Dus $\angle(MND, OMD) = \angle(m, n) \approx 77,5^\circ$.

Bladzijde 168

- 38 a Lijn m is een normaal van AMN en lijn n is een normaal van OAM .

MN snijdt de y -as in $(0, 9, 0)$ en de z -as in $(0, 0, 6)$.

$$AMN: \frac{x}{5} + \frac{y}{9} + \frac{z}{6} = 1$$

$$18x + 10y + 15z = 90$$

$$\text{Dus } \vec{r}_n = \vec{n}_{AMN} = \begin{pmatrix} 18 \\ 10 \\ 15 \end{pmatrix}.$$

$$\vec{r}_n = \vec{n}_{OAM} = \overrightarrow{OA} \times \overrightarrow{OM} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -20 \\ 15 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 18 \\ 10 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix} \right|}{\left| \begin{pmatrix} 18 \\ 10 \\ 15 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix} \right|} = \frac{|0 - 40 + 45|}{\sqrt{649} \cdot 5} = \frac{1}{\sqrt{649}}$$

Dus $\angle(AMN, OAM) = \angle(m, n) \approx 87,8^\circ$.

- b Lijn m is een normaal van EMN en lijn n is een normaal van BCD .

$$EMN: \frac{x}{r} + \frac{y}{9} + \frac{z}{6} = 1$$

$$E(5, 0, 4) \text{ invullen in } \frac{x}{r} + \frac{y}{9} + \frac{z}{6} = 1 \text{ geeft } \frac{5}{r} + \frac{0}{9} + \frac{4}{6} = 1$$

$$\frac{5}{r} + \frac{2}{3} = 1$$

$$\frac{5}{r} = \frac{1}{3}$$

$$r = 15$$

$$EMN: \frac{x}{15} + \frac{y}{9} + \frac{z}{6} = 1$$

$$6x + 10y + 15z = 90$$

$$\text{Dus } \vec{r}_m = \vec{n}_{EMN} = \begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix}.$$

$$\vec{r}_n = \vec{n}_{BCD} = \vec{r}_{BC} \times \vec{r}_{DC} = \begin{pmatrix} -5 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ -20 \\ -30 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} \right|}{\left| \begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} \right|} = \frac{|0 + 20 + 45|}{19 \cdot \sqrt{13}} = \frac{65}{19\sqrt{13}}$$

$$\text{Dus } \angle(EMN, BCD) = \angle(m, n) \approx 18,4^\circ.$$

39 a $\vec{r}_{DT} = \vec{t} - \vec{d} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$

Lijn n is een normaal van ABN .

$C(-2, -2, 0)$ en $T(0, 0, 3)$, dus $N(-1, 1, 1\frac{1}{2})$.

$$\vec{r}_n = \vec{n}_{ABN} = \vec{r}_{AN} \times \vec{r}_{AB} = \begin{pmatrix} -3 \\ 3 \\ 1\frac{1}{2} \end{pmatrix} \times \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \\ -12 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\cos(\angle(DT, n)) = \frac{\left| \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \right|} = \frac{|2 + 0 + 6|}{\sqrt{17} \cdot \sqrt{5}} = \frac{8}{\sqrt{85}}, \text{ dus } \angle(DT, n) \approx 29,8^\circ.$$

$$\text{Dus } \angle(DT, ABN) \approx 90^\circ - 29,8^\circ = 60,2^\circ.$$

- b $A(2, -2, 0)$ en $D(-2, -2, 0)$, dus $M(0, -2, 0)$.

$$\vec{r}_{MN} = \vec{n} - \vec{m} = \begin{pmatrix} -1 \\ 1 \\ 1\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 1\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix}$$

$$\vec{r}_{CT} = \vec{t} - \vec{c} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$$

$$\cos(\angle(MN, CT)) = \frac{\left| \begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix} \right|}{\left| \begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix} \right|} = \frac{|-4 - 12 + 9|}{7 \cdot \sqrt{17}} = \frac{7}{7\sqrt{17}} = \frac{1}{\sqrt{17}}, \text{ dus } \angle(MN, CT) \approx 76,0^\circ.$$

c Lijn m is een normaal van ABN en lijn n is een normaal van ACT .

$$\vec{r}_m = \vec{n}_{ABN} = \vec{r}_{AN} \times \vec{r}_{AB} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \text{ (zie a)}$$

$$\vec{r}_n = \vec{n}_{ACT} = \vec{r}_{AC} \times \vec{r}_{AT} = \begin{pmatrix} -4 \\ 4 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 12 \\ 12 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right|} = \frac{|1 + 0 + 0|}{\sqrt{5} \cdot \sqrt{2}} = \frac{1}{\sqrt{10}}$$

Dus $\angle(ABN, ACT) = \angle(m, n) \approx 71,6^\circ$.

d $\angle MBN = \angle(\overrightarrow{BM}, \overrightarrow{BN})$

$$\overrightarrow{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \\ 0 \end{pmatrix}$$

$$\overrightarrow{BN} = \vec{n} - \vec{b} = \begin{pmatrix} -1 \\ 1 \\ 1\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 1\frac{1}{2} \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{BM}, \overrightarrow{BN})) = \frac{\begin{pmatrix} -2 \\ -4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -1 \\ 1\frac{1}{2} \end{pmatrix}}{\left| \begin{pmatrix} -2 \\ -4 \\ 0 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ -1 \\ 1\frac{1}{2} \end{pmatrix} \right|} = \frac{6 + 4 + 0}{\sqrt{20} \cdot \sqrt{12\frac{1}{4}}} = \frac{10}{\sqrt{245}}$$

$$\angle MBN = \angle(\overrightarrow{BM}, \overrightarrow{BN}) \approx 50,3^\circ$$

