c De karakteristieke vergelijking is $\lambda^2 + 5\lambda - 6 = 0$
$(\lambda - 1)(\lambda + 6) = 0$
$\lambda = 1 \vee \lambda = -6$
Dus $y(t) = A\,e^t + B\,e^{-6t}$ is de algemene oplossing.
$y(0) = -2$ geeft $A + B = -2$
$y(t) = A\,e^t + B\,e^{-6t}$ geeft $y'(t) = A\,e^t - 6B\,e^{-6t}$
$y'(0) = 3$ geeft $A - 6B = 3$

$$\begin{cases} A + B = -2 \\ A - 6B = 3 \end{cases} \quad -$$
$$7B = -5$$

$$\left.\begin{array}{r} B = -\frac{5}{7} \\ A + B = -2 \end{array}\right\} A = -1\tfrac{2}{7}$$

De gevraagde oplossing is $y(t) = -1\frac{2}{7}\,e^t - \frac{5}{7}\,e^{-6t}$.

10 **a** Er moet gelden $D = 0$.
$m = 800$ en $c = 125\,000$ geeft $D = \dfrac{k^2 - 4 \cdot 125\,000 \cdot 800}{800^2} = \dfrac{k^2 - 400\,000\,000}{640\,000}$
$D = 0$ geeft $\dfrac{k^2 - 400\,000\,000}{640\,000} = 0$
$k^2 - 400\,000\,000$
$k^2 = 400\,000\,000$
$k = \sqrt{400\,000\,000}$
Dus $k = 20\,000$.

b $F = -20\,000\dfrac{du}{dt} - 125\,000u$ en $\dfrac{d^2u}{dt^2} = \dfrac{F}{800}$ geeft $\dfrac{d^2u}{dt^2} = -25\dfrac{du}{dt} - 156\frac{1}{4}u$ ofwel $\dfrac{d^2u}{dt^2} + 25\dfrac{du}{dt} + 156\frac{1}{4}u = 0$
De karakteristieke vergelijking is $\lambda^2 + 25\lambda + 156\frac{1}{4} = 0$.
$D = 0$, dus $\lambda = -\dfrac{25}{2} = -12\frac{1}{2}$.
De oplossingen zijn van de vorm $u(t) = (A + Bt) \cdot e^{-12\frac{1}{2}t}$.
$u(0) = 1$ geeft $A = 1$
$v(t) = \dfrac{du}{dt} = B\,e^{-12\frac{1}{2}t} + (A + Bt) \cdot -12\frac{1}{2}\,e^{-12\frac{1}{2}t}$
$v(0) = 0$ geeft $B - 12\frac{1}{2}A = 0$
$A = 1 \wedge B - 12\frac{1}{2}A = 0$ geeft $B = 12\frac{1}{2}$
Dus $u(t) = \left(1 + 12\frac{1}{2}t\right) \cdot e^{-12\frac{1}{2}t}$.

c Los op $u(t) = 0{,}001$.
Voer in $y_1 = \left(1 + 12\frac{1}{2}x\right) \cdot e^{-12\frac{1}{2}x}$ en $y_2 = 0{,}001$.
Intersect geeft $x \approx 0{,}739$.
Dus na ongeveer 0,7 seconde is de uitwijking minder dan 1 mm.

16 Complexe functies

Voorkennis Complexe getallen

Bladzijde 154

1 **a** $2\frac{1}{2}z - 2 - i = z + 1 + 5i$

$1\frac{1}{2}z = 3 + 6i$

$z = 2 + 4i$

b $1\frac{1}{4}z + 5 - 2i = z$

$\frac{1}{4}z = -5 + 2i$

$z = -20 + 8i$

c $(4 - i) \cdot z = 2 + i$

$z = \frac{2 + i}{4 - i} \cdot \frac{4 + i}{4 + i} = \frac{8 + 6i + i^2}{16 - i^2} = \frac{8 + 6i - 1}{16 + 1} = \frac{7 + 6i}{17} = \frac{7}{17} + \frac{6}{17}i$

d $(3 + i) \cdot z - 1 + 2i = 5 + i$

$(3 + i) \cdot z = 6 - i$

$z = \frac{6 - i}{3 + i} \cdot \frac{3 - i}{3 - i} = \frac{18 - 9i + i^2}{9 - i^2} = \frac{18 - 9i - 1}{9 + 1} = \frac{17 - 9i}{10} = 1\frac{7}{10} - \frac{9}{10}i$

e $z^2 - 2z + 5 = 0$

$(z - 1)^2 - 1 + 5 = 0$

$(z - 1)^2 = -4$

$(z - 1)^2 = 4i^2$

$z - 1 = 2i \lor z - 1 = -2i$

$z = 1 + 2i \lor z = 1 - 2i$

f $z^2 - 5z + 10 = 0$

$\left(z - 2\frac{1}{2}\right)^2 - 6\frac{1}{4} + 10 = 0$

$\left(z - 2\frac{1}{2}\right)^2 = -3\frac{3}{4}$

$\left(z - 2\frac{1}{2}\right)^2 = \frac{15}{4}i^2$

$z - 2\frac{1}{2} = \frac{1}{2}i\sqrt{15} \lor z - 2\frac{1}{2} = -\frac{1}{2}i\sqrt{15}$

$z = 2\frac{1}{2} + \frac{1}{2}i\sqrt{15} \lor z = 2\frac{1}{2} - \frac{1}{2}i\sqrt{15}$

Bladzijde 155

2 **a** $|3 - i| = \sqrt{3^2 + (-1)^2} = \sqrt{9 + 1} = \sqrt{10}$

$\text{Arg}(3 - i) \approx -0{,}32$

b $|-4 + 3i| = \sqrt{(-4)^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$\text{Arg}(-4 + 3i) \approx 2{,}50$

c $|5 - 2i| = \sqrt{5^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29}$

$\text{Arg}(5 - 2i) \approx -0{,}38$

d $|-6 - 8i| = \sqrt{(-6)^2 + (-8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

$\text{Arg}(-6 - 8i) \approx -2{,}21$

3 **a** $\left|\sqrt{3} + i\right| = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{3 + 1} = \sqrt{4} = 2$

$\text{Arg}\left(\sqrt{3} + i\right) = \frac{1}{6}\pi$

b $|5 - 5i| = \sqrt{5^2 + (-5)^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$

$\text{Arg}(5 - 5i) = -\frac{1}{4}\pi$

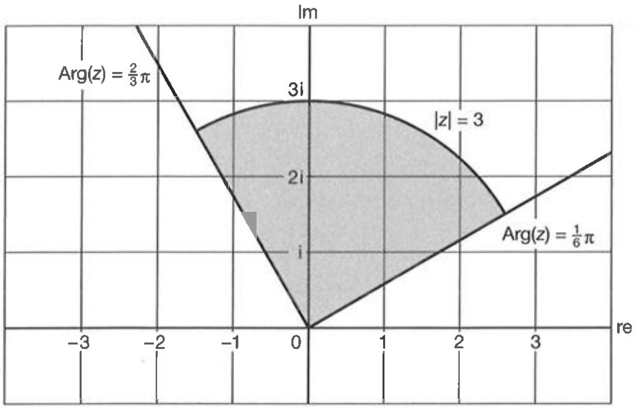
c $\left|-2 + 2i\sqrt{3}\right| = \sqrt{(-2)^2 + \left(2\sqrt{3}\right)^2} = \sqrt{4 + 12} = \sqrt{16} = 4$

$\text{Arg}\left(-2 + 2i\sqrt{3}\right) = \frac{2}{3}\pi$

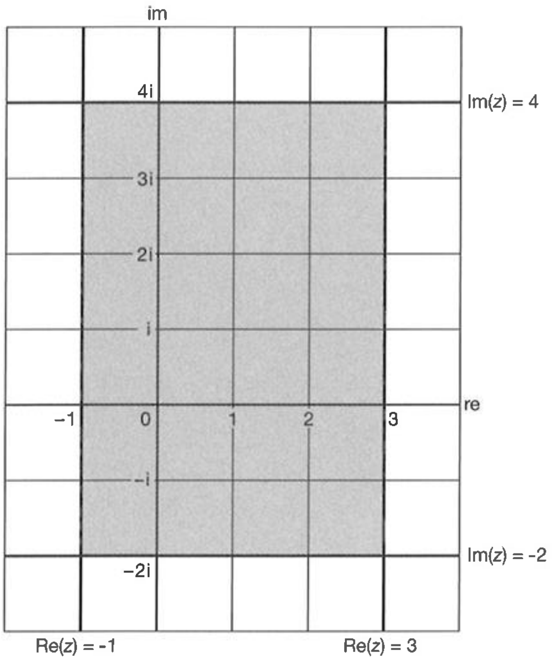
d $\left|-3 - 3i\sqrt{3}\right| = \sqrt{(-3)^2 + \left(-3\sqrt{3}\right)^2} = \sqrt{9 + 27} = \sqrt{36} = 6$

$\text{Arg}\left(-3 - 3i\sqrt{3}\right) = -\frac{2}{3}\pi$

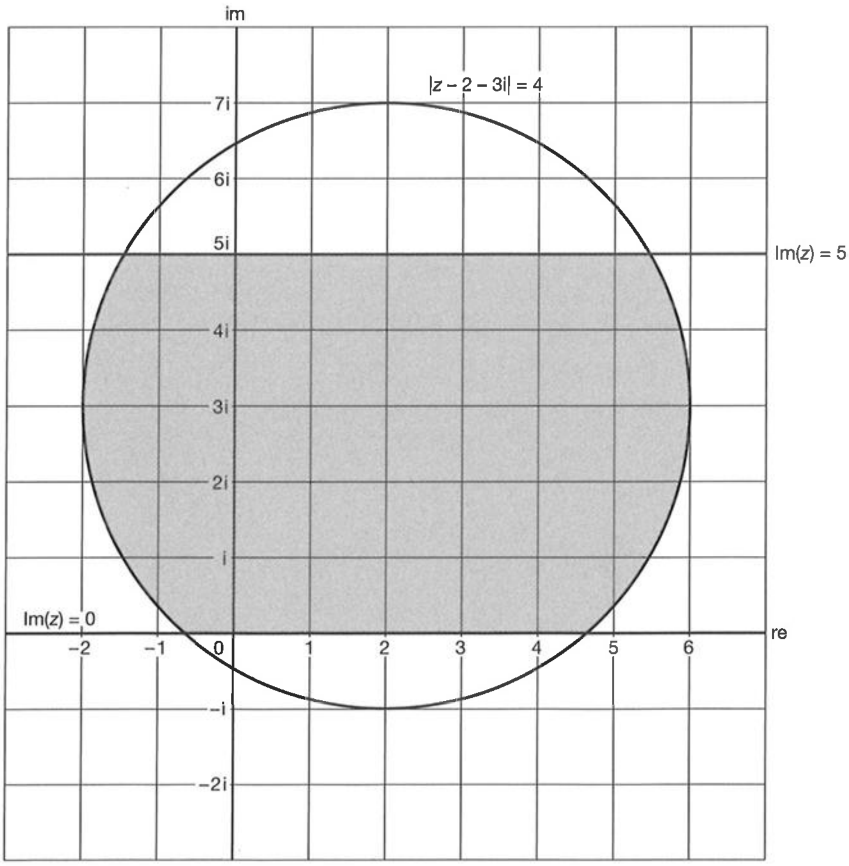
4 **a**


Im
Arg(z) = $\frac{2}{3}\pi$
3i
|z| = 3
2i
Arg(z) = $\frac{1}{6}\pi$
i
re
−3
−2
−1
0
1
2
3

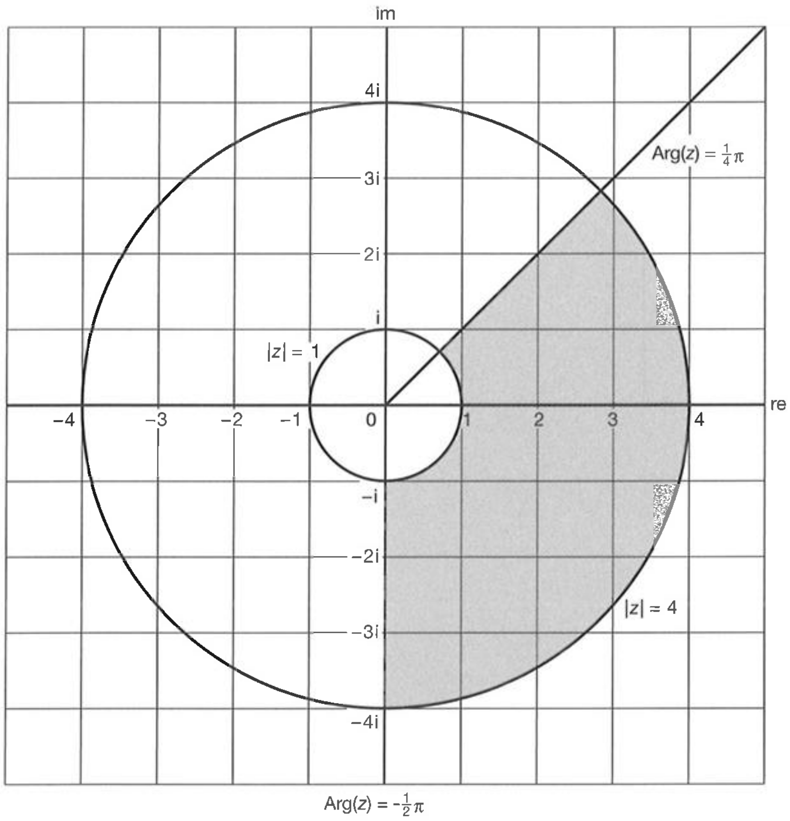
b


im
4i
Im(z) = 4
3i
2i
i
re
−1
0
1
2
3
−i
Im(z) = -2
−2i
Re(z) = -1
Re(z) = 3

c


im
|z − 2 − 3i| = 4
7i
6i
5i
Im(z) = 5
4i
3i
2i
i
Im(z) = 0
re
−2
−1
0
1
2
3
4
5
6
−i
−2i

d



16

Bladzijde 156

5 **a** $|z| = \sqrt{3^2 + 2^2} = \sqrt{13}$ en $\arg(z) \approx 0{,}59$ geeft $z = \sqrt{13}\,e^{0{,}59i}$

b $|z| = \sqrt{3^2 + (-4)^2} = 5$ en $\arg(z) \approx -0{,}93$ geeft $z = 5\,e^{-0{,}93i}$

c $|z| = \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2} = \sqrt{5}$ en $\arg(z) \approx 0{,}68$ geeft $z = \sqrt{5}\,e^{0{,}68i}$

Bladzijde 157

6 **a** $z = 3e^{-i} = 3(\cos(-1) + i\sin(-1)) = 3\cos(-1) + 3i\sin(-1) \approx 1{,}62 - 2{,}52i$
b $z = \pi^2 e^{2i} = \pi^2(\cos(2) + i\sin(2)) = \pi^2\cos(2) + \pi^2 i\sin(2) \approx -4{,}11 + 8{,}97i$
c $z = \sqrt{3}\,e^{\frac{1}{5}\pi i} = \sqrt{3}\left(\cos\left(\frac{1}{5}\pi\right) + i\sin\left(\frac{1}{5}\pi\right)\right) = \sqrt{3}\cos\left(\frac{1}{5}\pi\right) + i\sqrt{3}\sin\left(\frac{1}{5}\pi\right) \approx 1{,}40 + 1{,}02i$

7 **a** $z = 2e^{\frac{1}{4}\pi i} = 2\left(\cos\left(\frac{1}{4}\pi\right) + i\sin\left(\frac{1}{4}\pi\right)\right) = 2\left(\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}\right) = \sqrt{2} + i\sqrt{2}$

b $z = \sqrt{3}\,e^{\frac{1}{3}\pi i} = \sqrt{3}\left(\cos\left(\frac{1}{3}\pi\right) + i\sin\left(\frac{1}{3}\pi\right)\right) = \sqrt{3}\left(\frac{1}{2} + \frac{1}{2}i\sqrt{3}\right) = \frac{1}{2}\sqrt{3} + 1\frac{1}{2}i$

c $z = 10e^{-\frac{1}{6}\pi i} = 10\left(\cos\left(-\frac{1}{6}\pi\right) + i\sin\left(-\frac{1}{6}\pi\right)\right) = 10\left(\frac{1}{2}\sqrt{3} - \frac{1}{2}i\right) = 5\sqrt{3} - 5i$

8 **a** $z = (1 - i)^5 = \left(\sqrt{2}\,e^{-\frac{1}{4}\pi i}\right)^5 = 4\sqrt{2}\,e^{-1\frac{1}{4}\pi i} = 4\sqrt{2}\,e^{\frac{3}{4}\pi i}$

b $z = \left(1 - i\sqrt{3}\right)^{-4} = \left(2e^{-\frac{1}{3}\pi i}\right)^{-4} = \frac{1}{16}e^{1\frac{1}{3}\pi i} = \frac{1}{16}e^{-\frac{2}{3}\pi i}$

c $z = \dfrac{2 + 2i\sqrt{3}}{1 - i\sqrt{3}} = \dfrac{4e^{\frac{1}{3}\pi i}}{2e^{-\frac{1}{3}\pi i}} = 2e^{\frac{2}{3}\pi i}$

9 **a** $(z - 3)^2 = 1 + i\sqrt{3}$
$(z - 3)^2 = 2e^{\frac{1}{3}\pi i + k \cdot 2\pi i}$
$z - 3 = \sqrt{2}\,e^{\frac{1}{6}\pi i + k \cdot \pi i}$
$z - 3 = \sqrt{2}\,e^{\frac{1}{6}\pi i} \vee z - 3 = \sqrt{2}\,e^{1\frac{1}{6}\pi i}$

$z - 3 = \frac{1}{2}\sqrt{6} + \frac{1}{2}i\sqrt{2} \vee z - 3 = -\frac{1}{2}\sqrt{6} - \frac{1}{2}i\sqrt{2}$

$z = 3 + \frac{1}{2}\sqrt{6} + \frac{1}{2}i\sqrt{2} \vee z = 3 - \frac{1}{2}\sqrt{6} - \frac{1}{2}i\sqrt{2}$

b $(z - 2 + i)^3 = -8i$
$(z - 2 + i)^3 = 8e^{-\frac{1}{2}\pi i + k \cdot 2\pi i}$
$z - 2 + i = 2e^{-\frac{1}{6}\pi i + k \cdot \frac{2}{3}\pi i}$
$z - 2 + i = 2e^{-\frac{1}{6}\pi i} \vee z - 2 + i = 2e^{\frac{1}{2}\pi i} \vee z - 2 + i = 2e^{1\frac{1}{6}\pi i}$
$z - 2 + i = \sqrt{3} - i \vee z - 2 + i = 2i \vee z - 2 + i = -\sqrt{3} - i$
$z = 2 + \sqrt{3} - 2i \vee z = 2 + i \vee z = 2 - \sqrt{3} - 2i$

c $z^4 = \frac{81}{16}$

$z^4 = \frac{81}{16}\,e^{k \cdot 2\pi i}$

$z = 1\frac{1}{2}\,e^{k \cdot \frac{1}{2}\pi i}$

$z = 1\frac{1}{2} \vee z = 1\frac{1}{2}\,e^{\frac{1}{2}\pi i} \vee z = 1\frac{1}{2}\,e^{\pi i} \vee z = 1\frac{1}{2}\,e^{1\frac{1}{2}\pi i}$

$z = 1\frac{1}{2} \vee z = 1\frac{1}{2}i \vee z = -1\frac{1}{2} \vee z = -1\frac{1}{2}i$

10 **a** $\sqrt{-10i}$, dus los op $z^2 = -10i$

$z^2 = 10e^{-\frac{1}{2}\pi i + k \cdot 2\pi i}$

$z = \sqrt{10}e^{-\frac{1}{4}\pi i + k \cdot \pi i}$

$z = \sqrt{10}e^{-\frac{1}{4}\pi i} \vee z = \sqrt{10}e^{\frac{3}{4}\pi i}$

$z = \sqrt{5} - i\sqrt{5} \vee z = -\sqrt{5} + i\sqrt{5}$

Dus $\sqrt{-10i} = \sqrt{5} - i\sqrt{5} \vee \sqrt{-10i} = -\sqrt{5} + i\sqrt{5}$.

b $-\frac{1}{8i} = i^2 \cdot \frac{1}{8i} = \frac{1}{8}i$

$\sqrt[3]{-\frac{1}{8i}}$, dus los op $z^3 = -\frac{1}{8i}$

$z^3 = \frac{1}{8}i$

$z^3 = \frac{1}{8}\,e^{\frac{1}{2}\pi i + k \cdot 2\pi i}$

$z = \frac{1}{2}\,e^{\frac{1}{6}\pi i + k \cdot \frac{2}{3}\pi i}$

$z = \frac{1}{2}\,e^{\frac{1}{6}\pi i} \vee z = \frac{1}{2}\,e^{\frac{5}{6}\pi i} \vee z = \frac{1}{2}\,e^{1\frac{1}{2}\pi i}$

$z = \frac{1}{4}\sqrt{3} + \frac{1}{4}i \vee z = -\frac{1}{4}\sqrt{3} + \frac{1}{4}i \vee z = -\frac{1}{2}i$

Dus $\sqrt[3]{-\frac{1}{8i}} = \frac{1}{4}\sqrt{3} + \frac{1}{4}i \vee \sqrt[3]{-\frac{1}{8i}} = -\frac{1}{4}\sqrt{3} + \frac{1}{4}i \vee \sqrt[3]{-\frac{1}{8i}} = -\frac{1}{2}i$.

c $\sqrt[4]{-2 + 2i\sqrt{3}}$, dus los op $z^4 = -2 + 2i\sqrt{3}$

$z^4 = 16e^{\frac{2}{3}\pi i + k \cdot 2\pi i}$

$z = 2e^{\frac{1}{6}\pi i + k \cdot \frac{1}{2}\pi i}$

$z = 2e^{\frac{1}{6}\pi i} \vee z = 2e^{\frac{2}{3}\pi i} \vee z = 2e^{1\frac{1}{6}\pi i} \vee z = 2e^{1\frac{2}{3}\pi i}$

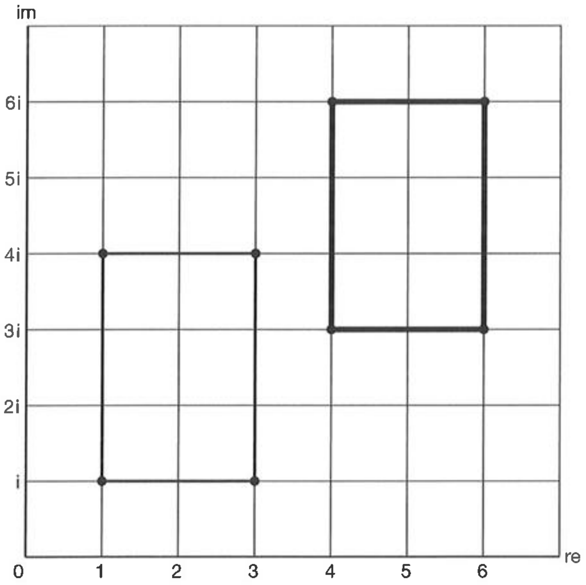
$z = \sqrt{3} + i \vee z = -1 + i\sqrt{3} \vee z = -\sqrt{3} - i \vee z = 1 - i\sqrt{3}$

Dus $\sqrt[4]{-2 + 2i\sqrt{3}} = \sqrt{3} + i \vee \sqrt[4]{-2 + 2i\sqrt{3}} = -1 + i\sqrt{3} \vee \sqrt[4]{-2 + 2i\sqrt{3}} = -\sqrt{3} - i \vee \sqrt[4]{-2 + 2i\sqrt{3}} = 1 - i\sqrt{3}$.