8.4 Afstanden in de ruimte

Bladzijde 170

40 a $\overrightarrow{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 6 \\ 0 \\ 12 \end{pmatrix} - \begin{pmatrix} 12 \\ 12 \\ 6 \end{pmatrix} = \begin{pmatrix} -6 \\ -12 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

V is het vlak door P , loodrecht op MN .

$$V: x + 2y - z = d \quad \left. \begin{array}{l} \text{door } P(0, 9, 12) \end{array} \right\} d = 0 + 18 - 12 = 6$$

Dus $V: x + 2y - z = 6$.

b $MN: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 12 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ snijden met $V: x + 2y - z = 6$ geeft

$$\begin{aligned} 12 + \lambda + 2 \cdot (12 + 2\lambda) - (6 - \lambda) &= 6 \\ 12 + \lambda + 24 + 4\lambda - 6 + \lambda &= 6 \\ 6\lambda &= -24 \\ \lambda &= -4 \end{aligned}$$

Het snijpunt is $Q(8, 4, 10)$.

c $d(P, Q) = |\overrightarrow{PQ}| = |\vec{q} - \vec{p}| = \left| \begin{pmatrix} 8 \\ 4 \\ 10 \end{pmatrix} - \begin{pmatrix} 0 \\ 9 \\ 12 \end{pmatrix} \right| = \left| \begin{pmatrix} 8 \\ -5 \\ -2 \end{pmatrix} \right| = \sqrt{8^2 + (-5)^2 + (-2)^2} = \sqrt{93}$

Omdat $\overrightarrow{PQ} \perp \overrightarrow{MN}$ is dit ook $d(P, MN)$.

Bladzijde 171

- 41 a**
- $C(0, 4, 0)$
- en
- $G(0, 4, 2)$
- , dus
- $M(0, 4, 1)$
- .

$$\overrightarrow{MP} = \vec{p} - \vec{m} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

$$d(M, P) = |\overrightarrow{MP}| = \left| \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} \right| = \sqrt{26}$$

$$\text{b } \vec{r}_{DG} = \vec{g} - \vec{d} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

V is het vlak door B , loodrecht op DG .

$$\left. \begin{array}{l} V: y = d \\ \text{door } B(4, 4, 0) \end{array} \right\} d = 4$$

Dus $V: y = 4$.

$$DG: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ snijden met } V \text{ geeft } \lambda = 4.$$

Het snijpunt is $G(0, 4, 2)$.

$$d(B, DG) = d(B, G) = |\overrightarrow{BG}| = \left| \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix} \right| = \sqrt{20} = 2\sqrt{5}$$

$$\text{c } \vec{r}_{MP} = \vec{p} - \vec{m} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

V is het vlak door F , loodrecht op MP .

$$\left. \begin{array}{l} V: 3x - 4y + z = d \\ \text{door } F(4, 4, 2) \end{array} \right\} d = 12 - 16 + 2 = -2$$

Dus $V: 3x - 4y + z = -2$.

$$MP: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} \text{ snijden met } V \text{ geeft } \begin{array}{l} 3 \cdot 3\lambda - 4 \cdot (4 - 4\lambda) + (1 + \lambda) = -2 \\ 9\lambda - 16 + 16\lambda + 1 + \lambda = -2 \\ 26\lambda = 13 \\ \lambda = \frac{1}{2} \end{array}$$

Het snijpunt is $S(1\frac{1}{2}, 2, 1\frac{1}{2})$.

$$d(F, MP) = d(F, S) = |\overrightarrow{FS}| = \left| \begin{pmatrix} -2\frac{1}{2} \\ -2 \\ -\frac{1}{2} \end{pmatrix} \right| = \sqrt{10\frac{1}{2}} = \frac{1}{2}\sqrt{42}$$

- 42 a**
- V
- is het vlak door
- P
- , loodrecht op
- k
- .

$$\left. \begin{array}{l} V: y + 2z = d \\ \text{door } P(8, 10, 0) \end{array} \right\} d = 10 + 0 = 10$$

Dus $V: y + 2z = 10$.

$$k: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \text{ snijden met } V \text{ geeft } \begin{array}{l} \lambda + 2 \cdot 2\lambda = 10 \\ 5\lambda = 10 \\ \lambda = 2 \end{array}$$

Het snijpunt is $S(4, 2, 4)$.

$$d(P, k) = d(P, S) = |\overrightarrow{PS}| = \left| \begin{pmatrix} -4 \\ -8 \\ 4 \end{pmatrix} \right| = \sqrt{96} = 4\sqrt{6}$$

- b V is het vlak door Q , loodrecht op l .

$$\left. \begin{array}{l} V: -x + 3z = d \\ \text{door } Q(1, 5, 6) \end{array} \right\} d = -1 + 18 = 17$$

$$\text{Dus } V: -x + 3z = 17.$$

$$l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} \text{ snijden met } V \text{ geeft } -(-\mu) + 3(-1 + 3\mu) = 17$$

$$10\mu = 20$$

$$\mu = 2$$

Het snijpunt is $S(-2, 1, 5)$.

$$d(Q, l) = d(Q, S) = |\overrightarrow{QS}| = \left| \begin{pmatrix} -3 \\ -4 \\ -1 \end{pmatrix} \right| = \sqrt{26}$$

- c V is het vlak door R , loodrecht op m .

$$\left. \begin{array}{l} V: x + 6y + z = d \\ \text{door } R(5, 4, 9) \end{array} \right\} d = 5 + 24 + 9 = 38$$

$$\text{Dus } V: x + 6y + z = 38.$$

$$m: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = v \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix} \text{ snijden met } V \text{ geeft } v + 6 \cdot 6v + v = 38$$

$$38v = 38$$

$$v = 1$$

Het snijpunt is $S(1, 6, 1)$.

$$d(R, m) = d(R, S) = |\overrightarrow{RS}| = \left| \begin{pmatrix} -4 \\ 2 \\ -8 \end{pmatrix} \right| = \sqrt{84} = 2\sqrt{21}$$

43 a $\vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix}$

V is het vlak door A , loodrecht op BC .

$$\left. \begin{array}{l} V: -3x + 2z = d \\ \text{door } A(-3, 0, 3) \end{array} \right\} d = 9 + 6 = 15$$

$$\text{Dus } V: -3x + 2z = 15.$$

$$BC: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix} \text{ snijden met } V \text{ geeft } -3 \cdot -3\lambda + 2(1 + 2\lambda) = 15$$

$$9\lambda + 2 + 4\lambda = 15$$

$$13\lambda = 13$$

$$\lambda = 1$$

Het snijpunt is $S(-3, 1, 3)$.

$$d(A, BC) = d(A, S) = \left| \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right| = \sqrt{1} = 1$$

b $O(\triangle ABC) = \frac{1}{2} \cdot d(B, C) \cdot d(A, BC) = \frac{1}{2} \cdot \sqrt{(-3)^2 + 0^2 + 2^2} \cdot 1 = \frac{1}{2} \sqrt{13}$

Bladzijde 172

- 44 $A(8, 0, 0)$ en $E(8, 0, 8)$ dus $M(8, 0, 4)$.
 $E(8, 0, 8)$ en $F(8, 8, 8)$ dus $N(8, 4, 8)$.

$$\vec{r}_{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 8 \\ 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

V is het vlak door $C(0, 8, 0)$, loodrecht op MN .

$$\left. \begin{array}{l} V: y + z = d \\ \text{door } C(0, 8, 0) \end{array} \right\} d = 8$$

Dus $V: y + z = 8$.

$$MN: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \text{ snijden met } V \text{ geeft } \lambda + 4 + \lambda = 8$$

$$2\lambda = 4$$

$$\lambda = 2$$

Het snijpunt is $S(8, 2, 6)$.

$$d(C, MN) = d(C, S) = |\overrightarrow{CS}| = \left| \begin{pmatrix} 8 \\ -6 \\ 6 \end{pmatrix} \right| = \sqrt{136} = 2\sqrt{34}$$

$$O(\triangle CMN) = \frac{1}{2} \cdot d(C, MN) \cdot d(M, N) = \frac{1}{2} \cdot 2\sqrt{34} \cdot \sqrt{4^2 + 4^2} = \sqrt{34} \cdot \sqrt{32} = \sqrt{17} \cdot \sqrt{64} = 8\sqrt{17}$$

45 a $\vec{r}_{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

$$\vec{r}_{AG} = \vec{g} - \vec{a} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{n}_{AMG} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$\left. \begin{array}{l} AMG: 2x + y + z = d \\ \text{door } A(4, 0, 0) \end{array} \right\} d = 8$$

Dus $AMG: 2x + y + z = 8$.

- b $B(4, 4, 0)$ en $F(4, 4, 4)$, dus $P(4, 4, 2)$.
 k is de lijn door P , loodrecht op AMG .

$$\vec{s}_k = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \text{ en } \vec{r}_k = \vec{n}_{AMG} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \text{ dus } k: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

- c k snijden met AMG geeft
 $2(4 + 2\lambda) + 4 + \lambda + 2 + \lambda = 8$
 $8 + 4\lambda + 4 + \lambda + 2 + \lambda = 8$
 $6\lambda = -6$
 $\lambda = -1$

Het snijpunt is $Q(2, 3, 1)$.

d $d(P, Q) = |\overrightarrow{PQ}| = \left| \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} \right| = \sqrt{(-2)^2 + (-1)^2 + (-1)^2} = \sqrt{6}$

$d(P, Q)$ is $d(P, AMG)$ omdat PQ loodrecht op AMG staat en Q in het vlak AMG ligt.

Bladzijde 173

46 a $\vec{s}_k = \vec{p} = \begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix}$ en $\vec{r}_k = \vec{n}_V = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, dus $k: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} + \lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

b k snijden met V geeft

$$a(x_p + a\lambda) + b(y_p + b\lambda) + c(z_p + c\lambda) = d$$

$$ax_p + a^2\lambda + by_p + b^2\lambda + cz_p + c^2\lambda = d$$

$$\lambda(a^2 + b^2 + c^2) = d - ax_p - by_p - cz_p$$

$$\lambda = \frac{d - ax_p - by_p - cz_p}{a^2 + b^2 + c^2}$$

c $d(P, Q) = |\overrightarrow{PQ}| = \left| \begin{pmatrix} a\lambda \\ b\lambda \\ c\lambda \end{pmatrix} \right| = \sqrt{a^2\lambda^2 + b^2\lambda^2 + c^2\lambda^2} = \sqrt{\lambda^2} \cdot \sqrt{a^2 + b^2 + c^2} = |\lambda| \sqrt{a^2 + b^2 + c^2}$

d $d(P, V) = d(P, Q) = |\lambda| \sqrt{a^2 + b^2 + c^2} = \left| \frac{d - ax_p - by_p - cz_p}{a^2 + b^2 + c^2} \right| \cdot \sqrt{a^2 + b^2 + c^2} =$

$$\frac{\sqrt{a^2 + b^2 + c^2} \cdot |d - ax_p - by_p - cz_p|}{a^2 + b^2 + c^2} = \frac{|d - ax_p - by_p - cz_p|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|ax_p + by_p + cz_p - d|}{\sqrt{a^2 + b^2 + c^2}}$$