16.1 Lineaire complexe functies

Bladzijde 158

1 **a**

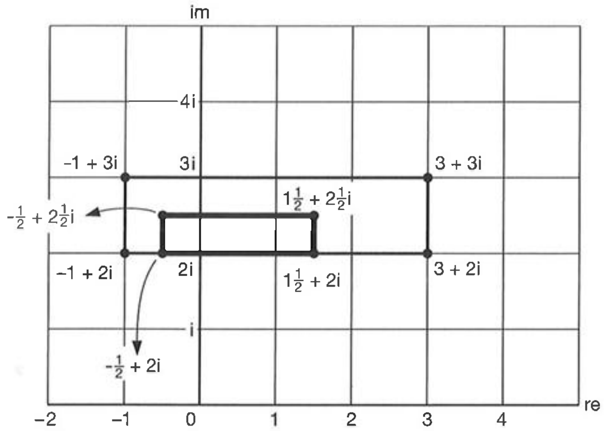


b $(1 + i) + (3 + 2i) = 4 + 3i$
$(3 + i) + (3 + 2i) = 6 + 3i$
$(3 + 4i) + (3 + 2i) = 6 + 6i$
$(1 + 4i) + (3 + 2i) = 4 + 6i$

c Zie de figuur van vraag a.
De rechthoek is 3 naar rechts en 2 omhoog verschoven.

Bladzijde 160

2 **a** Bij $f(z) = \frac{1}{2}z + i$ hoort de vermenigvuldiging ten opzichte van 0 met $\frac{1}{2}$ gevolgd door de translatie (0, 1).



b $f(z) = 0$ geeft $\frac{1}{2}z + i = 0$

$\frac{1}{2}z = -i$

$z = -2i$

Het nulpunt is $-2i$.

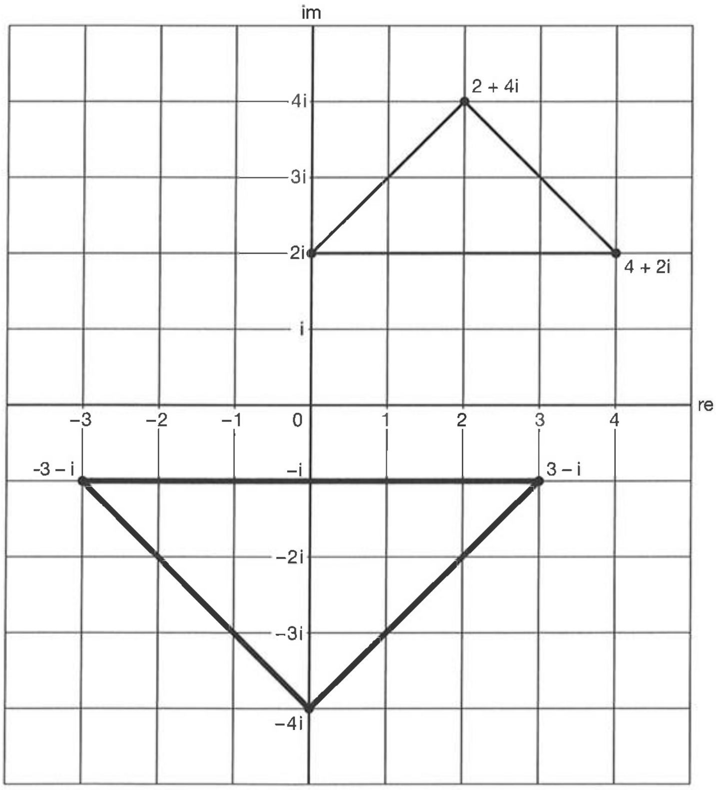
c $f(z) = z$ geeft $\frac{1}{2}z + i = z$

$-\frac{1}{2}z = -i$

$z = 2i$

Het dekpunt is $2i$.

3 **a** Bij $f(z) = -1\frac{1}{2}z + 3 + 2i$ hoort de vermenigvuldiging ten opzichte van 0 met $-1\frac{1}{2}$ gevolgd door de translatie (3, 2).



b $f(z) = 0$ geeft $-1\frac{1}{2}z + 3 + 2i = 0$

$-1\frac{1}{2}z = -3 - 2i$

$z = 2 + 1\frac{1}{3}i$

Het nulpunt is $2 + 1\frac{1}{3}i$.

c $f(z) = z$ geeft $-1\frac{1}{2}z + 3 + 2i = z$

$-2\frac{1}{2}z = -3 - 2i$

$z = 1\frac{1}{5} + \frac{4}{5}i$

Het dekpunt is $1\frac{1}{5} + \frac{4}{5}i$.

4 **a** $f(z) = 0$ geeft $3z + 2 - 4i = 0$

$3z = -2 + 4i$

$z = -\frac{2}{3} + 1\frac{1}{3}i$

Het nulpunt is $-\frac{2}{3} + 1\frac{1}{3}i$.

$f(z) = z$ geeft $3z + 2 - 4i = z$

$2z = -2 + 4i$

$z = -1 + 2i$

Het dekpunt is $-1 + 2i$.

b $g(z) = 0$ geeft $\frac{1}{3}z + 5 = 0$

$\frac{1}{3}z = -5$

$z = -15$

Het nulpunt is -15.

$g(z) = z$ geeft $\frac{1}{3}z + 5 = z$

$-\frac{2}{3}z = -5$

$z = 7\frac{1}{2}$

Het dekpunt is $7\frac{1}{2}$.

5 **a** $f(z) = z$ geeft $az + 5 - 2i = z$

$az - z = -5 + 2i$

$z(a - 1) = -5 + 2i$

$$z = \frac{-5 + 2i}{a - 1}$$

Voor $a = 1$ heeft de functie f geen dekpunt.

b $f(z) = 0$ geeft $az + 5 - 2i = 0$

$az = -5 + 2i$

$$z = \frac{-5 + 2i}{a}$$

Voor $a = 0$ heeft de functie f geen nulpunt.

6 **a** $f(1 + 2i) = 0$ geeft $3(1 + 2i) + a + bi = 0$

$3 + 6i + a + bi = 0$

$a + bi = -3 - 6i$

Dus $a = -3$ en $b = -6$.

b $f(1 + 2i) = 1 + 2i$ geeft $3(1 + 2i) + a + bi = 1 + 2i$

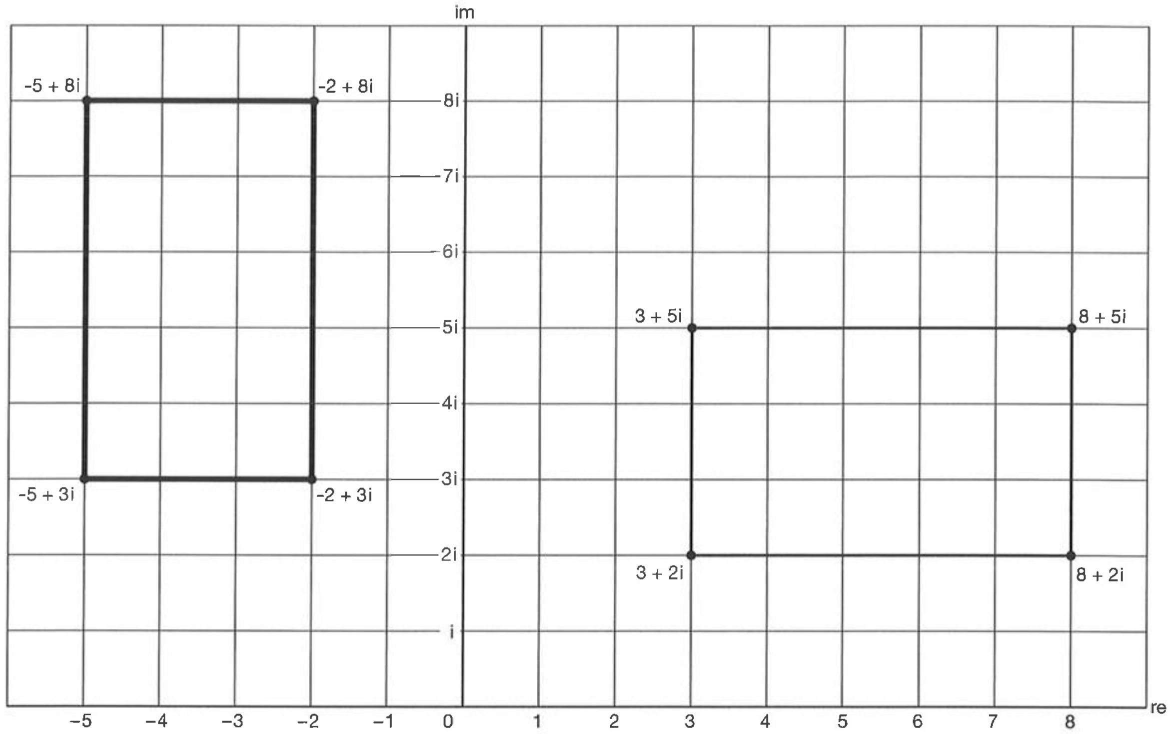
$3 + 6i + a + bi = 1 + 2i$

$a + bi = -2 - 4i$

Dus $a = -2$ en $b = -4$.

Bladzijde 161

7 **a,b**

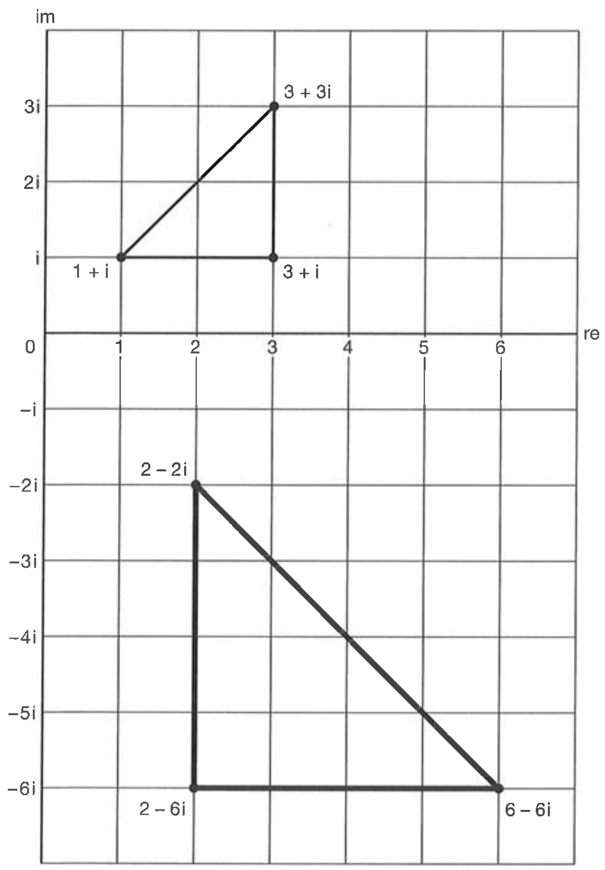


b $i(3 + 2i) = 3i - 2 = -2 + 3i$
$i(8 + 2i) = 8i - 2 = -2 + 8i$
$i(8 + 5i) = 8i - 5 = -5 + 8i$
$i(3 + 5i) = 3i - 5 = -5 + 3i$

c Rotatie om 0 over $\frac{1}{2}\pi$.

d Rotatie om 0 over $\arg\left(\frac{1}{2} + \frac{1}{2}i\sqrt{3}\right) = \frac{1}{3}\pi$.

8 **a,b**

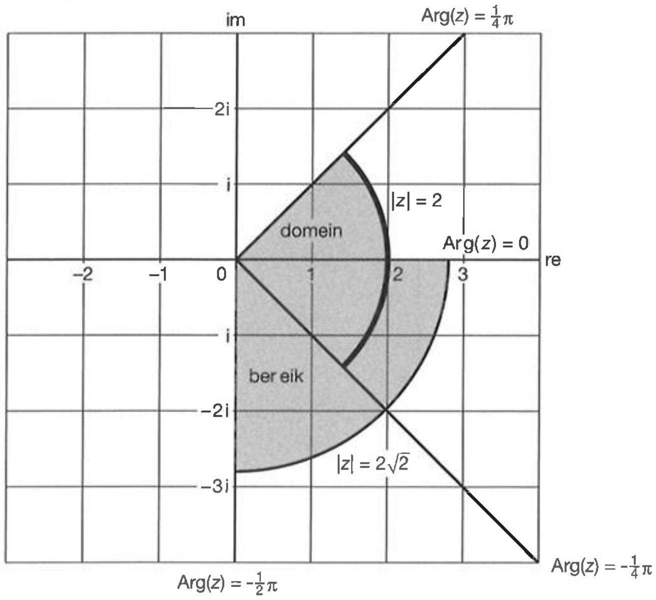


b $-2i(1 + i) = -2i + 2 = 2 - 2i$
$-2i(3 + i) = -6i + 2 = 2 - 6i$
$-2i(3 + 3i) = -6i + 6 = 6 - 6i$

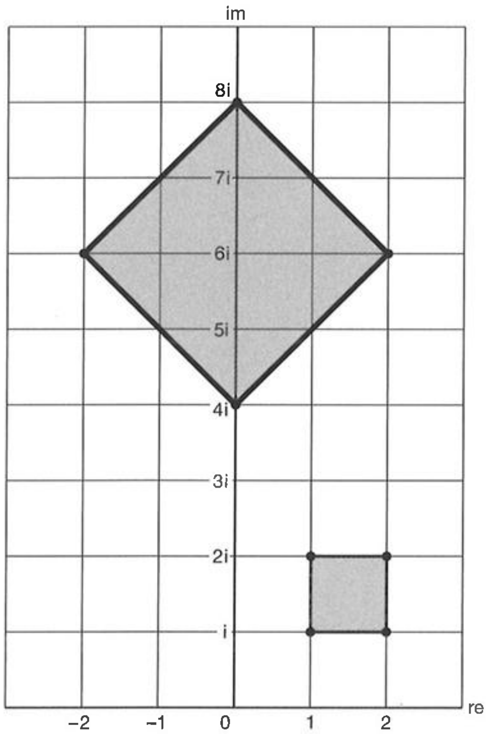
c De rotatie om 0 over $-\frac{1}{2}\pi$.
De vermenigvuldiging ten opzichte van 0 met 2.

Bladzijde 162

9



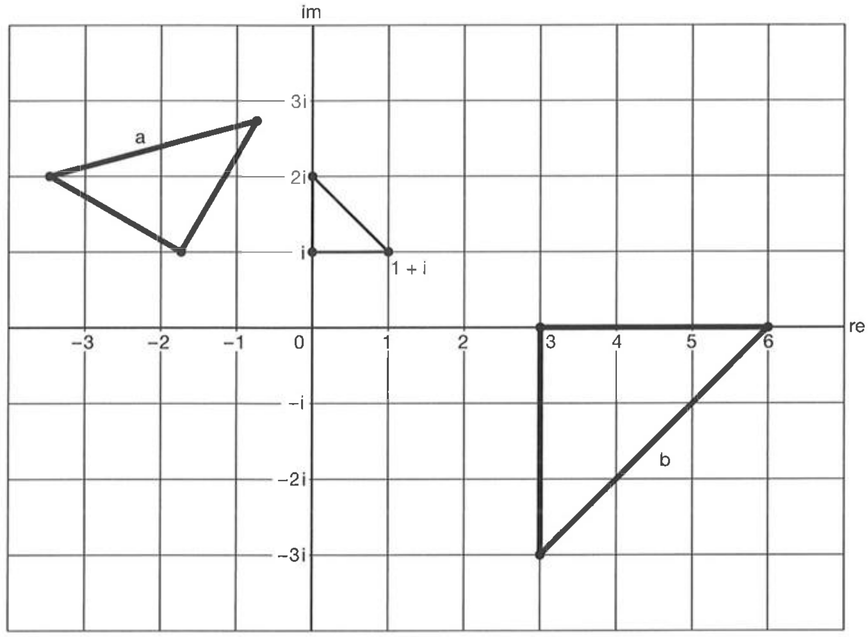
10 **a** $|2 + 2\text{i}| = 2\sqrt{2}$ en $\text{Arg}(2 + 2\text{i}) = \frac{1}{4}\pi$, dus rotatie om 0 over $\frac{1}{4}\pi$ en vermenigvuldiging ten opzichte van 0 met $2\sqrt{2}$.



b Het bereik is $2\sqrt{2} \leq |z| \leq 4\sqrt{2} \wedge \frac{5}{12}\pi \leq \text{Arg}(z) \leq \frac{7}{12}\pi$.

11 **a,b** $\left|1 + i\sqrt{3}\right| = 2$ en $\text{Arg}\left(1 + i\sqrt{3}\right) = \frac{1}{3}\pi$, dus rotatie om 0 over $\frac{1}{3}\pi$ en vermenigvuldiging ten opzichte van 0 met 2.

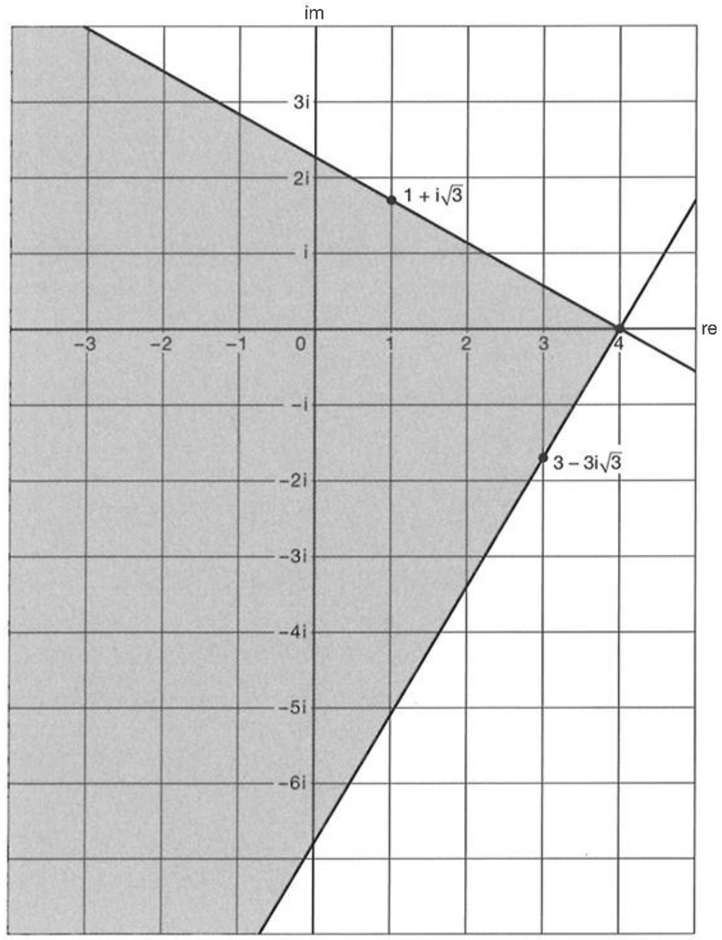
$|-3i| = 3$ en $\text{Arg}(-3i) = -\frac{1}{2}\pi$, dus rotatie om 0 over $-\frac{1}{2}\pi$ en vermenigvuldiging ten opzichte van 0 met 3.