Bladzijde 174

47 $\vec{r}_{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$

$$\vec{r}_{AG} = \vec{g} - \vec{a} = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ 3 \end{pmatrix}$$

$$\vec{n}_{AMG} = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} \times \begin{pmatrix} -4 \\ 6 \\ 3 \end{pmatrix} = \begin{pmatrix} -18 \\ -6 \\ -12 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

$$\left. \begin{array}{l} AMG: 3x + y + 2z = d \\ \text{door } A(4, 0, 0) \end{array} \right\} d = 12$$

$$\text{Dus } AMG: 3x + y + 2z = 12.$$

$$N(4, 6, 1\frac{1}{2}), \text{ dus } d(N, AMG) = \frac{|12 + 6 + 3 - 12|}{\sqrt{3^2 + 1^2 + 2^2}} = \frac{9}{\sqrt{14}} \approx 2,41.$$

48 $\vec{r}_{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\vec{r}_{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{n}_{ABM} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\left. \begin{array}{l} ABM: x + z = d \\ \text{door } A(4, 0, 0) \end{array} \right\} d = 4$$

$$\text{Dus } ABM: x + z = 4.$$

$$N(3, 3, 3), \text{ dus } d(N, ABM) = \frac{|3 + 0 + 3 - 4|}{\sqrt{1^2 + 0^2 + 1^2}} = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

- 49 Breng een *Oxyz*-assenstelsel aan zo, dat *O* het punt (0, 0, 0) is, *A* het punt (4, 0, 0), *C* het punt (0, 4, 0) en *D* het punt (0, 0, 4).

$$\vec{r}_{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\vec{n}_{BMD} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\left. \begin{array}{l} BMD: x + y + 2z = d \\ \text{door } B(4, 4, 0) \end{array} \right\} d = 4 + 4 + 0 = 8$$

Dus *BMD*: $x + y + 2z = 8$.

$$F(4, 4, 4), \text{ dus } d(F, BMD) = \frac{|4 + 4 + 8 - 8|}{\sqrt{1^2 + 1^2 + 2^2}} = \frac{8}{\sqrt{6}} = 1\frac{1}{3}\sqrt{6}.$$

50 $\vec{r}_{AF} = \vec{f} - \vec{a} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$$\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\vec{n}_V = \vec{r}_{AF} \times \vec{r}_{BD} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

Bladzijde 176

51 a $\vec{r}_{OB} = \vec{b} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$$\vec{r}_{GM} = \vec{m} - \vec{g} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

V is het vlak door *GM* en evenwijdig met *OB*.

$$\vec{n}_V = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

$$\left. \begin{array}{l} V: x - y + 4z = d \\ \text{door } G(0, 4, 4) \end{array} \right\} d = 0 - 4 + 16 = 12$$

Dus *V*: $x - y + 4z = 12$.

$$d(OB, GM) = d(O, V) = \frac{|0 - 0 + 0 - 12|}{\sqrt{1^2 + (-1)^2 + 4^2}} = \frac{12}{\sqrt{18}} = 2\sqrt{2}$$

$$\mathbf{b} \quad \vec{r}_{BG} = \vec{g} - \vec{b} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{r}_{CM} = \vec{m} - \vec{c} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

V is het vlak door CM en evenwijdig met BG .

$$\vec{n}_V = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$$

$$\left. \begin{array}{l} V: 2x + 3y + 2z = d \\ \text{door } C(0, 4, 0) \end{array} \right\} d = 12$$

$$\text{Dus } V: 2x + 3y + 2z = 12.$$

$$d(BG, CM) = d(B, V) = \frac{|8 + 12 + 0 - 12|}{\sqrt{2^2 + 3^2 + 2^2}} = \frac{8}{\sqrt{17}} = \frac{8}{17}\sqrt{17}$$

52 V is het vlak door l en evenwijdig met m .

$$\vec{n} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

$$\left. \begin{array}{l} V: x - 2y + 2z = d \\ \text{door } (-1, -2, 0) \end{array} \right\} d = -1 + 4 + 0 = 3$$

$$\text{Dus } V: x - 2y + 2z = 3.$$

$$P(4, 2, 3) \text{ ligt op } m.$$

$$d(l, m) = d(P, V) = \frac{|4 - 4 + 6 - 3|}{\sqrt{1^2 + (-2)^2 + 2^2}} = \frac{3}{\sqrt{9}} = 1$$

$$\mathbf{53} \quad \mathbf{a} \quad \vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix}$$

$$\vec{r}_{EG} = \vec{g} - \vec{e} = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$$

V is het vlak door BD en evenwijdig met EG .

$$\vec{n}_V = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} \times \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -9 \\ -6 \\ -24 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ 2 \\ 8 \end{pmatrix}$$

$$\left. \begin{array}{l} V: 3x + 2y + 8z = d \\ \text{door } B(4, 6, 0) \end{array} \right\} d = 12 + 12 + 0 = 24$$

$$\text{Dus } V: 3x + 2y + 8z = 24.$$

$$d(BD, EG) = d(E, V) = \frac{|12 + 0 + 24 - 24|}{\sqrt{3^2 + 2^2 + 8^2}} = \frac{12}{\sqrt{77}} = \frac{12}{77}\sqrt{77}$$

- b Van de kleinste driehoek AFP is AF de basis en $d(BD, AF)$ de hoogte.

$$\vec{r}_{BD} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} \text{ en } \vec{r}_{AF} = \vec{f} - \vec{a} = \begin{pmatrix} 4 \\ 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

V is het vlak door BD en evenwijdig met AF .

$$\vec{n}_V = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -12 \\ 4 \\ -8 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

$$V: 3x - y + 2z = d \quad \left. \begin{array}{l} \text{door } B(4, 6, 0) \end{array} \right\} d = 12 - 6 + 0 = 6$$

$$\text{Dus } V: 3x - y + 2z = 6.$$

$$d(BD, AF) = d(A, V) = \frac{|12 - 0 + 0 - 6|}{\sqrt{3^2 + (-1)^2 + 2^2}} = \frac{6}{\sqrt{14}} = \frac{3}{7}\sqrt{14}$$

$$d(A, F) = |\vec{AF}| = \left| \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} \right| = \sqrt{0^2 + 6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$$

$$\text{Oppervlakte kleinste driehoek is } \frac{1}{2} \cdot d(A, F) \cdot d(BD, AF) = \frac{1}{2} \cdot 3\sqrt{5} \cdot \frac{3}{7}\sqrt{14} = \frac{9}{14}\sqrt{70}.$$

- 54 a Kies het assenstelsel zo, dat je de volgende punten krijgt: $S(0, 0, 0)$, $A(2, -2, 0)$, $B(2, 2, 0)$, $C(-2, 2, 0)$, $D(-2, -2, 0)$ en $T = (0, 0, 3)$.

$$\vec{r}_{AT} = \vec{t} - \vec{a} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}$$

$$\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

V is het vlak door AT en evenwijdig met BD .

$$\vec{n}_V = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ -3 \\ 4 \end{pmatrix}$$

$$V: 3x - 3y + 4z = d \quad \left. \begin{array}{l} \text{door } A(2, -2, 0) \end{array} \right\} d = 6 + 6 + 0 = 12$$

$$\text{Dus } V: 3x - 3y + 4z = 12.$$

$$d(AT, BD) = d(B, V) = \frac{|6 - 6 + 0 - 12|}{\sqrt{3^2 + (-3)^2 + 4^2}} = \frac{12}{\sqrt{34}} = \frac{6}{17}\sqrt{34}$$

- b Van de kleinste driehoek BCP is BC de basis en $d(AT, BC)$ de hoogte.

$$\vec{r}_{AT} = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} \text{ en } \vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

V is het vlak door AT en evenwijdig met BC .

$$\vec{n}_V = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix}$$

$$V: 3y - 2z = d \quad \left. \begin{array}{l} \text{door } A(2, -2, 0) \end{array} \right\} d = -6$$

$$\text{Dus } V: 3y - 2z = -6.$$

$$d(AT, BC) = d(B, V) = \frac{|6 + 0 + 6|}{\sqrt{0^2 + 3^2 + (-2)^2}} = \frac{12}{\sqrt{13}} = \frac{12}{13}\sqrt{13}$$

$$d(B, C) = 4$$

$$\text{Oppervlakte kleinste driehoek is } \frac{1}{2} \cdot d(B, C) \cdot d(AT, BC) = \frac{1}{2} \cdot 4 \cdot \frac{12}{13}\sqrt{13} = \frac{11}{13}\sqrt{13}$$

55 Vliegtuig 1 vliegt volgens de lijn

$$l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 270 \\ 313 \\ 9,588 \end{pmatrix} + \lambda \begin{pmatrix} 120 - 270 \\ 487 - 313 \\ -0,012 - 9,588 \end{pmatrix} \text{ ofwel } l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 270 \\ 313 \\ 9,588 \end{pmatrix} + \lambda \begin{pmatrix} -150 \\ 174 \\ -9,6 \end{pmatrix}$$

Vliegtuig 2 vliegt volgens de lijn

$$m: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 87 \\ 313 \\ 10,202 \end{pmatrix} + \mu \begin{pmatrix} 230 - 87 \\ 560 - 313 \\ 0,002 - 10,202 \end{pmatrix} \text{ ofwel } m: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 87 \\ 313 \\ 10,202 \end{pmatrix} + \mu \begin{pmatrix} 143 \\ 247 \\ -10,2 \end{pmatrix}$$

V is het vlak door m en evenwijdig met l .

$$\vec{n}_V = \begin{pmatrix} -150 \\ 174 \\ -9,6 \end{pmatrix} \times \begin{pmatrix} 143 \\ 247 \\ -10,2 \end{pmatrix} = \begin{pmatrix} 596,4 \\ -2902,8 \\ -61932 \end{pmatrix}$$

$$V: 596,4x - 2902,8y - 61932z = d \quad \left. \begin{array}{l} \text{door } (87; 313; 10,202) \end{array} \right\} d = -1488519,864$$

$$\text{Dus } V: 596,4x - 2902,8y - 61932z = -1488519,864.$$

$$d(l, m) = d((270; 313; 9,588), V) = \frac{|596,4 \cdot 270 - 2902,8 \cdot 313 - 61932 \cdot 9,588 + 1488519,864|}{\sqrt{596,4^2 + (-2902,8)^2 + (-61932)^2}} \approx 2,3736$$

de kortst mogelijke afstand tussen de twee vliegtuigen is 2374 meter.