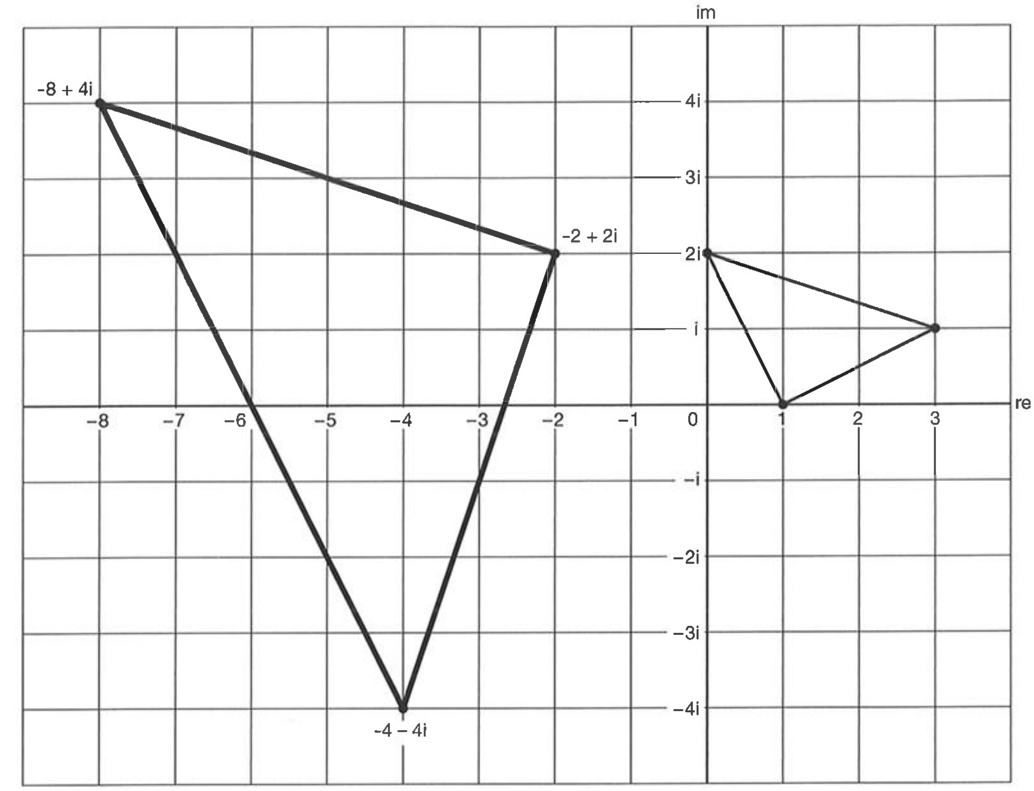
12 **a** $\left|\sqrt{3} - i\right| = 2$ en $\text{Arg}\left(\sqrt{3} - i\right) = -\frac{1}{6}\pi$, dus het bereik is $20 \leq z \leq 40 \wedge \frac{1}{3}\pi \leq \text{Arg}(z) \leq \frac{5}{6}\pi$.

b Het bereik is $z \geq 6 \wedge -\frac{1}{6}\pi \leq \text{Arg}(z) \leq \frac{1}{3}\pi$.

c De lijnen $\text{Re}(z) = \sqrt{3}$ en $\text{Im}(z) = 1$ snijden elkaar in $\sqrt{3} + i$.
$f\left(\sqrt{3} + i\right) = \left(\sqrt{3} - i\right)\left(\sqrt{3} + i\right) = 3 - i^2 = 3 + 1 = 4$
$\sqrt{3}$ ligt op $\text{Re}(z) = \sqrt{3}$.
$f\left(\sqrt{3}\right) = \left(\sqrt{3} - i\right) \cdot \sqrt{3} = 3 - i\sqrt{3}$
Het beeld van $\text{Re}(z) = \sqrt{3}$ is de lijn door 4 en $3 - i\sqrt{3}$.
i ligt op $\text{Im}(z) = 1$.
$f(i) = \left(\sqrt{3} - i\right) \cdot i = i\sqrt{3} - i^2 = 1 + i\sqrt{3}$
Het beeld van $\text{Im}(z) = 1$ is de lijn door 4 en $1 + i\sqrt{3}$.



13 **a** $|-2+2i| = 2\sqrt{2}$ en $\text{Arg}(-2+2i) = \frac{3}{4}\pi$, dus rotatie om 0 over $\frac{3}{4}\pi$ en vermenigvuldiging ten opzichte van 0 met $2\sqrt{2}$.
$f(1) = (-2+2i) \cdot 1 = -2+2i$
$f(3+i) = (-2+2i) \cdot (3+i) = -8+4i$
$f(2i) = (-2+2i) \cdot 2i = -4-4i$



b $f(z) = 3+2i$ geeft $(-2+2i)z = 3+2i$

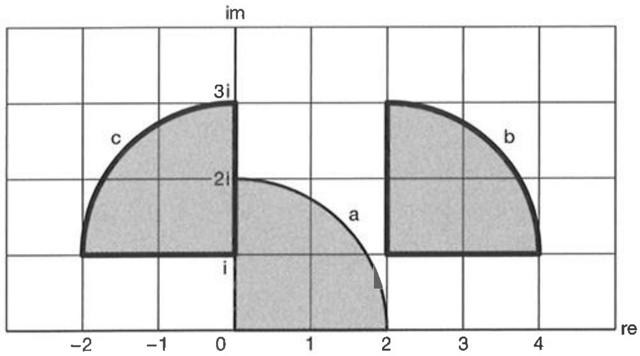
$$z = \frac{3+2i}{-2+2i} \cdot \frac{-2-2i}{-2-2i} = \frac{-6-10i-4i^2}{4-4i^2} = \frac{-2-10i}{8} = -\frac{1}{4} - 1\frac{1}{4}i$$

c $f(z) = f\left(\frac{1}{z}\right)$ geeft $(-2+2i)z = (-2+2i) \cdot \frac{1}{z}$

$$z = \frac{1}{z}$$
$$z^2 = 1$$
$$z = 1 \vee z = -1$$

Bladzijde 163

14 **a,b,c**



Bladzijde 164

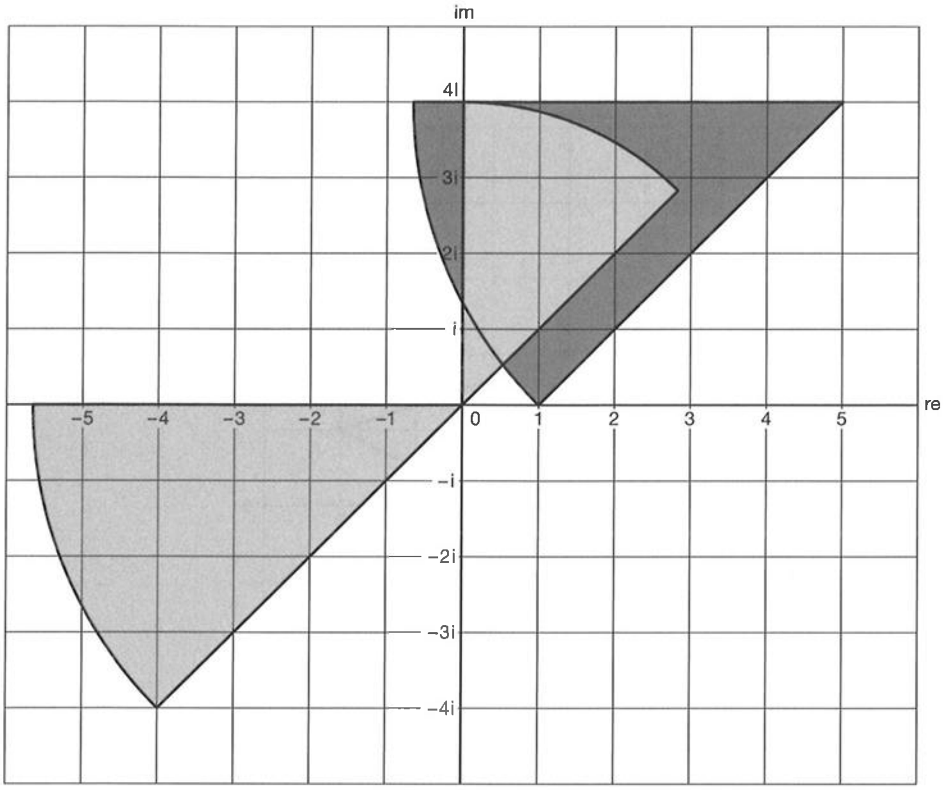
15 **a** $f(z) = 10+i$ geeft $(-1+i)z + 5 + 4i = 10 + i$

$$(-1+i)z = 5-3i$$
$$z = \frac{5-3i}{-1+i} \cdot \frac{-1-i}{-1-i} = \frac{-5-2i+3i^2}{1-i^2} = \frac{-8-2i}{2} = -4-i$$

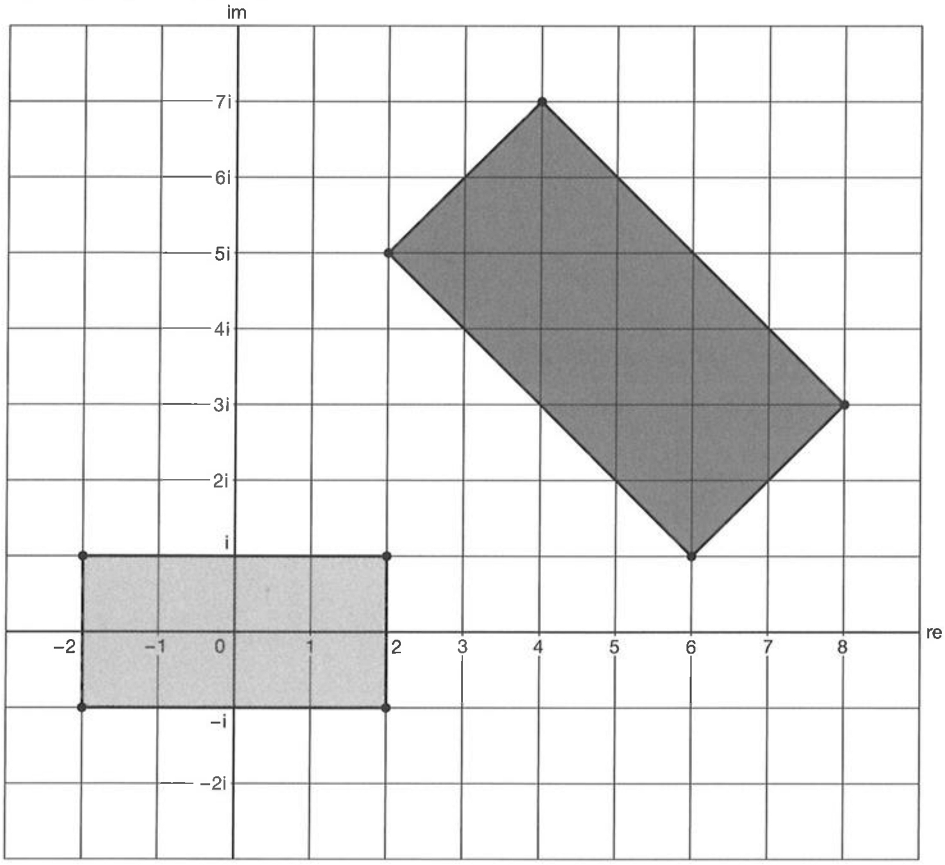
16

16

b $|-1 + i| = \sqrt{2}$ en $\text{Arg}(-1 + i) = \frac{3}{4}\pi$, dus rotatie om 0 over $\frac{3}{4}\pi$, vermenigvuldiging ten opzichte van 0 met $\sqrt{2}$ en translatie (5, 4).



c $f(-2 + i) = (-1 + i)(-2 + i) + 5 + 4i = 6 + i$
$f(-2 - i) = (-1 + i)(-2 - i) + 5 + 4i = 8 + 3i$
$f(2 - i) = (-1 + i)(2 - i) + 5 + 4i = 4 + 7i$
$f(2 + i) = (-1 + i)(2 + i) + 5 + 4i = 2 + 5i$



d Het beeld van V heeft zijde $\sqrt{10}$.
Dus V heeft zijde $\dfrac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$.
De oppervlakte van V is dus 5.

16 **a** $f(z) = 0$ geeft $(1 + i\sqrt{3})z - 2 + i = 0$

$$(1 + i\sqrt{3})z = 2 - i$$

$$z = \frac{2 - i}{1 + i\sqrt{3}} \cdot \frac{1 - i\sqrt{3}}{1 - i\sqrt{3}} = \frac{2 - 2i\sqrt{3} - i + i^2\sqrt{3}}{1 - 3i^2} = \frac{2 - \sqrt{3} - 2i\sqrt{3} - i}{4} = \tfrac{1}{2} - \tfrac{1}{4}\sqrt{3} + (-\tfrac{1}{4} - \tfrac{1}{2}\sqrt{3})i$$

Het nulpunt van f is $\tfrac{1}{2} - \tfrac{1}{4}\sqrt{3} + (-\tfrac{1}{4} - \tfrac{1}{2}\sqrt{3})i$.

$f(z) = z$ geeft $(1 + i\sqrt{3})z - 2 + i = z$

$$i\sqrt{3} \cdot z = 2 - i$$

$$z = \frac{2 - i}{i\sqrt{3}} \cdot \frac{i\sqrt{3}}{i\sqrt{3}} = \frac{2i\sqrt{3} - i^2\sqrt{3}}{3i^2} = \frac{2i\sqrt{3} + \sqrt{3}}{-3} = -\tfrac{1}{3}\sqrt{3} - \tfrac{2}{3}i\sqrt{3}$$

Het dekpunt van f is $-\tfrac{1}{3}\sqrt{3} - \tfrac{2}{3}i\sqrt{3}$.

b $g(z) = 0$ geeft $-2iz + 1 - 3i = 0$

$$-2iz = -1 + 3i$$

$$z = \frac{-1 + 3i}{-2i} \cdot \frac{i}{i} = \frac{-i + 3i^2}{-2i^2} = \frac{-i - 3}{2} = -1\tfrac{1}{2} - \tfrac{1}{2}i$$

Het nulpunt van g is $-1\tfrac{1}{2} - \tfrac{1}{2}i$.

$g(z) = z$ geeft $-2iz + 1 - 3i = z$

$$-z - 2iz = -1 + 3i$$

$$z + 2iz = 1 - 3i$$

$$z(1 + 2i) = 1 - 3i$$

$$z = \frac{1 - 3i}{1 + 2i} \cdot \frac{1 - 2i}{1 - 2i} = \frac{1 - 5i + 6i^2}{1 - 4i^2} = \frac{-5 - 5i}{5} = -1 - i$$

Het dekpunt van g is $-1 - i$.

17 **a** $f(z) = 0$ geeft $(\sqrt{3} + i)z + 1 + 2i = 0$

$$(\sqrt{3} + i)z = -1 - 2i$$

$$z = \frac{-1 - 2i}{\sqrt{3} + i} \cdot \frac{\sqrt{3} - i}{\sqrt{3} - i} = \frac{-\sqrt{3} + i - 2i\sqrt{3} + 2i^2}{3 - i^2} = \frac{-2 - \sqrt{3} + i - 2i\sqrt{3}}{4} = -\tfrac{1}{2} - \tfrac{1}{4}\sqrt{3} + (\tfrac{1}{4} - \tfrac{1}{2}\sqrt{3})i$$

Het nulpunt van f is $-\tfrac{1}{2} - \tfrac{1}{4}\sqrt{3} + (\tfrac{1}{4} - \tfrac{1}{2}\sqrt{3})i$.

$f(z) = z$ geeft $(\sqrt{3} + i)z + 1 + 2i = z$

$$(\sqrt{3} - 1 + i)z = -1 - 2i$$

$$z = \frac{-1 - 2i}{\sqrt{3} - 1 + i} \cdot \frac{\sqrt{3} - 1 - i}{\sqrt{3} - 1 - i} = \frac{-\sqrt{3} + 1 + i - 2i\sqrt{3} + 2i + 2i^2}{(\sqrt{3} - 1)^2 - i^2} = \frac{-\sqrt{3} + 1 + i - 2i\sqrt{3} + 2i - 2}{3 - 2\sqrt{3} + 1 + 1}$$

$$= \frac{-\sqrt{3} - 1 + 3i - 2i\sqrt{3}}{5 - 2\sqrt{3}} \cdot \frac{5 + 2\sqrt{3}}{5 + 2\sqrt{3}} = \frac{-5\sqrt{3} - 6 - 5 - 2\sqrt{3} + 15i + 6i\sqrt{3} - 10i\sqrt{3} - 12i}{25 - 12}$$

$$= \frac{-11 - 7\sqrt{3} + 3i - 4i\sqrt{3}}{13} = -\tfrac{11}{13} - \tfrac{7}{13}\sqrt{3} + (\tfrac{3}{13} - \tfrac{4}{13}\sqrt{3})i$$

Het dekpunt van f is $-\tfrac{11}{13} - \tfrac{7}{13}\sqrt{3} + (\tfrac{3}{13} - \tfrac{4}{13}\sqrt{3})i$.

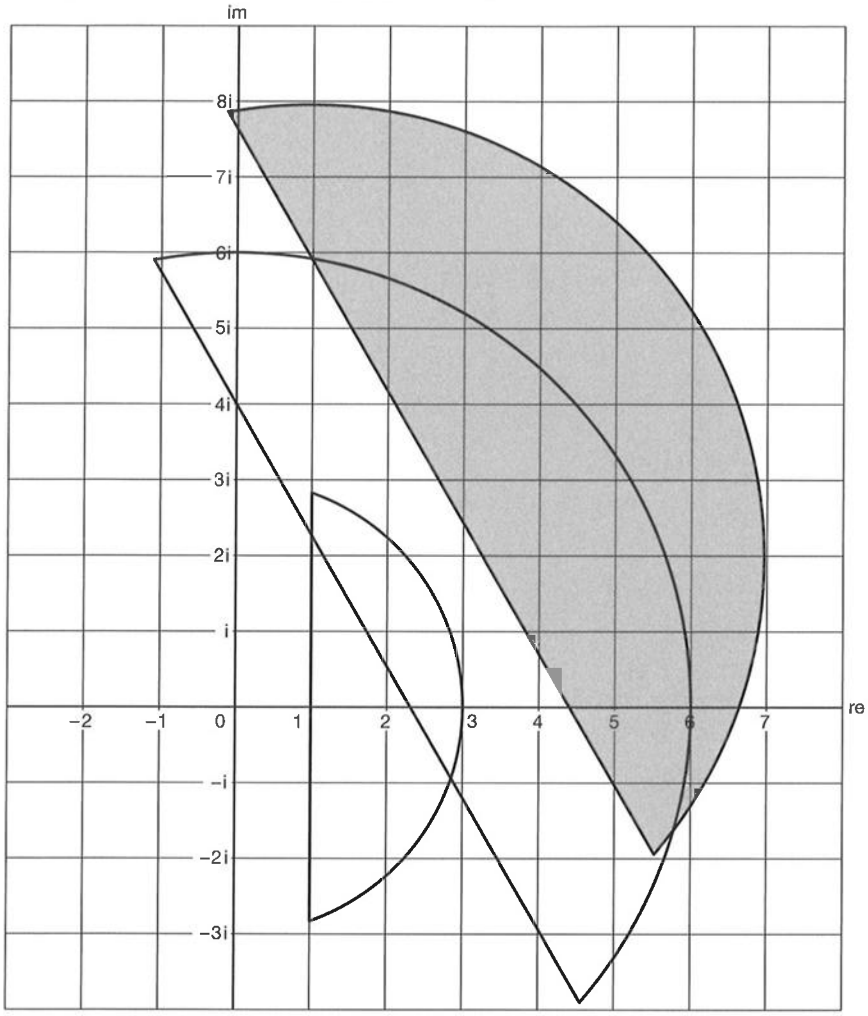
b $f(z) = 5 + 3i$ geeft $(\sqrt{3} + i)z + 1 + 2i = 5 + 3i$

$$(\sqrt{3} + i)z = 4 + i$$

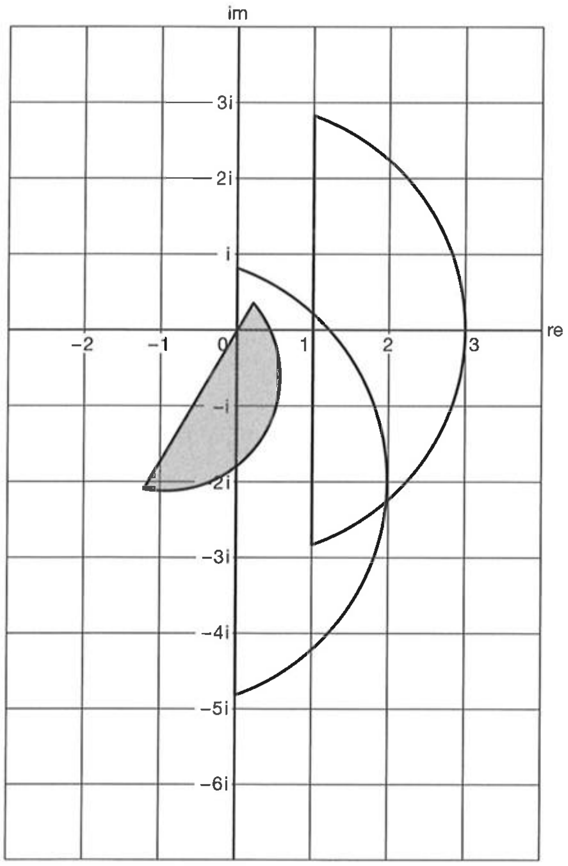
$$z = \frac{4 + i}{\sqrt{3} + i} \cdot \frac{\sqrt{3} - i}{\sqrt{3} - i} = \frac{4\sqrt{3} - 4i + i\sqrt{3} - i^2}{3 - i^2} = \frac{1 + 4\sqrt{3} - 4i + i\sqrt{3}}{4} = \tfrac{1}{4} + \sqrt{3} + (-1 + \tfrac{1}{4}\sqrt{3})i$$

Het origineel van $5 + 3i$ is $\tfrac{1}{4} + \sqrt{3} + (-1 + \tfrac{1}{4}\sqrt{3})i$.

c $|\sqrt{3}+\mathrm{i}| = 2$ en $\operatorname{Arg}(\sqrt{3}+\mathrm{i}) = \frac{1}{6}\pi$, dus rotatie om 0 over $\frac{1}{6}\pi$, vermenigvuldiging ten opzichte van 0 met 2 en translatie (1, 2).



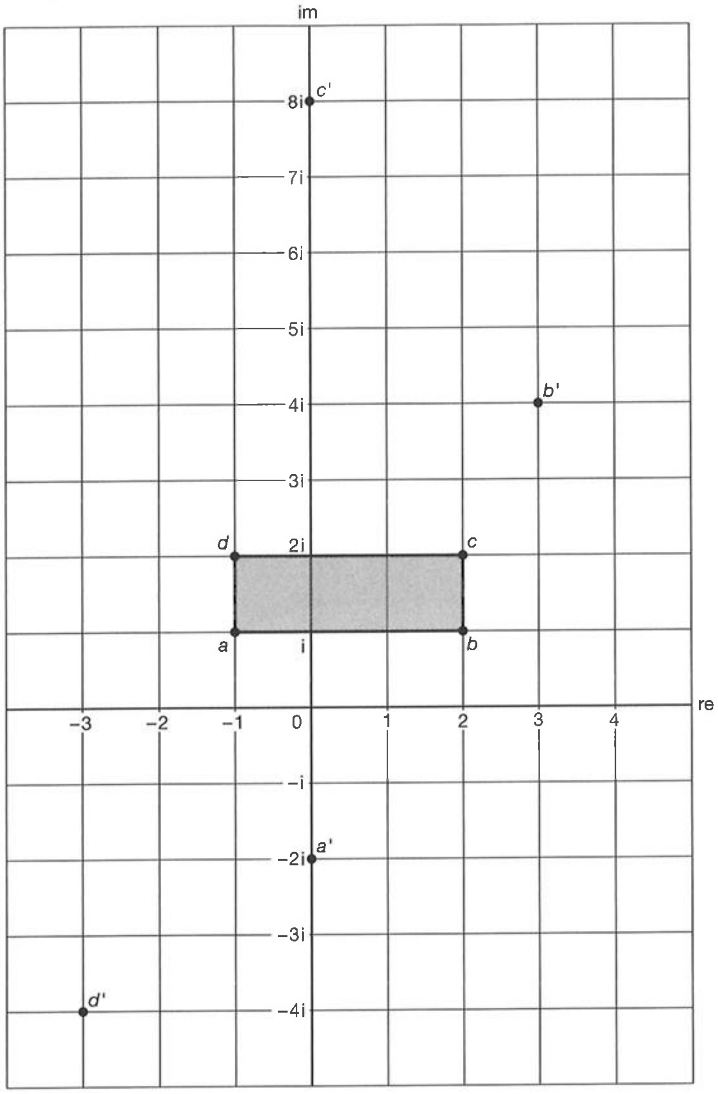
d Translatie (–1, –2), vermenigvuldiging ten opzichte van 0 met $\frac{1}{2}$ en rotatie om 0 over $-\frac{1}{6}\pi$.



16.2 Niet-lineaire complexe functies

Bladzijde 166

18 **a**



b $a' = (-1 + i)^2 = 1 - 2i - 1 = -2i$

$b' = (2 + i)^2 = 4 + 4i - 1 = 3 + 4i$

$c' = (2 + 2i)^2 = 4 + 8i - 4 = 8i$

$d' = (-1 + 2i)^2 = 1 - 4i - 4 = -3 - 4i$

c Nee, want $-2i$, $3 + 4i$, $8i$ en $-3 - 4i$ zijn niet de hoekpunten van een rechthoek.

Bladzijde 168

19 **a** $f(z) = 0$ geeft $z^2 + 2 = 0$

$z^2 = -2$

$z^2 = 2i^2$

$z = i\sqrt{2} \vee z = -i\sqrt{2}$

De nulpunten zijn $i\sqrt{2}$ en $-i\sqrt{2}$.

b $f(z) = z$ geeft $z^2 + 2 = z$

$z^2 - z + 2 = 0$

$\left(z - \frac{1}{2}\right)^2 - \frac{1}{4} + 2 = 0$

$\left(z - \frac{1}{2}\right)^2 = -1\frac{3}{4}$

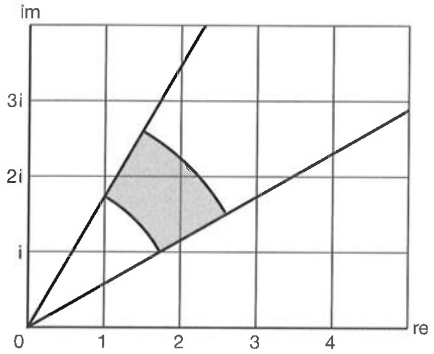
$\left(z - \frac{1}{2}\right)^2 = \frac{7}{4}i^2$

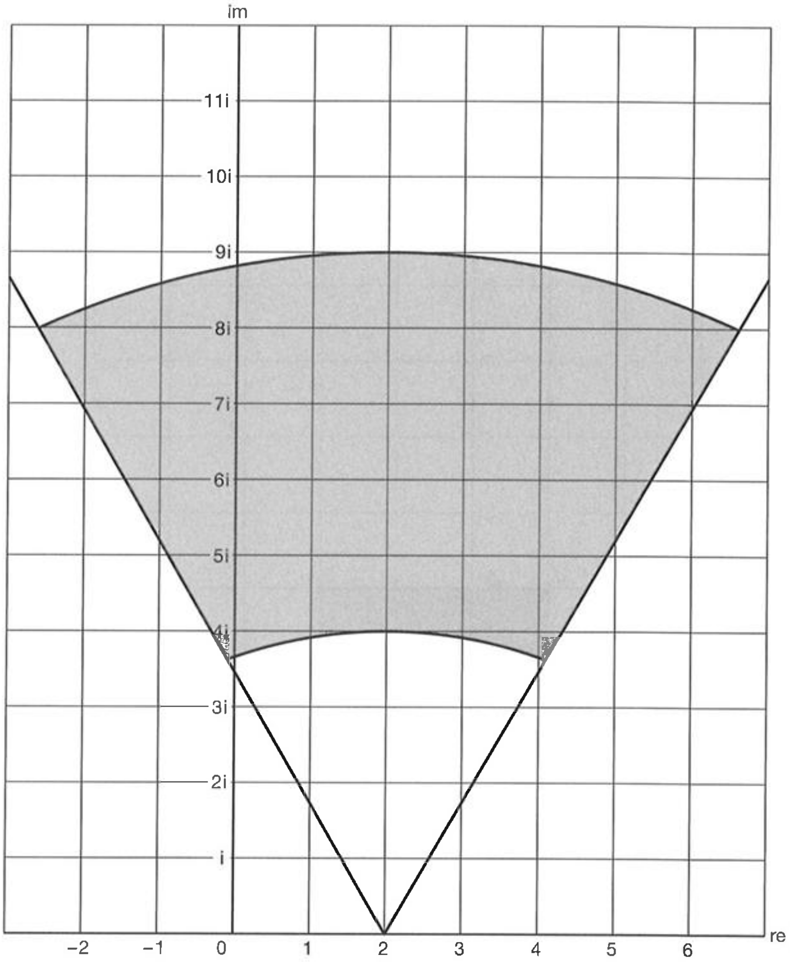
$z - \frac{1}{2} = \frac{1}{2}i\sqrt{7} \vee z - \frac{1}{2} = -\frac{1}{2}i\sqrt{7}$

$z = \frac{1}{2} + \frac{1}{2}i\sqrt{7} \vee z = \frac{1}{2} - \frac{1}{2}i\sqrt{7}$

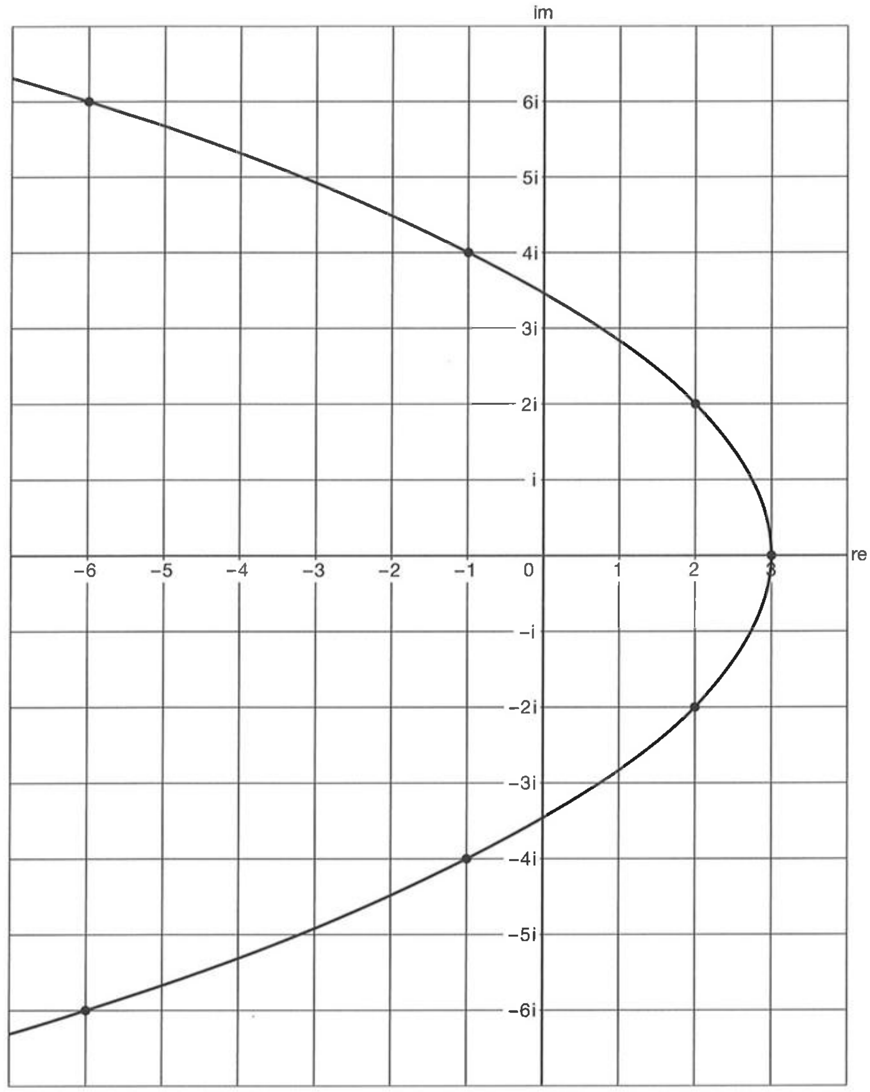
De dekpunten zijn $\frac{1}{2} + \frac{1}{2}i\sqrt{7}$ en $\frac{1}{2} - \frac{1}{2}i\sqrt{7}$.

c Eerst z^2: $|z^2| = |z|^2$, dus $4 \leq |z^2| \leq 9$ en $\arg(z^2) = 2 \cdot \arg(z)$, dus $\frac{1}{3}\pi \leq \arg(z^2) \leq \frac{2}{3}\pi$.
Dan de translatie (2, 0).





d $f(1) = 1^2 + 2 = 3$
$f(1 + i) = (1 + i)^2 + 2 = 2 + 2i$
$f(1 - i) = (1 - i)^2 + 2 = 2 - 2i$
$f(1 + 2i) = (1 + 2i)^2 + 2 = -1 + 4i$
$f(1 - 2i) = (1 - 2i)^2 + 2 = -1 - 4i$
$f(1 + 3i) = (1 + 3i)^2 + 2 = -6 + 6i$
$f(1 - 3i) = (1 - 3i)^2 + 2 = -6 - 6i$



20 **a** $f(z) = 0$ geeft $\mathrm{i}z^2 - 4 = 0$

$$\mathrm{i}z^2 = 4$$

$$z^2 = \frac{4}{\mathrm{i}} = -4\mathrm{i}$$

$$z^2 = 4\mathrm{e}^{-\frac{1}{2}\pi\mathrm{i} + k \cdot 2\pi\mathrm{i}}$$

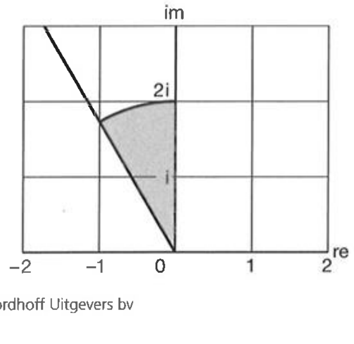
$$z = 2\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i} + k \cdot \pi\mathrm{i}}$$

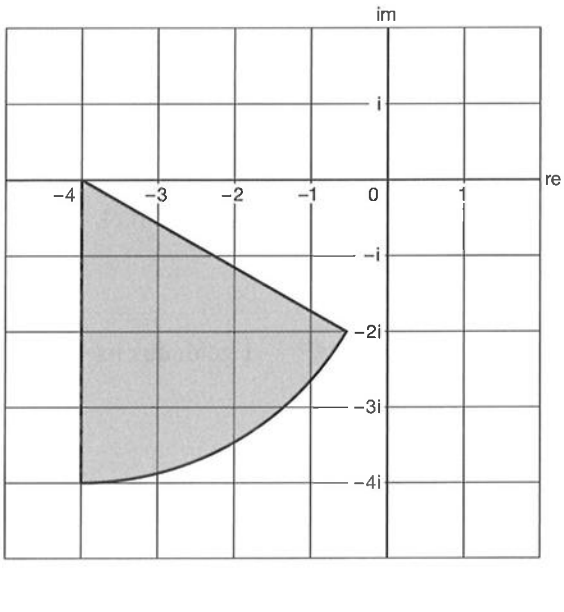
$$z = \sqrt{2} - \mathrm{i}\sqrt{2} \vee z = -\sqrt{2} + \mathrm{i}\sqrt{2}$$

De nulpunten zijn $\sqrt{2} - \mathrm{i}\sqrt{2}$ en $-\sqrt{2} + \mathrm{i}\sqrt{2}$.

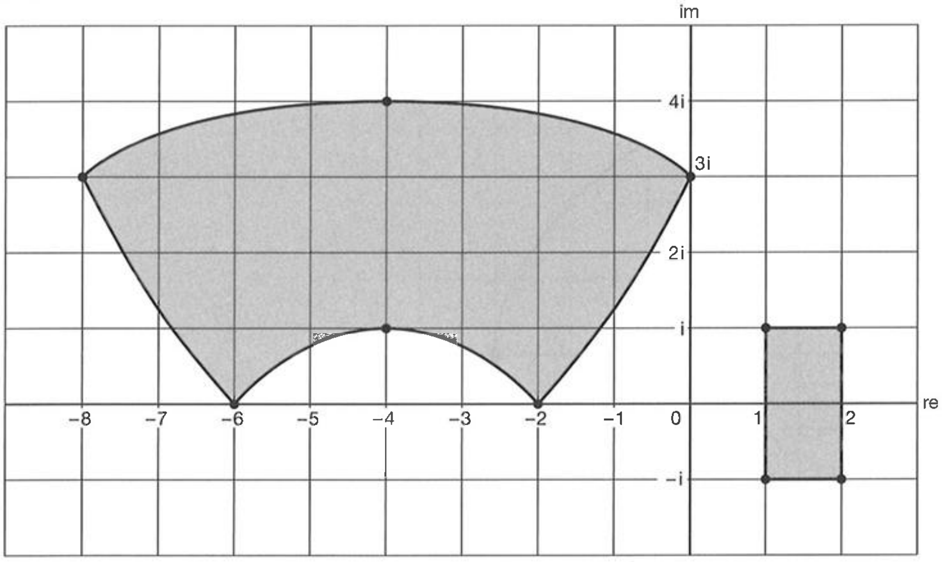
b Eerst $\mathrm{i}z^2$: $|\mathrm{i}z^2| = |\mathrm{i}| \cdot |z^2| = 1 \cdot |z|^2$, dus $|\mathrm{i}z^2| \leq 4$ en $\arg(\mathrm{i}z^2) = \arg(\mathrm{i}) + \arg(z^2) = \frac{1}{2}\pi + 2 \cdot \arg(z)$, dus $1\frac{1}{2}\pi \leq \arg(\mathrm{i}z^2) \leq 1\frac{5}{6}\pi$.

Dan de translatie $(-4, 0)$.





c $f(1 - i) = i(1 - i)^2 - 4 = -2$
$f(2 - i) = i(2 - i)^2 - 4 = 3i$
$f(2 + i) = i(2 + i)^2 - 4 = -8 + 3i$
$f(1 + i) = i(1 + i)^2 - 4 = -6$
$f(2) = -4 + 4i$
$f(1) = -4 + i$



Bladzijde 169

21 **a** Het beeld van $\text{Re}(z) = 1$ en van $\text{Re}(z) = -\text{Im}(z)$ gaat door $-2 - 2i$.
Dus $z = 1 - i$.
$f(1 - i) = (1 - i)^3 = (1 - i)^2 \cdot (1 - i) = -2i \cdot (1 - i) = -2i - 2 = -2 - 2i$ klopt

b $|z^3| = |z|^3$ en $\arg(z^3) = 3\arg(z)$.
Voor alle punten van een lijn door 0 geldt dus dat ze op één lijn liggen waarvan de hoek ten opzichte van de reële as drie keer zo groot is geworden.
Het beeld van de lijn is dus weer een rechte lijn.

c Voor deze punten moet gelden $\arg(z) = \frac{1}{3}\pi \vee \arg(z) = -\frac{1}{3}\pi$.
Dus $z = 1 + i\sqrt{3} \vee z = 1 - i\sqrt{3}$.

22 **a** $f(0) = e^0 = 1$
$f(1) = e^1 = e$
$f(-2) = e^{-2} = \dfrac{1}{e^2}$

b De functie $f(z) = e^z$ geeft voor elk reëel origineel een positief reëel beeld, dus de functie $f(z) = e^z$ beeldt de reële as af op de positieve reële as.

c $|f(\frac{1}{4}\pi i)| = |e^{\frac{1}{4}\pi i}| = |\cos(\frac{1}{4}\pi) + i\sin(\frac{1}{4}\pi)| = |\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}| = \sqrt{(\frac{1}{2}\sqrt{2})^2 + (\frac{1}{2}\sqrt{2})^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{1} = 1$
$\text{Arg}(f(\frac{1}{4}\pi i)) = \text{Arg}(e^{\frac{1}{4}\pi i}) = \frac{1}{4}\pi$

d $|f(\frac{3}{4}\pi i)| = |e^{\frac{3}{4}\pi i}| = |\cos(\frac{3}{4}\pi) + i\sin(\frac{3}{4}\pi)| = |-\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}| = \sqrt{(-\frac{1}{2}\sqrt{2})^2 + (\frac{1}{2}\sqrt{2})^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{1} = 1$
$\text{Arg}(f(\frac{3}{4}\pi i)) = \text{Arg}(e^{\frac{3}{4}\pi i}) = \frac{3}{4}\pi$

e $|f(2\frac{1}{4}\pi i)| = |e^{2\frac{1}{4}\pi i}| = |\cos(2\frac{1}{4}\pi) + i\sin(2\frac{1}{4}\pi)| = |\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}| = \sqrt{(\frac{1}{2}\sqrt{2})^2 + (\frac{1}{2}\sqrt{2})^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{1} = 1$
$\text{Arg}(f(2\frac{1}{4}\pi i)) = \text{Arg}(e^{2\frac{1}{4}\pi i}) = \frac{1}{4}\pi$

f $|f(\alpha i)| = |e^{\alpha i}| = 1$ en $\text{Arg}(f(\alpha i)) = \text{Arg}(e^{\alpha i}) = \alpha$
Dus het beeld van een zuiver imaginair getal ligt op de eenheidscirkel.

g $f(z + k \cdot 2\pi i) = e^{z + k \cdot 2\pi i} = e^z \cdot e^{k \cdot 2\pi i} = e^z \cdot (e^{2\pi i})^k = e^z \cdot 1^k = e^z$
Dus de functie $f(z) = e^z$ is periodiek.

23 **a** $\ln(e^{i\varphi}) = i\varphi$ met $\varphi = \pi$ geeft $\ln(e^{\pi i}) = \pi i$. En omdat $e^{\pi i} = -1$ geldt dus $\ln(-1) = \pi i$.

b $\ln(-1) = 3\pi i$, want $e^{3\pi i} = e^{\pi i} = -1$.

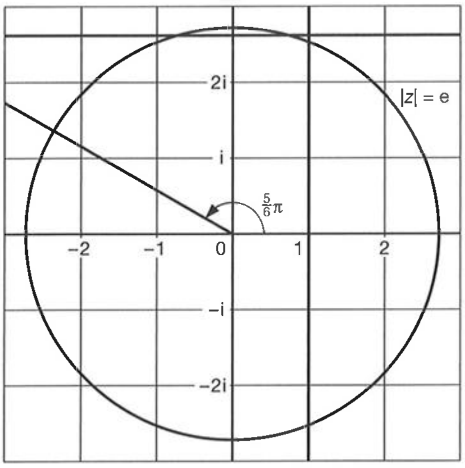
c $e^{\frac{1}{2}\pi i} = i$, dus $f(i) = \ln(i) = \ln(e^{\frac{1}{2}\pi i}) = \frac{1}{2}\pi i$.

Bladzijde 171

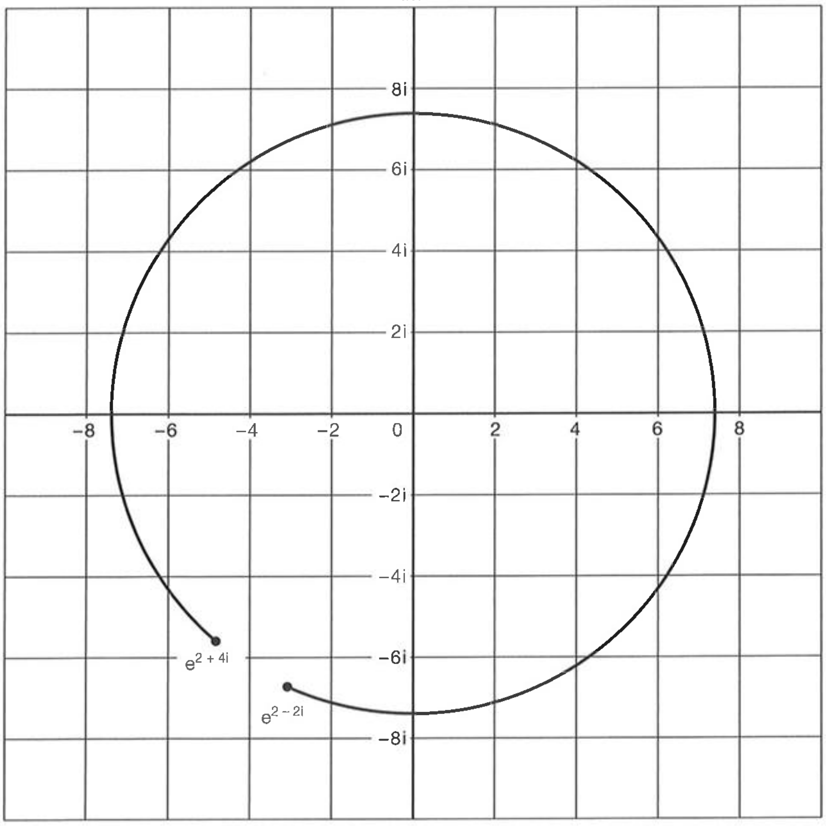
24 **a** $f(a + bi) = e^{a+bi} = e^a \cdot e^{bi} = e^a(\cos(b) + i\sin(b))$
Bij vaste a en variabele b krijg je dus de verzameling complexe getallen die op afstand e^a van 0 liggen en waarbij elke draaiingshoek b kan voorkomen.
Dus bij de functie $f(z) = e^z$ is het beeld van $a + bi$ met a vast de cirkel met middelpunt 0 en straal e^a.

b Bij vaste a en vaste b is $e^a(\cos(b) + i\sin(b))$ een punt in het complexe vlak met afstand e^a tot 0 en draaiingshoek b om 0 ten opzichte van de positieve reële as.
Bij vaste b en variabele a krijg je dus punten die alle hetzelfde argument hebben, maar waarvan de afstanden tot 0 verschillen. Deze punten vormen samen de halve lijn die een hoek van b radialen maakt met de positieve reële as.
Dus bij de functie $f(z) = e^z$ is het beeld van $a + bi$ met b vast de halve lijn die een hoek van b radialen maakt met de positieve reële as.