Diagnostische toets

Bladzijde 178

1 a $M\left(\frac{0+0}{2}, \frac{6+6}{2}, \frac{0+6}{2}\right) = M(0, 6, 3)$, dus $N\left(\frac{0+4}{2}, \frac{6+4}{2}, \frac{3+0}{2}\right) = N(2, 5, 1\frac{1}{2})$.

b $d(M, P) = |\overrightarrow{MP}| = \left| \begin{pmatrix} 4 \\ -2 \\ -3 \end{pmatrix} \right| = \sqrt{4^2 + (-2)^2 + (-3)^2} = \sqrt{16 + 4 + 9} = \sqrt{29}$

c $\angle DPG = \angle(\overrightarrow{PD}, \overrightarrow{PG})$

$$\overrightarrow{PD} = \vec{d} - \vec{p} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 5 \end{pmatrix}$$

$$\overrightarrow{PG} = \vec{g} - \vec{p} = \begin{pmatrix} 0 \\ 6 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 5 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{PD}, \overrightarrow{PG})) = \frac{\begin{pmatrix} -4 \\ -4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 2 \\ 5 \end{pmatrix}}{\left| \begin{pmatrix} -4 \\ -4 \\ 5 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -4 \\ 2 \\ 5 \end{pmatrix} \right|} = \frac{16 - 8 + 25}{\sqrt{57} \cdot \sqrt{45}} = \frac{33}{\sqrt{2565}}$$

$$\angle DPG = \angle(\overrightarrow{PD}, \overrightarrow{PG}) \approx 49,3^\circ$$

2 a $A(4, 0, 0)$ en $T(0, 0, 6)$ geeft $M(2, 0, 3)$.

$$\vec{n} = \vec{b} + \frac{1}{4}\overrightarrow{BT} = \vec{b} + \frac{1}{4}(\vec{t} - \vec{b}) = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} + \frac{1}{4}\left(\begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} -4 \\ -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 1\frac{1}{2} \end{pmatrix}$$

Dus $N(3, 3, 1\frac{1}{2})$.

$$\vec{p} = \vec{c} + \frac{3}{4}\overrightarrow{CT} = \vec{c} + \frac{3}{4}(\vec{t} - \vec{c}) = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} + \frac{3}{4}\left(\begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} + \frac{3}{4}\begin{pmatrix} 0 \\ -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 4\frac{1}{2} \end{pmatrix}$$

Dus $P(0, 1, 4\frac{1}{2})$.

$$\text{b } \vec{m} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \text{ en } \vec{n} - \vec{m} = \begin{pmatrix} 3 \\ 3 \\ 1\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -1\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}, \text{ dus } MN: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}.$$

$x_D = 0$ geeft $2 + 2\lambda = 0$, dus $\lambda = -1$.

$$\lambda = -1 \text{ geeft } \vec{d} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 6 \end{pmatrix}.$$

Dus $D(0, -6, 6)$.

$$\text{c } \vec{m} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \text{ en } \vec{p} - \vec{m} = \begin{pmatrix} 0 \\ 1 \\ 4\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix}, \text{ dus } MP: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix}.$$

$z_E = 0$ geeft $3 + 3\lambda = 0$, dus $\lambda = -1$.

$$\lambda = -1 \text{ geeft } \vec{e} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix}.$$

Dus $E(6, -2, 0)$.

- 3 a** ACD snijdt de x -as in $A(6, 0, 0)$, de y -as in $C(0, 8, 0)$ en de z -as in $D(0, 0, 5)$,

dus $ACD: \frac{x}{6} + \frac{y}{8} + \frac{z}{5} = 1$ ofwel $20x + 15y + 24z = 120$.

- b** CDF snijdt de y -as in $(0, 8, 0)$ en de z -as in $(0, 0, 5)$, dus $CDF: \frac{x}{r} + \frac{y}{8} + \frac{z}{5} = 1$.

$F(6, 8, 5)$ invullen in $\frac{x}{r} + \frac{y}{8} + \frac{z}{5} = 1$ geeft $\frac{6}{r} + \frac{8}{8} + \frac{5}{5} = 1$

$$\frac{6}{r} + 1 + 1 = 1$$

$$\frac{6}{r} = -1$$

$$r = -6$$

$CDF: \frac{x}{-6} + \frac{y}{8} + \frac{z}{5} = 1$ ofwel $CDF: -20x + 15y + 24z = 120$.

$$\text{Dus } \vec{n}_{CDF} = \begin{pmatrix} -20 \\ 15 \\ 24 \end{pmatrix}.$$

$$\text{c } OF \text{ is de lijn } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \begin{pmatrix} 6 \\ 8 \\ 5 \end{pmatrix}.$$

CMN snijdt de y -as in $(0, 8, 0)$ en de z -as in $(0, 0, 10)$, dus $CMN: \frac{x}{r} + \frac{y}{8} + \frac{z}{10} = 1$.

$N(6, 0, 2\frac{1}{2})$ invullen in $\frac{x}{r} + \frac{y}{8} + \frac{z}{10} = 1$ geeft $\frac{6}{r} + \frac{0}{8} + \frac{2\frac{1}{2}}{10} = 1$

$$\frac{6}{r} + 0 + \frac{1}{4} = 1$$

$$\frac{6}{r} = \frac{3}{4}$$

$$3r = 24$$

$$r = 8$$

$CMN: \frac{x}{8} + \frac{y}{8} + \frac{z}{10} = 1$ ofwel $CMN: 5x + 5y + 4z = 40$.

Substitutie van $x = 6\lambda$, $y = 8\lambda$ en $z = 5\lambda$ in $5x + 5y + 4z = 40$ geeft

$$5 \cdot 6\lambda + 5 \cdot 8\lambda + 4 \cdot 5\lambda = 40$$

$$30\lambda + 40\lambda + 20\lambda = 40$$

$$90\lambda = 40$$

$$\lambda = \frac{4}{9}$$

Dus het snijpunt is $S(2\frac{2}{3}, 3\frac{5}{9}, 2\frac{2}{9})$.