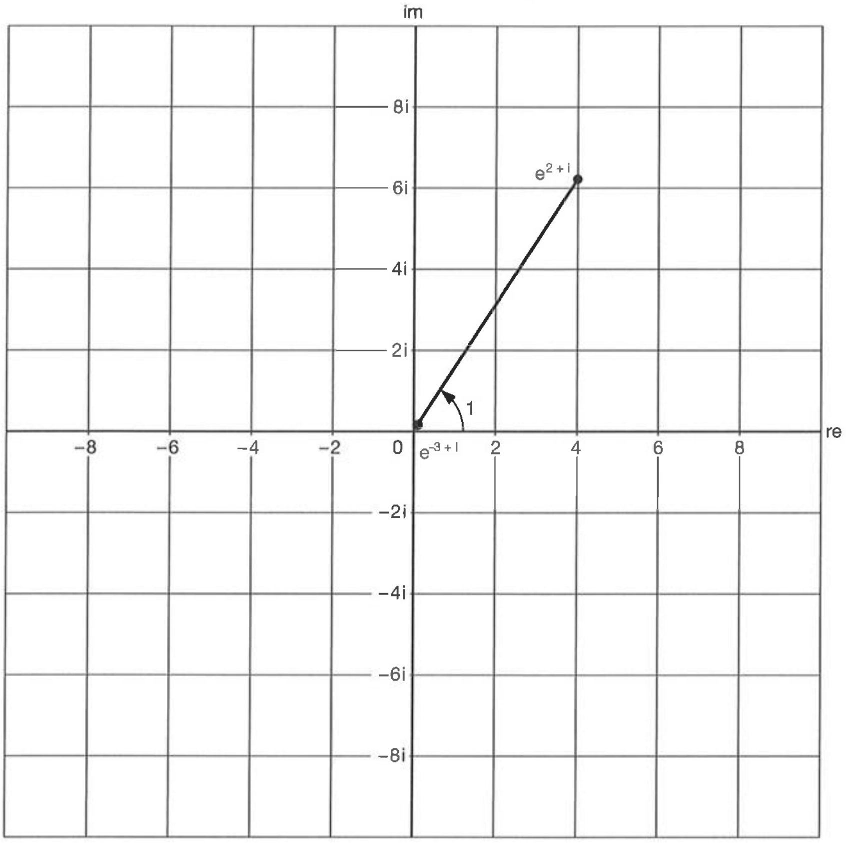
25 Het beeld van $\text{Re}(z) = 1$ is de cirkel met middelpunt 0 en straal e^1, ofwel de cirkel met vergelijking $|z| = e$.
Het beeld van $\text{Im}(z) = \frac{5}{6}\pi$ is de halve lijn met beginpunt 0 die een hoek van $\frac{5}{6}\pi$ radialen maakt met de positieve reële as, ofwel de halve lijn met vergelijking $\text{Arg}(z) = \frac{5}{6}\pi$.



26 **a** Het beeld van $\text{Re}(z) = 2$ is de cirkel met middelpunt 0 en straal e^2.
Het beeld van $\text{Im}(z) = 4$ heeft argument 4 radialen en het beeld van $\text{Im}(z) = -2$ heeft argument -2 radialen.
Dus het beeld van het lijnstuk bestaat uit de getallen z waarvoor geldt $|z| = e^2 \wedge -2 < \arg(z) < 4$.



b Het beeld van $\text{Im}(z) = 1$ bestaat uit getallen met argument 1 radiaal.
Het beeld van $\text{Re}(z) = -3$ heeft modulus e^{-3} en het beeld van $\text{Re}(z) = 2$ heeft modulus e^2.
Dus het beeld van het lijnstuk bestaat uit alle getallen z waarvoor geldt $e^{-3} \leq |z| < e^2$ en $\text{Arg}(z) = 1$.



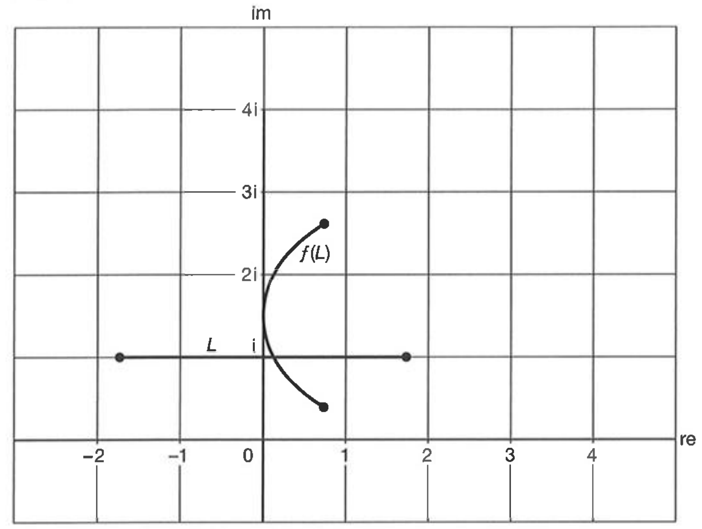
27 **a** $i^i = \left(e^{\frac{1}{2}\pi i}\right)^i = e^{\frac{1}{2}\pi i^2} = e^{-\frac{1}{2}\pi}$
b $i^{2i} = \left(e^{\frac{1}{2}\pi i}\right)^{2i} = e^{\pi i^2} = e^{-\pi}$
$i^{1+i} = i^1 \cdot i^i = i \cdot e^{-\frac{1}{2}\pi}$

28 **a** $f(-e) = \ln(-e) = \ln(-1) + \ln(e) = \ln(e^{\pi i}) + 1 = 1 + \pi i$
b $f(-e^2) = \ln(-e^2) = \ln(-1) + \ln(e^2) = \ln(e^{\pi i}) + 2 = 2 + \pi i$
c $f(ei) = \ln(ei) = \ln(e) + \ln(i) = 1 + \ln\left(e^{\frac{1}{2}\pi i}\right) = \frac{1}{2}\pi i + 1$
d $f(3i) = \ln(3i) = \ln(3) + \ln(i) = \ln(3) + \ln\left(e^{\frac{1}{2}\pi i}\right) = \frac{1}{2}\pi i + \ln(3)$
e $f(-3) = \ln(-3) = \ln(-1) + \ln(3) = \ln(e^{\pi i}) + \ln(3) = \ln(3) + \pi i$
f $f(-2i) = \ln(-2i) = \ln(2) + \ln(-i) = \ln(2) + \ln\left(e^{-\frac{1}{2}\pi i}\right) = \ln(2) - \frac{1}{2}\pi i$
g $f(2 - 2i) = \ln(2 - 2i) = \ln\left(2\sqrt{2}\, e^{-\frac{1}{4}\pi i}\right) = \ln\left(2\sqrt{2}\right) + \ln\left(e^{-\frac{1}{4}\pi i}\right) = 1\frac{1}{2}\ln(2) - \frac{1}{4}\pi i$
h $f\left(\sqrt{3} - i\right) = \ln\left(\sqrt{3} - i\right) = \ln\left(2e^{-\frac{1}{6}\pi i}\right) = \ln(2) + \ln\left(e^{-\frac{1}{6}\pi i}\right) = \ln(2) - \frac{1}{6}\pi i$
i $f(e + ei) = \ln(e + ei) = \ln(e(1 + i)) = \ln(e) + \ln(1 + i) = 1 + \ln\left(\sqrt{2}e^{\frac{1}{4}\pi i}\right) = 1 + \ln\left(\sqrt{2}\right) + \ln\left(e^{\frac{1}{4}\pi i}\right) = 1 + \frac{1}{2}\ln(2) + \frac{1}{4}\pi i$

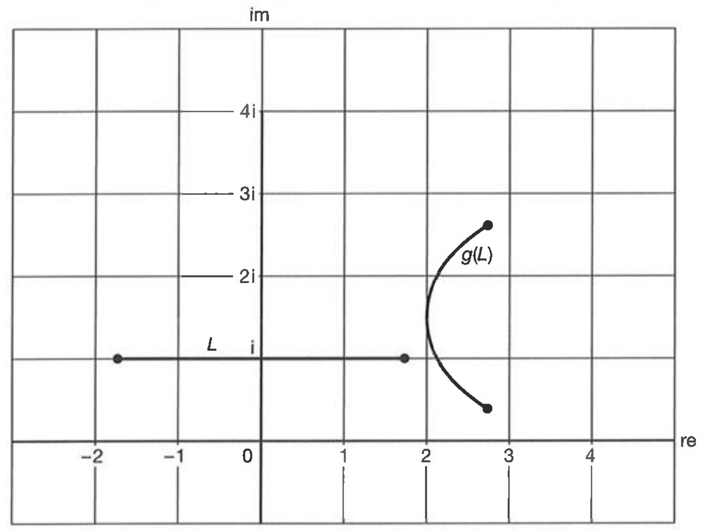
Bladzijde 172

29 **a** $f\left(-e\sqrt{e}\right) = \ln\left(-e\sqrt{e}\right) = \ln(-1) + \ln\left(e\sqrt{e}\right) = \ln(e^{\pi i}) + \ln\left(e^{1\frac{1}{2}}\right) = 1\frac{1}{2} + \pi i$
b $f\left(-i^2\sqrt{i}\right) = \ln\left(-i^2\sqrt{i}\right) = \ln\left(\sqrt{i}\right) = \frac{1}{2}\ln(i) = \frac{1}{2}\ln\left(e^{\frac{1}{2}\pi i}\right) = \frac{1}{2} \cdot \frac{1}{2}\pi i = \frac{1}{4}\pi i$
c $f\left(-\dfrac{1}{e}\right) = \ln\left(-\dfrac{1}{e}\right) = \ln(-1) - \ln(e) = \ln(e^{\pi i}) - 1 = -1 + \pi i$
d $f\left(\dfrac{2}{i\sqrt{i}}\right) = \ln\left(\dfrac{2}{i\sqrt{i}}\right) = \ln(2) - \ln\left(i\sqrt{i}\right) = \ln(2) - \ln\left(i^{1\frac{1}{2}}\right) = \ln(2) - 1\frac{1}{2}\ln(i) = \ln(2) - 1\frac{1}{2}\ln\left(e^{\frac{1}{2}\pi i}\right) =$
$\ln(2) - 1\frac{1}{2} \cdot \frac{1}{2}\pi i = \ln(2) - \frac{3}{4}\pi i$
e $f\left(\dfrac{2}{1 + i}\right) = \ln\left(\dfrac{2}{1 + i}\right) = \ln(2) - \ln(1 + i) = \ln(2) - \ln\left(\sqrt{2}e^{\frac{1}{4}\pi i}\right) = \ln(2) - \ln\left(\sqrt{2}\right) - \ln\left(e^{\frac{1}{4}\pi i}\right) =$
$\ln(2) - \frac{1}{2}\ln(2) - \frac{1}{4}\pi i = \frac{1}{2}\ln(2) - \frac{1}{4}\pi i$
f $f\left(\dfrac{e}{1 + i\sqrt{3}}\right) = \ln\left(\dfrac{e}{1 + i\sqrt{3}}\right) = \ln(e) - \ln\left(1 + i\sqrt{3}\right) = 1 - \ln\left(2e^{\frac{1}{3}\pi i}\right) = 1 - \ln(2) - \ln\left(e^{\frac{1}{3}\pi i}\right) = 1 - \ln(2) - \frac{1}{3}\pi i$

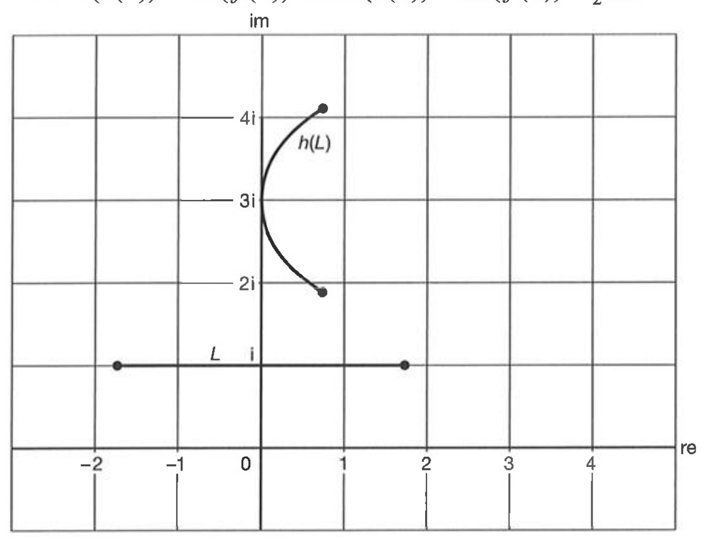
30 **a** $f(\sqrt{3} + \mathrm{i}) \approx 1{,}76 + 0{,}17\mathrm{i}$
$f(1 + \mathrm{i}) \approx 0{,}35 + 0{,}79\mathrm{i}$
$f(1) \approx 1{,}57\mathrm{i}$
$f(-1 + \mathrm{i}) \approx 0{,}35 + 2{,}36\mathrm{i}$
$f(-\sqrt{3} + \mathrm{i}) \approx 0{,}69 + 2{,}62\mathrm{i}$



b $g(\mathrm{e}^2 z) = \ln(\mathrm{e}^2 z) = \ln(\mathrm{e}^2) + \ln(z) = 2 + \ln(z)$,
dus $\mathrm{Re}(g(L)) = \mathrm{Re}(f(L)) + 2$ en $\mathrm{Im}(g(L)) = \mathrm{Im}(f(L))$.



c $h(\mathrm{i}z) = \ln(\mathrm{i}z) = \ln(\mathrm{i}) + \ln(z) = \ln\left(\mathrm{e}^{\frac{1}{2}\pi \mathrm{i}}\right) + \ln(z) = \frac{1}{2}\pi\mathrm{i} + \ln(z)$,
dus $\mathrm{Re}(h(L)) = \mathrm{Re}(f(L))$ en $\mathrm{Im}(h(L)) = \mathrm{Im}(f(L)) + \frac{1}{2}\pi\mathrm{i}$.



31 $\cos(\mathrm{i}) = \dfrac{\mathrm{e}^{\mathrm{i}^2} + \mathrm{e}^{-\mathrm{i}^2}}{2} = \dfrac{\mathrm{e}^{-1} + \mathrm{e}^{1}}{2} = \dfrac{\mathrm{e}^{-1}}{2} + \dfrac{\mathrm{e}}{2} = \dfrac{1}{2\mathrm{e}} + \dfrac{\mathrm{e}}{2} = \dfrac{\mathrm{e}}{2} + \dfrac{1}{2\mathrm{e}}$

Bladzijde 173

32 **a** $f(\mathrm{i}) = \sin(\mathrm{i}) = \dfrac{\mathrm{e}^{\mathrm{i}^2} - \mathrm{e}^{-\mathrm{i}^2}}{2\mathrm{i}} = \dfrac{\mathrm{e}^{-1} - \mathrm{e}^{1}}{2\mathrm{i}} = \dfrac{1}{2\mathrm{ei}} - \dfrac{\mathrm{e}}{2\mathrm{i}} = \left(\dfrac{\mathrm{e}}{2} - \dfrac{1}{2\mathrm{e}}\right)\mathrm{i}$

b $g\left(\frac{1}{6}\pi + \mathrm{i}\right) = \cos\left(\frac{1}{6}\pi + \mathrm{i}\right) = \dfrac{\mathrm{e}^{\mathrm{i}\cdot\left(\frac{1}{6}\pi + \mathrm{i}\right)} + \mathrm{e}^{-\mathrm{i}\cdot\left(\frac{1}{6}\pi + \mathrm{i}\right)}}{2} = \dfrac{\mathrm{e}^{-1+\frac{1}{6}\pi\mathrm{i}} + \mathrm{e}^{1-\frac{1}{6}\pi\mathrm{i}}}{2} = \dfrac{\mathrm{e}^{-1}\cdot\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}} + \mathrm{e}^{1}\cdot\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}}}{2} =$

$\dfrac{\mathrm{e}^{-1}\left(\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}\right) + \mathrm{e}\left(\frac{1}{2}\sqrt{3} - \frac{1}{2}\mathrm{i}\right)}{2} = \dfrac{\frac{1}{2}\sqrt{3}}{2\mathrm{e}} + \dfrac{\frac{1}{2}\mathrm{i}}{2\mathrm{e}} + \dfrac{\frac{1}{2}\mathrm{e}\sqrt{3}}{2} - \dfrac{\frac{1}{2}\mathrm{ei}}{2} = \dfrac{\sqrt{3}}{4\mathrm{e}} + \dfrac{\mathrm{e}\sqrt{3}}{4} + \left(\dfrac{1}{4\mathrm{e}} - \dfrac{\mathrm{e}}{4}\right)\mathrm{i}$

c $f\left(\frac{1}{4}\pi - 3\mathrm{i}\right) = \sin\left(\frac{1}{4}\pi - 3\mathrm{i}\right) = \dfrac{\mathrm{e}^{\mathrm{i}\cdot\left(\frac{1}{4}\pi - 3\mathrm{i}\right)} - \mathrm{e}^{-\mathrm{i}\cdot\left(\frac{1}{4}\pi - 3\mathrm{i}\right)}}{2\mathrm{i}} = \dfrac{\mathrm{e}^{3+\frac{1}{4}\pi\mathrm{i}} - \mathrm{e}^{-3-\frac{1}{4}\pi\mathrm{i}}}{2\mathrm{i}} = \dfrac{\mathrm{e}^{3}\cdot\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}} - \mathrm{e}^{-3}\cdot\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}}}{2\mathrm{i}} =$

$\dfrac{\mathrm{e}^{3}\left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right) - \mathrm{e}^{-3}\left(\frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}\right)}{2\mathrm{i}} = \dfrac{\frac{1}{2}\mathrm{e}^3\sqrt{2}}{2\mathrm{i}} + \dfrac{\frac{1}{2}\mathrm{e}^3\mathrm{i}\sqrt{2}}{2\mathrm{i}} - \dfrac{\frac{1}{2}\mathrm{e}^{-3}\sqrt{2}}{2\mathrm{i}} + \dfrac{\frac{1}{2}\mathrm{e}^{-3}\mathrm{i}\sqrt{2}}{2\mathrm{i}} = \dfrac{-\mathrm{e}^3\mathrm{i}\sqrt{2}}{4} + \dfrac{\mathrm{e}^3\sqrt{2}}{4} + \dfrac{\mathrm{i}\sqrt{2}}{4\mathrm{e}^3} + \dfrac{\sqrt{2}}{4\mathrm{e}^3} =$

$\dfrac{\mathrm{e}^{3}\sqrt{2}}{4} + \dfrac{\sqrt{2}}{4\mathrm{e}^{3}} + \left(\dfrac{\sqrt{2}}{4\mathrm{e}^{3}} - \dfrac{\mathrm{e}^{3}\sqrt{2}}{4}\right)\mathrm{i}$

d $g\left(\frac{1}{3}\pi + 2\mathrm{i}\right) = \cos\left(\frac{1}{3}\pi + 2\mathrm{i}\right) = \dfrac{\mathrm{e}^{\mathrm{i}\cdot\left(\frac{1}{3}\pi + 2\mathrm{i}\right)} + \mathrm{e}^{-\mathrm{i}\cdot\left(\frac{1}{3}\pi + 2\mathrm{i}\right)}}{2} = \dfrac{\mathrm{e}^{-2+\frac{1}{3}\pi\mathrm{i}} + \mathrm{e}^{2-\frac{1}{3}\pi\mathrm{i}}}{2} = \dfrac{\mathrm{e}^{-2}\cdot\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}} + \mathrm{e}^{2}\cdot\mathrm{e}^{-\frac{1}{3}\pi\mathrm{i}}}{2} =$

$\dfrac{\mathrm{e}^{-2}\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right) + \mathrm{e}^{2}\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right)}{2} = \dfrac{\frac{1}{2}}{2\mathrm{e}^2} + \dfrac{\frac{1}{2}\mathrm{i}\sqrt{3}}{2\mathrm{e}^2} + \dfrac{\frac{1}{2}\mathrm{e}^2}{2} - \dfrac{\frac{1}{2}\mathrm{e}^2\mathrm{i}\sqrt{3}}{2} = \dfrac{1}{4\mathrm{e}^2} + \dfrac{\mathrm{e}^2}{4} + \left(\dfrac{\sqrt{3}}{4\mathrm{e}^2} - \dfrac{\mathrm{e}^2\sqrt{3}}{4}\right)\mathrm{i}$

33 **a** $\sin(2z) = \dfrac{\mathrm{e}^{2\mathrm{i}z} - \mathrm{e}^{-2\mathrm{i}z}}{2\mathrm{i}}$

$2\sin(z)\cos(z) = 2\cdot\dfrac{\mathrm{e}^{\mathrm{i}z} - \mathrm{e}^{-\mathrm{i}z}}{2\mathrm{i}}\cdot\dfrac{\mathrm{e}^{\mathrm{i}z} + \mathrm{e}^{-\mathrm{i}z}}{2} = \dfrac{2((\mathrm{e}^{\mathrm{i}z})^2 - (\mathrm{e}^{-\mathrm{i}z})^2)}{4\mathrm{i}} = \dfrac{\mathrm{e}^{2\mathrm{i}z} - \mathrm{e}^{-2\mathrm{i}z}}{2\mathrm{i}}$

Dus voor elk complex getal z geldt $\sin(2z) = 2\sin(z)\cos(z)$.

b $\sin^2(z) + \cos^2(z) = \left(\dfrac{\mathrm{e}^{\mathrm{i}z} - \mathrm{e}^{-\mathrm{i}z}}{2\mathrm{i}}\right)^2 + \left(\dfrac{\mathrm{e}^{\mathrm{i}z} + \mathrm{e}^{-\mathrm{i}z}}{2}\right)^2 = \dfrac{\mathrm{e}^{2\mathrm{i}z} - 2\mathrm{e}^{0} + \mathrm{e}^{-2\mathrm{i}z}}{-4} + \dfrac{\mathrm{e}^{2\mathrm{i}z} + 2\mathrm{e}^{0} + \mathrm{e}^{-2\mathrm{i}z}}{4} =$

$\dfrac{-\mathrm{e}^{2\mathrm{i}z} + 2\mathrm{e}^{0} - \mathrm{e}^{-2\mathrm{i}z} + \mathrm{e}^{2\mathrm{i}z} + 2\mathrm{e}^{0} + \mathrm{e}^{-2\mathrm{i}z}}{4} = \dfrac{4}{4} = 1$

Dus voor elk complex getal z geldt $\sin^2(z) + \cos^2(z) = 1$.