4 a $\overrightarrow{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\overrightarrow{BN} = \vec{n} - \vec{b} = \begin{pmatrix} 6 \\ 0 \\ 2\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -8 \\ 2\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -16 \\ 5 \end{pmatrix}$$

$$\vec{n}_{BCN} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -16 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \\ -16 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 5 \\ 16 \end{pmatrix}$$

$$\left. \begin{array}{l} BCN: 5y + 16z = d \\ \text{door } C(0, 8, 0) \end{array} \right\} d = 40$$

$$\text{Dus } BCN: 5y + 16z = 40.$$

b $\overrightarrow{BF} = \vec{f} - \vec{b} = \begin{pmatrix} 6 \\ 8 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$\overrightarrow{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -8 \\ 5 \end{pmatrix}$$

$$\vec{n}_{BFD} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} -6 \\ -8 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}$$

$$\left. \begin{array}{l} BFD: 4x - 3y = d \\ \text{door } B(6, 8, 0) \end{array} \right\} d = 24 - 24 = 0$$

$$\text{Dus } BFD: 4x - 3y = 0.$$

c $\overrightarrow{OM} = \vec{m} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}$ en $\overrightarrow{ON} = \vec{n} = \begin{pmatrix} 6 \\ 0 \\ 2\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 12 \\ 0 \\ 5 \end{pmatrix}$

$$\vec{n}_{OMN} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix} \times \begin{pmatrix} 12 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 20 \\ 60 \\ -48 \end{pmatrix} \triangleq \begin{pmatrix} 5 \\ 15 \\ -12 \end{pmatrix}$$

$$\left. \begin{array}{l} OMN: 5x + 15y - 12z = d \\ \text{door } O(0, 0, 0) \end{array} \right\} d = 0$$

$$\text{Dus } OMN: 5x + 15y - 12z = 0.$$

Bladzijde 179

5 $\vec{r}_l = \vec{r}_k \times \vec{r}_m = \begin{pmatrix} 0 \\ -3 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ en $\vec{s}_l = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$

$$l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} + v \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

$$\text{6 a } \vec{r}_{OF} = \vec{f} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$$

$$\vec{r}_{AG} = \vec{g} - \vec{a} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$$

$$\cos(\angle(OF, AG)) = \frac{\left| \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix} \right|} = \frac{|-4 + 9 + 4|}{\sqrt{17} \cdot \sqrt{17}} = \frac{9}{17}$$

Dus $\angle(OF, AG) \approx 58,0^\circ$.

$$\text{b } \vec{r}_{DF} = \vec{f} - \vec{d} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}$$

$$\vec{r}_{AG} = \begin{pmatrix} -4 \\ 6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$$

$$\cos(\angle(DF, AG)) = \frac{\left| \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix} \right|}{\left| \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix} \right|} = \frac{|-8 + 18 + 2|}{\sqrt{53} \cdot \sqrt{17}} = \frac{12}{\sqrt{901}}$$

Dus $\angle(DF, AG) \approx 66,4^\circ$.

$$\text{c } \angle ADG = \angle(\overrightarrow{DA}, \overrightarrow{DG})$$

$$\overrightarrow{DA} = \vec{a} - \vec{d} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$$

$$\overrightarrow{DG} = \vec{g} - \vec{d} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{DA}, \overrightarrow{DG})) = \frac{\left| \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix} \right|}{\left| \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix} \right|} = \frac{0 + 0 - 3}{\sqrt{25} \cdot \sqrt{37}} = \frac{-3}{5\sqrt{37}}$$

$$\angle ADG = \angle(\overrightarrow{DA}, \overrightarrow{DG}) \approx 95,7^\circ$$

7 a $\vec{r}_{CE} = \vec{e} - \vec{c} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix}$

Lijn n is een normaal van BDF .

$$\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix}$$

$$\vec{r}_{BF} = \vec{f} - \vec{b} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{BDF} = \vec{r}_{BD} \times \vec{r}_{BF} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}$$

$$\cos(\angle(CE, n)) = \frac{\left| \begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} \right|} = \frac{|-12 - 12 + 0|}{\sqrt{61} \cdot \sqrt{13}} = \frac{24}{\sqrt{793}}$$

Dus $\angle(CE, n) \approx 31,5^\circ$ en $\angle(CE, BDF) \approx 90^\circ - 31,5^\circ = 58,5^\circ$.

b Lijn m is een normaal van DEF en lijn n is een normaal van AFC .

$$\vec{r}_{DE} = \vec{e} - \vec{d} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{r}_{DF} = \vec{f} - \vec{d} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}$$

$$\vec{r}_m = \vec{n}_{DEF} = \vec{r}_{DE} \times \vec{r}_{DF} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 6 \end{pmatrix}$$

$$\vec{r}_{AF} = \vec{f} - \vec{a} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$$

$$\vec{r}_{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{AFC} = \vec{r}_{AF} \times \vec{r}_{AC} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} \times \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 0 \\ -1 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix} \right|}{\left| \begin{pmatrix} 0 \\ -1 \\ 6 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix} \right|} = \frac{|0 + 2 + 18|}{\sqrt{37} \cdot \sqrt{22}} = \frac{20}{\sqrt{814}}$$

Dus $\angle(DEF, AFC) = \angle(m, n) \approx 45,5^\circ$.

c Lijn m is een normaal van BGD en lijn n is een normaal van OBE .

$$\vec{r}_{BG} = \vec{g} - \vec{b} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix}$$

$$\vec{r}_m = \vec{n}_{BGD} = \vec{r}_{BG} \times \vec{r}_{BD} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \\ 6 \end{pmatrix}$$

$$\vec{r}_{OB} = \vec{b} = \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$$\vec{r}_{OE} = \vec{e} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{OBE} = \vec{r}_{OB} \times \vec{r}_{OE} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ -6 \\ -12 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 6 \\ -1 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix} \right|}{\left| \begin{pmatrix} 6 \\ -1 \\ 6 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix} \right|} = \frac{|18 + 2 - 24|}{\sqrt{73} \cdot \sqrt{29}} = \frac{4}{\sqrt{2117}}$$

Dus $\angle(BGD, OBE) = \angle(m, n) \approx 85,0^\circ$.