16.3 Meetkunde met complexe getallen

Bladzijde 175

34 **a** $\vec{b} = \vec{a} + \vec{c}$, dus $b = a + c$.

b $\vec{d} = \vec{c} + \vec{a}^*$ met $\vec{a}^*$ het beeld van $\vec{a}$ na draaiing over 90°, dus $d = c + \mathrm{i}a$.

c $e = a + c + \mathrm{i}a$

d $m = c + \frac{1}{2}a + \frac{1}{2}\mathrm{i}a = \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\right)a + c$

Bladzijde 176

35 **a** $\vec{e} = \vec{a} + \vec{c} + \vec{a}_L$

$\vec{m} = \frac{1}{2}\vec{a} + \vec{c} + \frac{1}{2}\vec{a}_L$

b Met complexe getallen kun je op dezelfde manier ook draaien over hoeken anders dan veelvouden van 90°.

36 **a** $c = a + (\cos(90°) + \mathrm{i}\sin(90°))(b - a) = a + (0 + \mathrm{i})(b - a) = (1 - \mathrm{i})a + \mathrm{i}b$

b $c = a + (\cos(-90°) + \mathrm{i}\sin(-90°))(b - a) = a + (0 - \mathrm{i})(b - a) = (1 + \mathrm{i})a - \mathrm{i}b$

c $c = a + (\cos(-60°) + \mathrm{i}\sin(-60°))(b - a) = a + \left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right)(b - a) = \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)a + \left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right)b$

d $c = a + (\cos(30°) + \mathrm{i}\sin(30°))(b - a) = a + \left(\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}\right)(b - a) = \left(1 - \frac{1}{2}\sqrt{3} - \frac{1}{2}\mathrm{i}\right)a + \left(\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}\right)b$

e $c = a + (\cos(45°) + \mathrm{i}\sin(45°))(b - a) = a + \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right)(b - a) = \left(1 - \frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}\right)a + \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right)b$

f $c = a + (\cos(135°) + \mathrm{i}\sin(135°))(b - a) = a + \left(-\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right)(b - a) = \left(1 + \frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}\right)a + \left(-\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right)b$

Bladzijde 177

37 **a** $(e-c)^*$ is $e-c$ gedraaid over 90°.
$g = c-(e-c)^* = c - \text{i}(e-c) = c - \text{i}e + \text{i}c = (1+\text{i})c - \text{i}e$
$c = b + d$ en $e = d + (\cos(60°) + \text{i}\sin(60°))b = d + \left(\frac{1}{2} + \frac{1}{2}\text{i}\sqrt{3}\right)b$
Dit geeft $g = (1+\text{i})(b+d) - \text{i}\left(d + \left(\frac{1}{2} + \frac{1}{2}\text{i}\sqrt{3}\right)b\right)$
$= b + d + \text{i}b + \text{i}d - \text{i}d - \frac{1}{2}\text{i}b + \frac{1}{2}b\sqrt{3}$
$= \left(1 + \frac{1}{2}\sqrt{3} + \frac{1}{2}\text{i}\right)b + d$

b $f = e + (c-e)\text{i} = c\text{i} + (1-\text{i})e$
$c = b + d$ en $e = d + \left(\frac{1}{2} + \frac{1}{2}\text{i}\sqrt{3}\right)b$ (zie onderdeel a)
Dit geeft $f = (b+d)\text{i} + (1-\text{i})\left(d + \left(\frac{1}{2} + \frac{1}{2}\text{i}\sqrt{3}\right)b\right)$
$= \text{i}b + \text{i}d + d + \frac{1}{2}b + \frac{1}{2}\text{i}b\sqrt{3} - \text{i}d - \frac{1}{2}\text{i}b + \frac{1}{2}b\sqrt{3}$
$= \left(\frac{1}{2} + \frac{1}{2}\sqrt{3} + \frac{1}{2}\text{i} + \frac{1}{2}\text{i}\sqrt{3}\right)b + d$

38 **a** $c = b + (c-b) = b + (d-a) = b + d - a$
$e = d - \text{i}(a-d) = -\text{i}a + (1+\text{i})d$
$f = a + \text{i}(d-a) = (1-\text{i})a + \text{i}d$
$g = d + \text{i}(c-d) = d + \text{i}(b-a) = -\text{i}a + \text{i}b + d$
$m = a + \frac{1}{2}(d-a) + \frac{1}{2}\text{i}(d-a) = \left(\frac{1}{2} - \frac{1}{2}\text{i}\right)a + \left(\frac{1}{2} + \frac{1}{2}\text{i}\right)d$
$n = d + \frac{1}{2}(c-d) + \frac{1}{2}\text{i}(h-c)$
$= d + \frac{1}{2}(b-a) + \frac{1}{2}\text{i}(c-d)$
$= d + \frac{1}{2}(b-a) + \frac{1}{2}\text{i}(b-a)$
$= d + \frac{1}{2}b - \frac{1}{2}a + \frac{1}{2}\text{i}b - \frac{1}{2}\text{i}a$
$= \left(-\frac{1}{2} - \frac{1}{2}\text{i}\right)a + \left(\frac{1}{2} + \frac{1}{2}\text{i}\right)b + d$

b $g = d + \text{i}(b-a)$, dus $g - a = d - a + \text{i}(b-a)$

c $e = d - \text{i}(a-d)$ en $c = b + d - a$, dus
$e - c = d - \text{i}(a-d) - (b+d-a) = d - \text{i}(a-d) - b - d + a = a - b - \text{i}(a-d) = (1-\text{i})a - b + \text{i}d$
$\text{i}(g-a) = \text{i}(d - a + \text{i}(b-a)) = \text{i}d - \text{i}a - (b-a) = (1-\text{i})a - b + \text{i}d$
Dus $\text{i}(g-a) = e - c$.

d De lijnstukken AG en CE zijn even lang en staan loodrecht op elkaar.

39 $c = a + (\cos(45°) + \text{i}\sin(45°))(b-a)$
$= a + \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\text{i}\sqrt{2}\right)(b-a)$
$= \left(1 - \frac{1}{2}\sqrt{2} - \frac{1}{2}\text{i}\sqrt{2}\right)a + \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\text{i}\sqrt{2}\right)b$
$m = \frac{1}{2}(b+c)$
$= \frac{1}{2}\left(b + \left(1 - \frac{1}{2}\sqrt{2} - \frac{1}{2}\text{i}\sqrt{2}\right)a + \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\text{i}\sqrt{2}\right)b\right)$
$= \frac{1}{2}\left(\left(1 - \frac{1}{2}\sqrt{2} - \frac{1}{2}\text{i}\sqrt{2}\right)a + \left(1 + \frac{1}{2}\sqrt{2} + \frac{1}{2}\text{i}\sqrt{2}\right)b\right)$
$= \left(\frac{1}{2} - \frac{1}{4}\sqrt{2} - \frac{1}{4}\text{i}\sqrt{2}\right)a + \left(\frac{1}{2} + \frac{1}{4}\sqrt{2} + \frac{1}{4}\text{i}\sqrt{2}\right)b$
Dus $p = \frac{1}{2} - \frac{1}{4}\sqrt{2} - \frac{1}{4}\text{i}\sqrt{2}$ en $q = \frac{1}{2} + \frac{1}{4}\sqrt{2} + \frac{1}{4}\text{i}\sqrt{2}$.

40 $d = a + (\cos(60°) + \text{i}\sin(60°))(b-a)$
$= a + \left(\frac{1}{2} + \frac{1}{2}\text{i}\sqrt{3}\right)(b-a)$
$= \left(\frac{1}{2} - \frac{1}{2}\text{i}\sqrt{3}\right)a + \left(\frac{1}{2} + \frac{1}{2}\text{i}\sqrt{3}\right)b$
$c = b + (\cos(-90°) + \text{i}\sin(-90°))(d-b)$
$= b - \text{i}d + \text{i}b$
$= (1+\text{i})b - \text{i}d$
$= (1+\text{i})b - \text{i}\left(\left(\frac{1}{2} - \frac{1}{2}\text{i}\sqrt{3}\right)a + \left(\frac{1}{2} + \frac{1}{2}\text{i}\sqrt{3}\right)b\right)$
$= (1+\text{i})b + \left(-\frac{1}{2}\text{i} - \frac{1}{2}\sqrt{3}\right)a + \left(-\frac{1}{2}\text{i} + \frac{1}{2}\sqrt{3}\right)b$
$= \left(-\frac{1}{2}\sqrt{3} - \frac{1}{2}\text{i}\right)a + \left(1 + \frac{1}{2}\sqrt{3} + \frac{1}{2}\text{i}\right)b$

$m = \frac{1}{2}(c + d)$
$= \frac{1}{2}\left(\left(-\frac{1}{2}\sqrt{3} - \frac{1}{2}\mathrm{i}\right)a + \left(1 + \frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}\right)b + \left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right)a + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)b\right)$
$= \frac{1}{2}\left(\left(\frac{1}{2} - \frac{1}{2}\sqrt{3} - \frac{1}{2}\mathrm{i} - \frac{1}{2}\mathrm{i}\sqrt{3}\right)a + \left(1\frac{1}{2} + \frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)b\right)$
$= \left(\frac{1}{4} - \frac{1}{4}\sqrt{3} - \frac{1}{4}\mathrm{i} - \frac{1}{4}\mathrm{i}\sqrt{3}\right)a + \left(\frac{3}{4} + \frac{1}{4}\sqrt{3} + \frac{1}{4}\mathrm{i} + \frac{1}{4}\mathrm{i}\sqrt{3}\right)b$
Dus $p = -\frac{1}{4} - \frac{1}{4}\sqrt{3} - \frac{1}{4}\mathrm{i} - \frac{1}{4}\mathrm{i}\sqrt{3}$ en $q = \frac{3}{4} + \frac{1}{4}\sqrt{3} + \frac{1}{4}\mathrm{i} + \frac{1}{4}\mathrm{i}\sqrt{3}$.

Bladzijde 178

41 $e = a + (\cos(45°) + \mathrm{i}\sin(45°))(c - a)$
$= a + \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right)(c - a)$
$= \left(1 - \frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}\right)a + \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right)c$
$d = b + (\cos(-45°) + \mathrm{i}\sin(-45°))(c - b)$
$= b + \left(\frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}\right)(c - b)$
$= \left(1 - \frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right)b + \left(\frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}\right)c$
$m = \frac{1}{2}(d + e)$
$= \frac{1}{2}\left(\left(1 - \frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right)b + \left(\frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}\right)c + \left(1 - \frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}\right)a + \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right)c\right)$
$= \frac{1}{2}\left(\left(1 - \frac{1}{2}\sqrt{2} - \frac{1}{2}\mathrm{i}\sqrt{2}\right)a + \left(1 - \frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{i}\sqrt{2}\right)b + c\sqrt{2}\right)$
$= \left(\frac{1}{2} - \frac{1}{4}\sqrt{2} - \frac{1}{4}\mathrm{i}\sqrt{2}\right)a + \left(\frac{1}{2} - \frac{1}{4}\sqrt{2} + \frac{1}{4}\mathrm{i}\sqrt{2}\right)b + \frac{1}{2}c\sqrt{2}$
Dus $p = \frac{1}{2} - \frac{1}{4}\sqrt{2} - \frac{1}{4}\mathrm{i}\sqrt{2}$, $q = \frac{1}{2} - \frac{1}{4}\sqrt{2} + \frac{1}{4}\mathrm{i}\sqrt{2}$ en $r = \frac{1}{2}\sqrt{2}$.

42 **a** $q = c + (\cos(60°) + \mathrm{i}\sin(60°))(d - c) = c + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(d - c)$
$r = a + (\cos(60°) + \mathrm{i}\sin(60°))(d - a) = a + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(d - a)$
$q - r = c + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(d - c) - a - \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(d - a)$
$= c + \frac{1}{2}d - \frac{1}{2}c + \frac{1}{2}\mathrm{i}d\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3} - a - \frac{1}{2}d + \frac{1}{2}a - \frac{1}{2}\mathrm{i}d\sqrt{3} + \frac{1}{2}\mathrm{i}a\sqrt{3}$
$= \frac{1}{2}c - \frac{1}{2}a + \left(\frac{1}{2}a\sqrt{3} - \frac{1}{2}c\sqrt{3}\right)\mathrm{i}$

b $p = c + (\cos(60°) + \mathrm{i}\sin(60°))(b - c) = c + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(b - c)$
$s = a + (\cos(60°) + \mathrm{i}\sin(60°))(b - a) = a + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(b - a)$
$p - s = c + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(b - c) - a - \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(b - a)$
$= c + \frac{1}{2}b - \frac{1}{2}c + \frac{1}{2}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3} - a - \frac{1}{2}b + \frac{1}{2}a - \frac{1}{2}\mathrm{i}b\sqrt{3} + \frac{1}{2}\mathrm{i}a\sqrt{3}$
$= \frac{1}{2}c - \frac{1}{2}a + \left(\frac{1}{2}a\sqrt{3} - \frac{1}{2}c\sqrt{3}\right)\mathrm{i}$

c $p - q = c + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(b - c) - c - \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(d - c)$
$= c + \frac{1}{2}b - \frac{1}{2}c + \frac{1}{2}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3} - c - \frac{1}{2}d + \frac{1}{2}c - \frac{1}{2}\mathrm{i}d\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3}$
$= \frac{1}{2}b - \frac{1}{2}d + \left(\frac{1}{2}b\sqrt{3} - \frac{1}{2}d\sqrt{3}\right)\mathrm{i}$
$s - r = a + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(b - a) - a - \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(d - a)$
$= a + \frac{1}{2}b - \frac{1}{2}a + \frac{1}{2}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}a\sqrt{3} - a - \frac{1}{2}d + \frac{1}{2}a - \frac{1}{2}\mathrm{i}d\sqrt{3} + \frac{1}{2}\mathrm{i}a\sqrt{3}$
$= \frac{1}{2}b - \frac{1}{2}d + \left(\frac{1}{2}b\sqrt{3} - \frac{1}{2}d\sqrt{3}\right)\mathrm{i}$
$p - q = s - r$, dus $PQ = RS$ en $PQ \parallel RS$.

d Een parallellogram is een vierhoek met twee paar evenwijdige zijden.
$PS \parallel QR$ en $PQ \parallel RS$, dus $PQRS$ is een parallellogram.

43 **a** $p = c + (\cos(60°) + \mathrm{i}\sin(60°))(b - c) = c + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(b - c) = \frac{1}{2}b + \frac{1}{2}c + \frac{1}{2}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3}$
$q = a + (\cos(60°) + \mathrm{i}\sin(60°))(c - a) = a + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(c - a) = \frac{1}{2}a + \frac{1}{2}c - \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3}$
$r = b + (\cos(60°) + \mathrm{i}\sin(60°))(a - b) = b + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)(a - b) = \frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}b\sqrt{3}$
$d = \frac{1}{3}(b + c + p) = \frac{1}{3}\left(b + c + \frac{1}{2}b + \frac{1}{2}c + \frac{1}{2}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3}\right) = \frac{1}{3}\left(1\frac{1}{2}b + 1\frac{1}{2}c + \frac{1}{2}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3}\right)$
$e = \frac{1}{3}(a + c + q) = \frac{1}{3}\left(a + c + \frac{1}{2}a + \frac{1}{2}c - \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3}\right) = \frac{1}{3}\left(1\frac{1}{2}a + 1\frac{1}{2}c - \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3}\right)$
$f = \frac{1}{3}(a + b + r) = \frac{1}{3}\left(a + b + \frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}b\sqrt{3}\right) = \frac{1}{3}\left(1\frac{1}{2}a + 1\frac{1}{2}b + \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}b\sqrt{3}\right)$

$d + (\cos(60°) + \mathrm{i}\sin(60°))(e - d) =$
$\frac{1}{3}\left(1\frac{1}{2}b + 1\frac{1}{2}c + \frac{1}{2}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3}\right) + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)\left(\frac{1}{3}\left(1\frac{1}{2}a - 1\frac{1}{2}b - \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}b\sqrt{3} + \mathrm{i}c\sqrt{3}\right)\right) =$
$\frac{1}{3}\left(1\frac{1}{2}b + 1\frac{1}{2}c + \frac{1}{2}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3} + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)\left(1\frac{1}{2}a - 1\frac{1}{2}b - \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}b\sqrt{3} + \mathrm{i}c\sqrt{3}\right)\right) =$
$\frac{1}{3}\left(1\frac{1}{2}b + 1\frac{1}{2}c + \frac{1}{2}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3} + \frac{3}{4}a - \frac{3}{4}b - \frac{1}{4}\mathrm{i}a\sqrt{3} - \frac{1}{4}\mathrm{i}b\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3} + \frac{3}{4}\mathrm{i}a\sqrt{3} - \frac{3}{4}\mathrm{i}b\sqrt{3} + \frac{3}{4}a + \frac{3}{4}b - 1\frac{1}{2}c\right) =$
$\frac{1}{3}\left(1\frac{1}{2}a + 1\frac{1}{2}b + \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}b\sqrt{3}\right)$
Dus $f = d + (\cos(60°) + \mathrm{i}\sin(60°))(e - d)$ ... (1).

$e + (\cos(60°) + \mathrm{i}\sin(60°))(f - e) =$
$\frac{1}{3}\left(1\frac{1}{2}a + 1\frac{1}{2}c - \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3}\right) + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)\left(\frac{1}{3}\left(1\frac{1}{2}b - 1\frac{1}{2}c + \mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3}\right)\right) =$
$\frac{1}{3}\left(1\frac{1}{2}a + 1\frac{1}{2}c - \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3} + \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{3}\right)\left(1\frac{1}{2}b - 1\frac{1}{2}c + \mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3}\right)\right) =$
$\frac{1}{3}\left(1\frac{1}{2}a + 1\frac{1}{2}c - \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3} + \frac{3}{4}b - \frac{3}{4}c + \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{4}\mathrm{i}b\sqrt{3} - \frac{1}{4}\mathrm{i}c\sqrt{3} + \frac{3}{4}\mathrm{i}b\sqrt{3} - \frac{3}{4}\mathrm{i}c\sqrt{3} - 1\frac{1}{2}a + \frac{3}{4}b + \frac{3}{4}c =\right.$
$\frac{1}{3}\left(1\frac{1}{2}b + 1\frac{1}{2}c + \frac{1}{2}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3}\right.$
Dus $d = e + (\cos(60°) + \mathrm{i}\sin(60°))(f - e)$ ... (2).
Uit (1) en (2) volgt dat driehoek *DEF* een gelijkzijdige driehoek is.
Hiermee is de stelling bewezen.

b Als de drie gelijkzijdige driehoeken naar binnen zijn gericht geldt
$p = c + (\cos(-60°) + \mathrm{i}\sin(-60°))(b - c) = c + \left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right)(b - c) = \frac{1}{2}b + \frac{1}{2}c - \frac{1}{2}\mathrm{i}b\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3}$
$q = a + (\cos(-60°) + \mathrm{i}\sin(-60°))(c - a) = a + \left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right)(c - a) = \frac{1}{2}a + \frac{1}{2}c + \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3}$
$r = b + (\cos(-60°) + \mathrm{i}\sin(-60°))(a - b) = b + \left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right)(a - b) = \frac{1}{2}a + \frac{1}{2}b - \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}b\sqrt{3}$
$d = \frac{1}{3}(b + c + p) = \frac{1}{3}\left(b + c + \frac{1}{2}b + \frac{1}{2}c - \frac{1}{2}\mathrm{i}b\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3}\right) = \frac{1}{3}\left(1\frac{1}{2}b + 1\frac{1}{2}c - \frac{1}{2}\mathrm{i}b\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3}\right)$
$e = \frac{1}{3}(a + c + q) = \frac{1}{3}\left(a + c + \frac{1}{2}a + \frac{1}{2}c + \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3}\right) = \frac{1}{3}\left(1\frac{1}{2}a + 1\frac{1}{2}c + \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3}\right)$
$f = \frac{1}{3}(a + b + r) = \frac{1}{3}\left(a + b + \frac{1}{2}a + \frac{1}{2}b - \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}b\sqrt{3}\right) = \frac{1}{3}\left(1\frac{1}{2}a + 1\frac{1}{2}b - \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}b\sqrt{3}\right)$
$d + (\cos(-60°) + \mathrm{i}\sin(-60°))(e - d) =$
$\frac{1}{3}\left(1\frac{1}{2}b + 1\frac{1}{2}c - \frac{1}{2}\mathrm{i}b\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3}\right) + \left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right)\left(\frac{1}{3}\left(1\frac{1}{2}a - 1\frac{1}{2}b + \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}b\sqrt{3} - \mathrm{i}c\sqrt{3}\right)\right) =$
$\frac{1}{3}\left(1\frac{1}{2}b + 1\frac{1}{2}c - \frac{1}{2}\mathrm{i}b\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3} + \left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right)\left(1\frac{1}{2}a - 1\frac{1}{2}b + \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}b\sqrt{3} - \mathrm{i}c\sqrt{3}\right)\right) =$
$\frac{1}{3}\left(1\frac{1}{2}b + 1\frac{1}{2}c - \frac{1}{2}\mathrm{i}b\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3} + \frac{3}{4}a - \frac{3}{4}b + \frac{1}{4}\mathrm{i}a\sqrt{3} + \frac{1}{4}\mathrm{i}b\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3} - \frac{3}{4}\mathrm{i}a\sqrt{3} + \frac{3}{4}\mathrm{i}b\sqrt{3} + \frac{3}{4}a + \frac{3}{4}b - 1\frac{1}{2}c\right) =$
$\frac{1}{3}\left(1\frac{1}{2}a + 1\frac{1}{2}b - \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}b\sqrt{3}\right)$
Dus $f = d + (\cos(-60°) + \mathrm{i}\sin(-60°))(e - d)$ ... (1).

$e + (\cos(-60°) + \mathrm{i}\sin(-60°))(f - e) =$
$\frac{1}{3}\left(1\frac{1}{2}a + 1\frac{1}{2}c + \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3}\right) + \left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right)\left(\frac{1}{3}\left(1\frac{1}{2}b - 1\frac{1}{2}c - \mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}b\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3}\right)\right) =$
$\frac{1}{3}\left(1\frac{1}{2}a + 1\frac{1}{2}c + \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3} + \left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{3}\right)\left(1\frac{1}{2}b - 1\frac{1}{2}c - \mathrm{i}a\sqrt{3} + \frac{1}{2}\mathrm{i}b\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3}\right)\right) =$
$\frac{1}{3}\left(1\frac{1}{2}a + 1\frac{1}{2}c + \frac{1}{2}\mathrm{i}a\sqrt{3} - \frac{1}{2}\mathrm{i}c\sqrt{3} + \frac{3}{4}b - \frac{3}{4}c - \frac{1}{2}\mathrm{i}a\sqrt{3} + \frac{1}{4}\mathrm{i}b\sqrt{3} + \frac{1}{4}\mathrm{i}c\sqrt{3} - \frac{3}{4}\mathrm{i}b\sqrt{3} + \frac{3}{4}\mathrm{i}c\sqrt{3} - 1\frac{1}{2}a + \frac{3}{4}b + \frac{3}{4}c =\right.$
$\frac{1}{3}\left(1\frac{1}{2}b + 1\frac{1}{2}c - \frac{1}{2}\mathrm{i}b\sqrt{3} + \frac{1}{2}\mathrm{i}c\sqrt{3}\right)$
Dus $d = e + (\cos(-60°) + \mathrm{i}\sin(-60°))(f - e)$ ... (2).
Uit (1) en (2) volgt dat driehoek *DEF* een gelijkzijdige driehoek is.
Hiermee is bewezen dat de stelling ook geldig is als de drie gelijkzijdige driehoeken naar binnen zijn gericht.

16.4 Derdegraadsvergelijkingen

Bladzijde 180

44 **a** $x = 1$ geeft $1^3 + 1^2 + 1 - 3 = 1 + 1 + 1 - 3 = 0$ klopt.
Dus $x = 1$ is een oplossing van de vergelijking $x^3 + x^2 + x - 3 = 0$.

b $x = -1 + \mathrm{i}\sqrt{2}$ geeft $\left(-1 + \mathrm{i}\sqrt{2}\right)^3 + \left(-1 + \mathrm{i}\sqrt{2}\right)^2 + \left(-1 + \mathrm{i}\sqrt{2}\right) - 3 =$
$\left(-1 + \mathrm{i}\sqrt{2}\right)\left(\left(-1 + \mathrm{i}\sqrt{2}\right)^2 + \left(-1 + \mathrm{i}\sqrt{2}\right) + 1\right) - 3 =$
$\left(-1 + \mathrm{i}\sqrt{2}\right)\left(1 - 2\mathrm{i}\sqrt{2} - 2 - 1 + \mathrm{i}\sqrt{2} + 1\right) - 3 =$
$\left(-1 + \mathrm{i}\sqrt{2}\right)\left(-1 - \mathrm{i}\sqrt{2}\right) - 3 =$
$1 + 2 - 3 = 0$ klopt.
Dus $x = -1 + \mathrm{i}\sqrt{2}$ is een oplossing van de vergelijking $x^3 + x^2 + x - 3 = 0$.

Bladzijde 181

45 **a** Voer in $y_1 = x^3 - 4x^2 + 4x - 16$.
De optie zero (TI) of ROOT (Casio) geeft $x = 4$.
$z - 4 \,/\, z^3 - 4z^2 + 4z - 16 \,\backslash\, z^2 + 4$
$\underline{z^3 - 4z^2}$ –
$4z - 16$
$\underline{4z - 16}$ –
0

$z^2 + 4 = 0$
$z^2 = -4$
$z^2 = 4i^2$
$z = 2i \vee z = -2i$
Dus $z = 4 \vee z = 2i \vee z = -2i$.

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b Voer in $y_1 = x^3 + 2x - 12$.
De optie zero (TI) of ROOT (Casio) geeft $x = 2$.
$z - 2 \,/\, z^3 \quad + 2z - 12 \,\backslash\, z^2 + 2z + 6$
$\underline{z^3 - 2z^2}$ –
$2z^2 + 2z$
$\underline{2z^2 - 4z}$ –
$6z - 12$
$\underline{6z - 12}$ –
0

$z^2 + 2z + 6 = 0$
$(z + 1)^2 - 1 + 6 = 0$
$(z + 1)^2 = -5$
$(z + 1)^2 = 5i^2$
$z + 1 = i\sqrt{5} \vee z + 1 = -i\sqrt{5}$
$z = -1 + i\sqrt{5} \vee z = -1 - i\sqrt{5}$
Dus $z = 2 \vee z = -1 + i\sqrt{5} \vee z = -1 - i\sqrt{5}$.

c $2z^3 + 3z^2 - 8z = 30$ ofwel $z^3 + 1\frac{1}{2}z^2 - 4z - 15 = 0$
Voer in $y_1 = x^3 + 1\frac{1}{2}x^2 - 4x - 15$.
De optie zero (TI) of ROOT (Casio) geeft $x = 2\frac{1}{2}$.

$z - 2\frac{1}{2} \,/\, z^3 + 1\frac{1}{2}z^2 - 4z - 15 \,\backslash\, z^2 + 4z + 6$

$\underline{z^3 - 2\frac{1}{2}z^2}$ –
$4z^2 - 4z$
$\underline{4z^2 - 10z}$ –
$6z - 15$
$\underline{6z - 15}$ –
0

$z^2 + 4z + 6 = 0$
$(z + 2)^2 - 4 + 6 = 0$
$(z + 2)^2 = -2$
$(z + 2)^2 = 2i^2$
$z + 2 = i\sqrt{2} \vee z + 2 = -i\sqrt{2}$
$z = -2 + i\sqrt{2} \vee z = -2 - i\sqrt{2}$
Dus $z = 2\frac{1}{2} \vee z = -2 + i\sqrt{2} \vee z = -2 - i\sqrt{2}$.

d $2z^3 - z^2 + 3z + 2 = 0$ ofwel $z^3 - \frac{1}{2}z^2 + 1\frac{1}{2}z + 1 = 0$
Voer in $y_1 = x^3 - \frac{1}{2}x^2 + 1\frac{1}{2}x + 1$.
De optie zero (TI) of ROOT (Casio) geeft $x = -\frac{1}{2}$.
$z + \frac{1}{2} \,/\, z^3 - \frac{1}{2}z^2 + 1\frac{1}{2}z + 1 \,\backslash\, z^2 - z + 2$

$\underline{z^3 + \frac{1}{2}z^2}$ –
$-z^2 + 1\frac{1}{2}z$

$\underline{-z^2 - \frac{1}{2}z}$ –
$2z + 1$
$\underline{2z + 1}$ –
0

$z^2 - z + 2 = 0$
$\left(z - \frac{1}{2}\right)^2 - \frac{1}{4} + 2 = 0$
$\left(z - \frac{1}{2}\right)^2 = -1\frac{3}{4}$
$\left(z - \frac{1}{2}\right)^2 = \frac{7}{4}\mathrm{i}^2$
$z - \frac{1}{2} = \frac{1}{2}\mathrm{i}\sqrt{7} \vee z - \frac{1}{2} = -\frac{1}{2}\mathrm{i}\sqrt{7}$
$z = \frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{7} \vee z = \frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{7}$
Dus $z = -\frac{1}{2} \vee z = \frac{1}{2} + \frac{1}{2}\mathrm{i}\sqrt{7} \vee z = \frac{1}{2} - \frac{1}{2}\mathrm{i}\sqrt{7}$.

46 **a** $(u + v)^3 + (-3uv)(u + v) = q$
$u^3 + 3u^2v + 3uv^2 + v^3 - 3u^2v - 3uv^2 = q$
$u^3 + v^3 = q$

b $v = -\dfrac{p}{3u}$ geeft $u^3 + \left(-\dfrac{p}{3u}\right) = q$
$u^3 - \dfrac{p^3}{27u^3} = q$
$u^6 - \frac{1}{27}p^3 = qu^3$
$u^6 - qu^3 - \frac{1}{27}p^3 = 0$

Bladzijde 182

47 **a** $(u + v)^3 + 6(u + v) = 20$
$u^3 + 3u^2v + 3uv^2 + v^3 + 6u + 6v = 20$
$u^3 + 3u \cdot uv + 3v \cdot uv + v^3 + 6u + 6v = 20$
$u^3 + 3u \cdot -2 + 3v \cdot -2 + v^3 + 6u + 6v = 20$
$u^3 - 6u - 6v + v^3 + 6u + 6v = 20$
$u^3 + v^3 = 20$

b $v = -\dfrac{2}{u}$ geeft $u^3 + \left(-\dfrac{2}{u}\right)^3 = 20$
$u^3 - \dfrac{8}{u^3} = 20$
$u^6 - 8 = 20u^3$
$u^6 - 20u^3 - 8 = 0$

c Stel $x = u^3$.
$x^2 - 20x - 8 = 0$
$D = (-20)^2 - 4 \cdot 1 \cdot -8 = 432$
$x = \dfrac{20 - \sqrt{432}}{2} \vee x = \dfrac{20 + \sqrt{432}}{2}$
$u^3 = \dfrac{20 - \sqrt{432}}{2} \vee u^3 = \dfrac{20 + \sqrt{432}}{2}$

d $u^3 = 10 + 6\sqrt{3}$ geeft $10 + 6\sqrt{3} + v^3 = 20$
$v^3 = 20 - 10 - 6\sqrt{3}$
$v^3 = 10 - 6\sqrt{3}$

e $u^3 = 10 + 6\sqrt{3}$ geeft $u = \sqrt[3]{10 + 6\sqrt{3}}$
$v^3 = 10 - 6\sqrt{3}$ geeft $v = \sqrt[3]{10 - 6\sqrt{3}}$
$z = u + v = \sqrt[3]{10 + 6\sqrt{3}} + \sqrt[3]{10 - 6\sqrt{3}}$

f De GR geeft $\sqrt[3]{10 + 6\sqrt{3}} + \sqrt[3]{10 - 6\sqrt{3}} = 2$.
$z = 2$ geeft $2^3 + 6 \cdot 2 = 8 + 12 = 20$ klopt.
Dus $z = 2$ is een oplossing van de vergelijking $z^3 + 6z = 20$.

Bladzijde 183

48 Verder rekenen met $u^3 = 10 - 6\sqrt{3}$ geeft $v^3 = 20 - u^3 = 20 - \left(10 - 6\sqrt{3}\right) = 10 + 6\sqrt{3}$.

Dit geeft $z = u + v = \sqrt[3]{10 - 6\sqrt{3}} + \sqrt[3]{10 + 6\sqrt{3}}$ en dus hetzelfde resultaat.

Verder rekenen met $u^3 = -1$ geeft $v^3 = 63 - u^3 = 63 - -1 = 64$.

Dit geeft $z = u + v = \sqrt[3]{-1} + \sqrt[3]{64} = -1 + 4 = 3$ en dus hetzelfde resultaat.

49 **a** $z = u + v$ en $36 = -3uv$ geeft $u^3 + v^3 = 208$
Uit $-3uv = 36$ volgt $v = -\dfrac{12}{u}$.

$$\left.\begin{array}{l} u^3 + v^3 = 208 \\ v = -\dfrac{12}{u} \end{array}\right\} u^3 + \left(-\frac{12}{u}\right)^3 = 208$$

$$u^3 - \frac{1728}{u^3} = 208$$

$$u^6 - 208u^3 - 1728 = 0$$

$D = 50\,176$, dus $\sqrt{D} = 224$

$u^3 = \dfrac{208 + 224}{2} = 216$ is een oplossing

$v^3 = 208 - u^3 = 208 - 216 = -8$

$z = \sqrt[3]{216} + \sqrt[3]{-8} = 6 - 2 = 4$

$$\begin{array}{l} z - 4 \,/\, z^3 \qquad + 36z - 208 \,\backslash\, z^2 + 4z + 52 \\ \quad \underline{z^3 - 4z^2} \; - \\ \qquad\quad 4z^2 + 36z \\ \qquad\quad \underline{4z^2 - 16z} \; - \\ \qquad\qquad\quad 52z - 208 \\ \qquad\qquad\quad \underline{52z - 208} \; - \\ \qquad\qquad\qquad\quad 0 \end{array}$$

$z^2 + 4z + 52 = 0$
$(z + 2)^2 - 4 + 52 = 0$
$(z + 2)^2 = -48$
$(z + 2)^2 = 48i^2$
$z + 2 = 4i\sqrt{3} \vee z + 2 = -4i\sqrt{3}$
$z = -2 + 4i\sqrt{3} \vee z = -2 - 4i\sqrt{3}$
Dus $z = 4 \vee z = -2 + 4i\sqrt{3} \vee z = -2 - 4i\sqrt{3}$.

b $z = u + v$ en $-18 = -3uv$ geeft $u^3 + v^3 = 108$
Uit $-3uv = -18$ volgt $v = \dfrac{6}{u}$.

$$\left.\begin{array}{l} u^3 + v^3 = 108 \\ v = \dfrac{6}{u} \end{array}\right\} u^3 + \left(\frac{6}{u}\right)^3 = 108$$

$$u^3 + \frac{216}{u^3} = 108$$

$u^6 - 108u^3 + 216 = 0$
$D = 10\,800$, dus $\sqrt{D} = 60\sqrt{3}$

$u^3 = \dfrac{108 + 60\sqrt{3}}{2} = 54 + 30\sqrt{3}$ is een oplossing

$v^3 = 108 - u^3 = 108 - \left(54 + 30\sqrt{3}\right) = 54 - 30\sqrt{3}$

$z = \sqrt[3]{54 + 30\sqrt{3}} + \sqrt[3]{54 - 30\sqrt{3}} = 6$

$$\begin{array}{l} z - 6 \,/\, z^3 \qquad - 18z - 108 \,\backslash\, z^2 + 6z + 18 \\ \quad \underline{z^3 - 6z^2} \; - \\ \qquad\quad 6z^2 - 18z \\ \qquad\quad \underline{6z^2 - 36z} \; - \\ \qquad\qquad\quad 18z - 108 \\ \qquad\qquad\quad \underline{18z - 108} \; - \\ \qquad\qquad\qquad\quad 0 \end{array}$$

$z^2 + 6z + 18 = 0$
$(z + 3)^2 - 9 + 18 = 0$
$(z + 3)^2 = -9$
$(z + 3)^2 = 9i^2$
$z + 3 = 3i \vee z + 3 = -3i$
$z = -3 + 3i \vee z = -3 - 3i$
Dus $z = 6 \vee z = -3 + 3i \vee z = -3 - 3i$.

c $z = u + v$ en $2\frac{1}{4} = -3uv$ geeft $u^3 + v^3 = 3\frac{1}{4}$
Uit $-3uv = 2\frac{1}{4}$ volgt $v = -\dfrac{3}{4u}$.

$$\left.\begin{array}{l} u^3 + v^3 = 3\frac{1}{4} \\ v = -\dfrac{3}{4u} \end{array}\right\} u^3 + \left(-\dfrac{3}{4u}\right)^3 = 3\frac{1}{4}$$

$u^3 - \dfrac{27}{64u^3} = 3\frac{1}{4}$

$u^6 - 3\frac{1}{4}u^3 - \frac{27}{64} = 0$

$D = 12\frac{1}{4}$, dus $\sqrt{D} = 3\frac{1}{2}$

$u^3 = \dfrac{3\frac{1}{4} + 3\frac{1}{2}}{2} = 3\frac{3}{8}$ is een oplossing

$v^3 = 3\frac{1}{4} - u^3 = 3\frac{1}{4} - 3\frac{3}{8} = -\frac{1}{8}$

$z = \sqrt[3]{3\frac{3}{8}} + \sqrt[3]{-\frac{1}{8}} = 1\frac{1}{2} - \frac{1}{2} = 1$

$$\begin{array}{l} z - 1 \,/\, z^3 \qquad + 2\frac{1}{4}z - 3\frac{1}{4} \,\backslash\, z^2 + z + 3\frac{1}{4} \\ \qquad \underline{z^3 - z^2} \;- \\ \qquad\quad z^2 + 2\frac{1}{4}z \\ \qquad\quad \underline{z^2 - \quad z} \;- \\ \qquad\qquad 3\frac{1}{4}z - 3\frac{1}{4} \\ \qquad\qquad \underline{3\frac{1}{4}z - 3\frac{1}{4}} \;- \\ \qquad\qquad\qquad 0 \end{array}$$

$z^2 + z + 3\frac{1}{4} = 0$

$\left(z + \frac{1}{2}\right)^2 - \frac{1}{4} + 3\frac{1}{4} = 0$

$\left(z + \frac{1}{2}\right)^2 = -3$

$\left(z + \frac{1}{2}\right)^2 = 3i^2$

$z + \frac{1}{2} = i\sqrt{3} \vee z + \frac{1}{2} = -i\sqrt{3}$

$z = -\frac{1}{2} + i\sqrt{3} \vee z = -\frac{1}{2} - i\sqrt{3}$

Dus $z = 1 \vee z = -\frac{1}{2} + i\sqrt{3} \vee z = -\frac{1}{2} - i\sqrt{3}$.

d $z = u + v$ en $81 = -3uv$ geeft $u^3 + v^3 = 702$
Uit $-3uv = 81$ volgt $v = -\dfrac{27}{u}$.

$$\left.\begin{array}{l} u^3 + v^3 = 702 \\ v = -\dfrac{27}{u} \end{array}\right\} u^3 + \left(-\dfrac{27}{u}\right)^3 = 702$$

$u^3 - \dfrac{19683}{u^3} = 702$

$u^6 - 702u^3 - 19683 = 0$

$D = 571\,536$, dus $\sqrt{D} = 756$

$u^3 = \dfrac{702 + 756}{2} = 729$ is een oplossing

$v^3 = 702 - u^3 = 702 - 729 = -27$

$z = \sqrt[3]{729} + \sqrt[3]{-27} = 9 - 3 = 6$

$$\begin{array}{l} z - 6 \,/\, z^3 \qquad + 81z - 702 \,\backslash\, z^2 + 6z + 117 \\ \qquad \underline{z^3 - 6z^2} \;- \\ \qquad\quad 6z^2 + 81z \\ \qquad\quad \underline{6z^2 - 36z} \;- \\ \qquad\qquad 117z - 702 \\ \qquad\qquad \underline{117z - 702} \;- \\ \qquad\qquad\qquad 0 \end{array}$$

$z^2 + 6z + 117 = 0$
$(z + 3)^2 - 9 + 117 = 0$
$(z + 3)^2 = -108$
$(z + 3)^2 = 108i^2$
$z + 3 = 6i\sqrt{3} \vee z + 3 = -6i\sqrt{3}$
$z = -3 + 6i\sqrt{3} \vee z = -3 - 6i\sqrt{3}$
Dus $z = 6 \vee z = -3 + 6i\sqrt{3} \vee z = -3 - 6i\sqrt{3}$.

Bladzijde 184

50 **a** $z^3 - 15z = 4$ ofwel $z^3 - 15z - 4 = 0$
Voer in $y_1 = x^3 - 15x - 4$.
De optie zero (TI) of ROOT (Casio) geeft $x = 4$.
Dus $z = 4$ is de gehele oplossing.
b $u^6 - 4u^3 + 125 = 0$
Stel $u^3 = x$.
$x^2 - 4x + 125 = 0$
$(x-2)^2 - 4 + 125 = 0$
$(x-2)^2 = -121$
$(x-2)^2 = 121i^2$
$x - 2 = 11i \vee x - 2 = -11i$
$x = 2 + 11i \vee x = 2 - 11i$
$u^3 = 2 + 11i \vee u^3 = 2 - 11i$
Dus $u = \sqrt[3]{2 + 11i}$ is een oplossing.
$z = u + v$ en $-15 = -3uv$ geeft $u^3 + v^3 = 4$, dus $v^3 = 4 - u^3 = 4 - (2 + 11i) = 2 - 11i$.
Dus $v = \sqrt[3]{2 - 11i}$.
c De GR geeft $\sqrt[3]{2 + 11i} + \sqrt[3]{2 - 11i} = 4$.
Dus je krijgt de oplossing $z = 4$.