8 a $\vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 9 \\ 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix}$

V is het vlak door A , loodrecht op BC .

$$\left. \begin{array}{l} V: 4x + 5y = d \\ \text{door } A(-4, 0, 1) \end{array} \right\} d = -16$$

Dus $V: 4x + 5y = -16$.

$$BC: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} \text{ snijden met } V \text{ geeft } \begin{aligned} 4(5 + 4\lambda) + 5(1 + 5\lambda) &= -16 \\ 20 + 16\lambda + 5 + 25\lambda &= -16 \\ 41\lambda &= -41 \\ \lambda &= -1 \end{aligned}$$

Het snijpunt is $S(1, -4, 3)$.

$$d(A, BC) = d(A, S) = |\vec{AS}| = \left| \begin{pmatrix} 5 \\ -4 \\ 2 \end{pmatrix} \right| = \sqrt{45} = 3\sqrt{5}$$

b $O(\triangle ABC) = \frac{1}{2} \cdot d(B, C) \cdot d(A, BC) = \frac{1}{2} \cdot \sqrt{4^2 + 5^2 + 0^2} \cdot 3\sqrt{5} = \frac{1}{2} \cdot \sqrt{41} \cdot 3\sqrt{5} = \frac{1}{2} \sqrt{205}$

9 a $\vec{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 5 \end{pmatrix}$

$$d(B, M) = |\vec{BM}| = \left| \begin{pmatrix} -6 \\ -4 \\ 5 \end{pmatrix} \right| = \sqrt{(-6)^2 + (-4)^2 + 5^2} = \sqrt{36 + 16 + 25} = \sqrt{77} \approx 8,77$$

b $\vec{r}_{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 6 \\ 0 \\ 2\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \\ -2\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 12 \\ -8 \\ -5 \end{pmatrix}$

$$\vec{r}_{OF} = \vec{f} = \begin{pmatrix} 6 \\ 8 \\ 5 \end{pmatrix}$$

V is het vlak door MN en evenwijdig met OF .

$$\vec{n}_V = \begin{pmatrix} 12 \\ -8 \\ -5 \end{pmatrix} \times \begin{pmatrix} 6 \\ 8 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ -90 \\ 144 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -5 \\ 8 \end{pmatrix}$$

$$\left. \begin{array}{l} V: -5y + 8z = d \\ \text{door } M(0, 4, 5) \end{array} \right\} d = -20 + 40 = 20$$

Dus $V: -5y + 8z = 20$.

$$d(MN, OF) = d(O, V) = \frac{|0 + 0 - 20|}{\sqrt{(-5)^2 + 8^2}} = \frac{20}{\sqrt{89}} \approx 2,12$$

c $\vec{r}_{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix}$

V is het vlak door D , loodrecht op AC .

$$\left. \begin{array}{l} V: -3x + 4y = d \\ \text{door } D(0, 0, 5) \end{array} \right\} d = 0$$

Dus $V: -3x + 4y = 0$.

$$AC: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix} \text{ snijden met } -3x + 4y = 0 \text{ geeft } \begin{array}{l} -3(6 - 3\lambda) + 4 \cdot 4\lambda = 0 \\ -18 + 9\lambda + 16\lambda = 0 \\ 25\lambda = 18 \\ \lambda = \frac{18}{25} \end{array}$$

Het snijpunt is $S(3\frac{21}{25}, 2\frac{22}{25}, 0)$.

$$d(D, AC) = d(D, S) = |\vec{DS}| = \left| \begin{pmatrix} 3\frac{21}{25} \\ 2\frac{22}{25} \\ -5 \end{pmatrix} \right| = \sqrt{(3\frac{21}{25})^2 + (2\frac{22}{25})^2 + (-5)^2} = \sqrt{48,04} \approx 6,93$$

d $\vec{r}_{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 5 \end{pmatrix}$

$$\vec{r}_{BN} = \vec{n} - \vec{b} = \begin{pmatrix} 6 \\ 0 \\ 2\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -8 \\ 2\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -16 \\ 5 \end{pmatrix}$$

$$\vec{n}_{BMN} = \begin{pmatrix} -6 \\ -4 \\ 5 \end{pmatrix} \times \begin{pmatrix} 0 \\ -16 \\ 5 \end{pmatrix} = \begin{pmatrix} 60 \\ 30 \\ 96 \end{pmatrix} \triangleq \begin{pmatrix} 10 \\ 5 \\ 16 \end{pmatrix}$$

$$\left. \begin{array}{l} BMN: 10x + 5y + 16z = d \\ \text{door } B(6, 8, 0) \end{array} \right\} d = 60 + 40 + 0 = 100$$

Dus $BMN: 10x + 5y + 16z = 100$.

$$d(A, BMN) = \frac{|60 + 0 + 0 - 100|}{\sqrt{10^2 + 5^2 + 16^2}} = \frac{40}{\sqrt{381}} \approx 2,05$$