Bladzijde 186

51 **a** Substitutie van $z = y - \frac{1}{3}a$ in $z^3 + az^2 + bz + c = 0$ geeft

$$\left(y - \tfrac{1}{3}a\right)^3 + a\left(y - \tfrac{1}{3}a\right)^2 + b\left(y - \tfrac{1}{3}a\right) + c = 0$$
$$y^3 - ay^2 + \tfrac{1}{3}a^2y - \tfrac{1}{27}a^3 + a\left(y^2 - \tfrac{2}{3}ay + \tfrac{1}{9}a^2\right) + by - \tfrac{1}{3}ab + c = 0$$
$$y^3 - ay^2 + \tfrac{1}{3}a^2y - \tfrac{1}{27}a^3 + ay^2 - \tfrac{2}{3}a^2y + \tfrac{1}{9}a^3 + by - \tfrac{1}{3}ab + c = 0$$
$$y^3 + by - \tfrac{1}{3}a^2y + \tfrac{2}{27}a^3 - \tfrac{1}{3}ab + c = 0$$
$$y^3 + \left(b - \tfrac{1}{3}a^2\right)y = -\tfrac{2}{27}a^3 + \tfrac{1}{3}ab - c$$

b Stel $p = b - \frac{1}{3}a^2$ en $q = -\frac{2}{27}a^3 + \frac{1}{3}ab - c$.
Dit geeft $y^3 + py = q$.
Stel $y = u + v$ en $p = -3uv$.
Uit $p = -3uv$ volgt $v = -\dfrac{p}{3u}$.

$$u^3 + \left(-\frac{p}{3u}\right)^3 = q$$
$$u^3 - \frac{p^3}{27u^3} = q$$
$$u^3 - qu^3 - \tfrac{1}{27}p^3 = 0$$

$D = q^2 + \frac{4}{27}p^3 = 4\left(\frac{1}{4}q^2 + \frac{1}{27}p^3\right) = 4\left(\left(\frac{1}{2}q\right)^2 + \left(\frac{1}{3}p\right)^3\right)$, dus $\sqrt{D} = 2\sqrt{\left(\frac{1}{2}q\right)^2 + \left(\frac{1}{3}p\right)^3}$

$u^3 = \dfrac{q + 2\sqrt{\left(\frac{1}{2}q\right)^2 + \left(\frac{1}{3}p\right)^3}}{2} = \frac{1}{2}q + \sqrt{\left(\frac{1}{2}q\right)^2 + \left(\frac{1}{3}p\right)^3}$ is een oplossing.

$$v^3 = q - u^3 = q - \left(\tfrac{1}{2}q + \sqrt{\left(\tfrac{1}{2}q\right)^2 + \left(\tfrac{1}{3}p\right)^3}\right) = \tfrac{1}{2}q - \sqrt{\left(\tfrac{1}{2}q\right)^2 + \left(\tfrac{1}{3}p\right)^3}$$
$$y = u + v = \sqrt[3]{\tfrac{1}{2}q + \sqrt{\left(\tfrac{1}{2}q\right)^2 + \left(\tfrac{1}{3}p\right)^3}} + \sqrt[3]{\tfrac{1}{2}q - \sqrt{\left(\tfrac{1}{2}q\right)^2 + \left(\tfrac{1}{3}p\right)^3}}$$
$$z = y - \tfrac{1}{3}a = -\tfrac{1}{3}a + \sqrt[3]{\tfrac{1}{2}q + \sqrt{\left(\tfrac{1}{2}q\right)^2 + \left(\tfrac{1}{3}p\right)^3}} + \sqrt[3]{\tfrac{1}{2}q - \sqrt{\left(\tfrac{1}{2}q\right)^2 + \left(\tfrac{1}{3}p\right)^3}}$$

Dus $z = -\frac{1}{3}a + \sqrt[3]{\frac{1}{2}q + \sqrt{\left(\frac{1}{2}q\right)^2 + \left(\frac{1}{3}p\right)^3}} + \sqrt[3]{\frac{1}{2}q - \sqrt{\left(\frac{1}{2}q\right)^2 + \left(\frac{1}{3}p\right)^3}}$ met $p = b - \frac{1}{3}a^2$ en
$q = -\frac{2}{27}a^3 + \frac{1}{3}ab - c$ is een reële oplossing van $z^3 + az^2 + bz + c = 0$.

52 **a** $z^3 + 6z^2 + 12z + 9 = 0$
$a = 6$, $b = 12$ en $c = 9$
$p = b - \frac{1}{3}a^2 = 12 - \frac{1}{3} \cdot 6^2 = 0$ en $q = -\frac{2}{27}a^3 + \frac{1}{3}ab - c = -\frac{2}{27} \cdot 6^3 + \frac{1}{3} \cdot 6 \cdot 12 - 9 = -1$
$z = -\frac{1}{3}a + \sqrt[3]{\frac{1}{2}q + \sqrt{\left(\frac{1}{2}q\right)^2 + \left(\frac{1}{3}p\right)^3}} + \sqrt[3]{\frac{1}{2}q - \sqrt{\left(\frac{1}{2}q\right)^2 + \left(\frac{1}{3}p\right)^3}} = -\frac{1}{3} \cdot 6 + \sqrt[3]{-\frac{1}{2} + \sqrt{\frac{1}{4} + 0}} + \sqrt[3]{-\frac{1}{2} - \sqrt{\frac{1}{4} + 0}} = -3$

$$\begin{array}{l} z+3\,/\,z^3+6z^2+12z+9\,\backslash\,z^2+3z+3 \\ \quad \underline{z^3+3z^2}\;- \\ \qquad 3z^2+12z \\ \qquad \underline{3z^2+\;\,9z}\;- \\ \qquad\qquad 3z+9 \\ \qquad\qquad \underline{3z+9}\;- \\ \qquad\qquad\quad 0 \end{array}$$

$z^2+3z+3=0$

$\left(z+1\frac{1}{2}\right)^2-2\frac{1}{4}+3=0$

$\left(z+1\frac{1}{2}\right)^2=-\frac{3}{4}$

$\left(z+1\frac{1}{2}\right)^2=\frac{3}{4}i^2$

$z+1\frac{1}{2}=\frac{1}{2}i\sqrt{3} \vee z+1\frac{1}{2}=-\frac{1}{2}i\sqrt{3}$

$z=-1\frac{1}{2}+\frac{1}{2}i\sqrt{3} \vee z=-1\frac{1}{2}-\frac{1}{2}i\sqrt{3}$

Dus $z=-3 \vee z=-1\frac{1}{2}+\frac{1}{2}i\sqrt{3} \vee z=-1\frac{1}{2}-\frac{1}{2}i\sqrt{3}$.

b $z^3-12z^2-392=0$

$a=-12$, $b=0$ en $c=-392$

$p=b-\frac{1}{3}a^2=0-\frac{1}{3}\cdot(-12)^2=-48$ en $q=-\frac{2}{27}a^3+\frac{1}{3}ab-c=-\frac{2}{27}\cdot(-12)^3+\frac{1}{3}\cdot -12\cdot 0--392=520$

$z=-\frac{1}{3}a+\sqrt[3]{\frac{1}{2}q+\sqrt{\left(\frac{1}{2}q\right)^2+\left(\frac{1}{3}p\right)^3}}+\sqrt[3]{\frac{1}{2}q-\sqrt{\left(\frac{1}{2}q\right)^2+\left(\frac{1}{3}p\right)^3}}=$

$-\frac{1}{3}\cdot -12+\sqrt[3]{260+\sqrt{260^2+(-16)^3}}+\sqrt[3]{260-\sqrt{260^2+(-16)^3}}=14$

$$\begin{array}{l} z-14\,/\,z^3-12z^2\qquad\quad -392\,\backslash\,z^2+2z+28 \\ \quad \underline{z^3-14z^2}\;- \\ \qquad\quad 2z^2 \\ \qquad \underline{2z^2-28z}\;- \\ \qquad\qquad 28z-392 \\ \qquad\qquad \underline{28z-392}\;- \\ \qquad\qquad\qquad 0 \end{array}$$

$z^2+2z+28=0$

$(z+1)^2-1+28=0$

$(z+1)^2=-27$

$(z+1)^2=27i^2$

$z+1=3i\sqrt{3} \vee z+1=-3i\sqrt{3}$

$z=-1+3i\sqrt{3} \vee z=-1-3i\sqrt{3}$

Dus $z=14 \vee z=-1+3i\sqrt{3} \vee z=-1-3i\sqrt{3}$.

c $z^3+3z^2+4z-28=0$

$a=3$, $b=4$ en $c=-28$

$p=b-\frac{1}{3}a^2=4-\frac{1}{3}\cdot 3^2=1$ en $q=-\frac{2}{27}a^3+\frac{1}{3}ab-c=-\frac{2}{27}\cdot 3^3+\frac{1}{3}\cdot 3\cdot 4--28=30$

$z=-\frac{1}{3}a+\sqrt[3]{\frac{1}{2}q+\sqrt{\left(\frac{1}{2}q\right)^2+\left(\frac{1}{3}p\right)^3}}+\sqrt[3]{\frac{1}{2}q-\sqrt{\left(\frac{1}{2}q\right)^2+\left(\frac{1}{3}p\right)^3}}=$

$-\frac{1}{3}\cdot 3+\sqrt[3]{15+\sqrt{15^2+\left(\frac{1}{3}\right)^3}}+\sqrt[3]{15-\sqrt{15^2+\left(\frac{1}{3}\right)^3}}=2$

$$\begin{array}{l} z-2\,/\,z^3+3z^2+\;4z-28\,\backslash\,z^2+5z+14 \\ \quad \underline{z^3-2z^2}\;- \\ \qquad 5z^2+\;4z \\ \qquad \underline{5z^2-10z}\;- \\ \qquad\qquad 14z-28 \\ \qquad\qquad \underline{14z-28}\;- \\ \qquad\qquad\quad 0 \end{array}$$

$z^2+5z+14=0$

$\left(z+2\frac{1}{2}\right)^2-6\frac{1}{4}+14=0$

$\left(z+2\frac{1}{2}\right)^2=-7\frac{3}{4}$

$\left(z+2\frac{1}{2}\right)^2=\frac{31}{4}i^2$

$z+2\frac{1}{2}=\frac{1}{2}i\sqrt{31} \vee z+2\frac{1}{2}=-\frac{1}{2}i\sqrt{31}$

$z=-2\frac{1}{2}+\frac{1}{2}i\sqrt{31} \vee z=-2\frac{1}{2}-\frac{1}{2}i\sqrt{31}$

Dus $z=2 \vee z=-2\frac{1}{2}+\frac{1}{2}i\sqrt{31} \vee z=-2\frac{1}{2}-\frac{1}{2}i\sqrt{31}$.

53 **a** De inhoud is 48, dus er moet gelden $abc = 48$.
De totale oppervlakte van de zijvlakken is 124, dus er moet gelden $2ab + 2ac + 2bc = 124$.
De lichaamsdiagonalen hebben lengte 101, dus er moet gelden $a^2 + b^2 + c^2 = 101$.
Dus om de lengte van de ribben a, b en c van de balk te berekenen moet het stelsel
$\begin{cases} abc = 48 \\ 2ab + 2ac + 2bc = 124 \\ a^2 + b^2 + c^2 = 101 \end{cases}$ worden opgelost.

b Als de lengte van één van de ribben gelijk is aan x geldt $x = a \vee x = b \vee x = c$
ofwel $x - a = 0 \vee x - b = 0 \vee x - c = 0$.
Hieruit volgt $(x - a)(x - b)(x - c) = 0$.
Dus als de lengte van één van de ribben gelijk is aan x geldt
$(x - a)(x - b)(x - c) = 0$
$(x^2 - bx - ax + ab)(x - c) = 0$
$x^3 - cx^2 - bx^2 + bcx - ax^2 + acx + abx - abc = 0$
$x^3 - (a + b + c)x^2 + (ab + ac + bc)x - abc = 0$

c $(a + b + c)^2 = (a + b + c)(a + b + c) = a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2 =$
$a^2 + b^2 + c^2 + 2ab + 2ac + 2bc = 101 + 124 = 225$

d $(a + b + c)^2 = 225$ geeft $a + b + c = 15$
$2ab + 2ac + 2bc = 124$ geeft $ab + ac + bc = 62$
$abc = 48$
Dit geeft de vergelijking $x^3 - 15x^2 + 62x - 48 = 0$.
$a = -15$, $b = 62$ en $c = -48$
$p = b - \frac{1}{3}a^2 = 62 - \frac{1}{3} \cdot (-15)^2 = -13$ en $q = -\frac{2}{27}a^3 + \frac{1}{3}ab - c = -\frac{2}{27} \cdot (-15)^3 + \frac{1}{3} \cdot -15 \cdot 62 - -48 = -12$
$x = -\frac{1}{3}a + \sqrt[3]{\frac{1}{2}q + \sqrt{\left(\frac{1}{2}q\right)^2 + \left(\frac{1}{3}p\right)^3}} + \sqrt[3]{\frac{1}{2}q - \sqrt{\left(\frac{1}{2}q\right)^2 + \left(\frac{1}{3}p\right)^3}} =$
$-\frac{1}{3} \cdot -15 + \sqrt[3]{-6 + \sqrt{(-6)^2 + \left(-\frac{13}{3}\right)^3}} + \sqrt[3]{-6 - \sqrt{(-6)^2 + \left(-\frac{13}{3}\right)^3}} = 8$

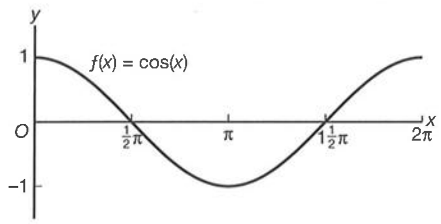
$x - 8 \,/\, x^3 - 15x^2 + 62x - 48 \,\backslash\, x^2 - 7x + 6$
$\quad x^3 - 8x^2 \quad -$
$\quad\quad -7x^2 + 62x$
$\quad\quad -7x^2 + 56x \quad -$
$\quad\quad\quad 6x - 48$
$\quad\quad\quad 6x - 48 \quad -$
$\quad\quad\quad\quad 0$

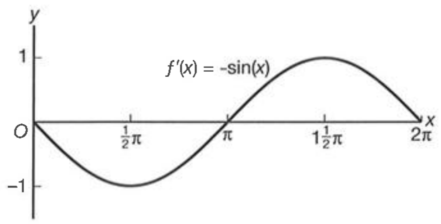
$x^2 - 7x + 6 = 0$
$(x - 1)(x - 6) = 0$
$x = 1 \vee x = 6$
Dus $x = 1 \vee x = 6 \vee x = 8$.
De afmetingen van de balk zijn 8 bij 6 bij 1.

16.5 Wisselspanning

Bladzijde 188

54 **a** $f(x) = \cos(x)$ geeft $f'(x) = -\sin(x)$





b De grafiek van $f'(x) = -\sin(x)$ ontstaat uit de grafiek van $f(x) = \cos(x)$ door de translatie $\left(-\frac{1}{2}\pi, 0\right)$.

55 a $f(t) = 6\cos(100\pi t)$ geeft $f'(t) = -600\pi\sin(100\pi t)$
$= -600\pi\cos\left(100\pi t - \tfrac{1}{2}\pi\right)$
$= 600\pi\cos\left(100\pi t - \tfrac{1}{2}\pi + \pi\right)$
$= 600\pi\cos\left(100\pi t + \tfrac{1}{2}\pi\right)$

b $\mathrm{i} = \mathrm{e}^{\frac{1}{2}\pi\mathrm{i}}$, dus $\mathrm{i} \cdot 600\pi\mathrm{e}^{\mathrm{i}\cdot 100\pi t} = \mathrm{e}^{\frac{1}{2}\pi\mathrm{i}} \cdot 600\pi\mathrm{e}^{\mathrm{i}\cdot 100\pi t} = 600\pi\mathrm{e}^{\mathrm{i}\cdot 100\pi t + \frac{1}{2}\pi\mathrm{i}} = 600\pi\mathrm{e}^{\mathrm{i}\cdot(100\pi t + \frac{1}{2}\pi)}$

c $600\pi\mathrm{e}^{\mathrm{i}\cdot(100\pi t + \frac{1}{2}\pi)} = 600\pi\left(\cos\left(100\pi t + \tfrac{1}{2}\pi\right) + \mathrm{i}\sin\left(100\pi t + \tfrac{1}{2}\pi\right)\right) = 600\pi\cos\left(100\pi t + \tfrac{1}{2}\pi\right) + 600\pi\mathrm{i}\sin\left(100\pi t + \tfrac{1}{2}\pi\right)$
$\mathrm{Re}(g'(t)) = \mathrm{Re}\left(600\pi\mathrm{e}^{\mathrm{i}\cdot(100\pi t + \frac{1}{2}\pi)}\right)$
$= \mathrm{Re}\left(600\pi\cos\left(100\pi t + \tfrac{1}{2}\pi\right) + 600\pi\sin\left(100\pi t + \tfrac{1}{2}\pi\right)\right)$
$= 600\pi\cos\left(100\pi t + \tfrac{1}{2}\pi\right)$
$= f'(t)$

Bladzijde 190

56 a $U = \left(2 + 2\mathrm{i}\sqrt{3}\right) \cdot \mathrm{e}^{\mathrm{i}\cdot 100\pi t} = 4\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}} \cdot \mathrm{e}^{\mathrm{i}\cdot 100\pi t} = 4\mathrm{e}^{\mathrm{i}\cdot(100\pi t + \frac{1}{3}\pi)}$

De reële spanning is $\mathrm{Re}(U) = 4\cos\left(100\pi t + \tfrac{1}{3}\pi\right)$.

b $U = (4 + 4\mathrm{i}) \cdot \mathrm{e}^{\mathrm{i}\cdot 5t} = 4\sqrt{2}\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}} \cdot \mathrm{e}^{\mathrm{i}\cdot 5t} = 4\sqrt{2}\mathrm{e}^{\mathrm{i}\cdot(5t + \frac{1}{4}\pi)}$

De reële spanning is $\mathrm{Re}(U) = 4\sqrt{2}\cos\left(5t + \tfrac{1}{4}\pi\right)$.

c $U = (1 - \mathrm{i}) \cdot \mathrm{e}^{\mathrm{i}\cdot 10t + 2} = \sqrt{2}\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}} \cdot \mathrm{e}^{\mathrm{i}\cdot 10t} \cdot \mathrm{e}^2 = \mathrm{e}^2\sqrt{2}\mathrm{e}^{\mathrm{i}\cdot(10t - \frac{1}{4}\pi)}$

De reële spanning is $\mathrm{Re}(U) = \mathrm{e}^2\sqrt{2}\cos\left(10t - \tfrac{1}{4}\pi\right)$.

57 a $U = (10 + 10\mathrm{i}) \cdot \mathrm{e}^{\mathrm{i}\cdot 20\pi t} = 10\sqrt{2}\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}} \cdot \mathrm{e}^{\mathrm{i}\cdot 20\pi t} = 10\sqrt{2}\mathrm{e}^{\mathrm{i}\cdot(20\pi t + \frac{1}{4}\pi)}$
$\arg(U) - \arg(I) = 20\pi t + \tfrac{1}{4}\pi - 20\pi t = \tfrac{1}{4}\pi$
Het faseverschil is $\tfrac{1}{4}\pi$ rad.

b $\dfrac{U_{\max}}{I_{\max}} = \dfrac{10\sqrt{2}}{2} = 5\sqrt{2}$

58 a $U = 12\cos(ct)$ met $c = 2\pi f = 2\pi \cdot 100 = 200\pi$.
Dus $U = 12\cos(200\pi t)$.

b $U = 12(\cos(200\pi t) + \mathrm{i}\sin(200\pi t)) = 12\mathrm{e}^{\mathrm{i}\cdot 200\pi t}$

c $I_1 = \dfrac{12\mathrm{e}^{\mathrm{i}\cdot 200\pi t}}{4} = 3\mathrm{e}^{\mathrm{i}\cdot 200\pi t}$

$I_2 = \dfrac{12\mathrm{e}^{\mathrm{i}\cdot 200\pi t}}{6} = 2\mathrm{e}^{\mathrm{i}\cdot 200\pi t}$

d $I = 3\mathrm{e}^{\mathrm{i}\cdot 200\pi t} + 2\mathrm{e}^{\mathrm{i}\cdot 200\pi t} = 5\mathrm{e}^{\mathrm{i}\cdot 200\pi t}$

Bladzijde 191

59 a Als de condensator volledig is opgeladen, verandert U niet meer.
Verder is C een constante.
Dus volgens $Q = C \cdot U$ verandert Q dan niet meer.
Dus $I = \dfrac{\mathrm{d}Q}{\mathrm{d}t} = 0$.
Er loopt dus geen stroom meer vanaf het moment dat de condensator volledig is opgeladen.

b $U = 220(\cos(100\pi t) + \mathrm{i}\sin(100\pi t)) = 220\mathrm{e}^{\mathrm{i}\cdot 100\pi t}$

c $Q = C \cdot U$ geeft $0{,}002 \cdot 220\mathrm{e}^{\mathrm{i}\cdot 100\pi t} = 0{,}44\mathrm{e}^{\mathrm{i}\cdot 100\pi t}$
$I = \dfrac{\mathrm{d}Q}{\mathrm{d}t} = 44\pi\mathrm{i}\,\mathrm{e}^{\mathrm{i}\cdot 100\pi t}$

Bladzijde 193

60 a $I_1 = \dfrac{U}{R} = \dfrac{50\mathrm{e}^{\mathrm{i}\cdot 30\pi t}}{3} = 16\tfrac{2}{3}\mathrm{e}^{\mathrm{i}\cdot 30\pi t}$
$Q = C \cdot U = 0{,}05\mathrm{e}^{\mathrm{i}\cdot 30\pi t}$ geeft $I_2 = \dfrac{\mathrm{d}Q}{\mathrm{d}t} = 1{,}5\pi\mathrm{i}\,\mathrm{e}^{\mathrm{i}\cdot 30\pi t}$
$I = I_1 + I_2 = 16\tfrac{2}{3}\mathrm{e}^{\mathrm{i}\cdot 30\pi t} + 1{,}5\pi\mathrm{i}\,\mathrm{e}^{\mathrm{i}\cdot 30\pi t} = \left(16\tfrac{2}{3} + 1{,}5\pi\mathrm{i}\right)\mathrm{e}^{\mathrm{i}\cdot 30\pi t}$

b $Z = \dfrac{U_{\max}}{I_{\max}} = \dfrac{50}{\left|16\tfrac{2}{3} + 1{,}5\pi\mathrm{i}\right|} = \dfrac{50}{\sqrt{\left(16\tfrac{2}{3}\right)^2 + (1{,}5\pi)^2}} \approx 2{,}89\ \Omega$

c $\arg(I) - \arg(U) = \arg\left(16\tfrac{2}{3} + 1{,}5\pi\mathrm{i}\right) + 30\pi t - 30\pi t = \arg\left(16\tfrac{2}{3} + 1{,}5\pi\mathrm{i}\right) \approx 0{,}28$ rad
I loopt 0,28 rad voor op U.

61 $U = U_{max} \cdot e^{i \cdot \omega t}$ geeft $\frac{dU}{dt} = \omega \cdot U_{max} \cdot e^{i \cdot (\omega t + \frac{1}{2}\pi)}$

$Q = C \cdot U$ met C een constante geeft $\frac{dQ}{dt} = C \cdot \frac{dU}{dt}$.

Dus $I = \frac{dQ}{dt} = C \cdot \frac{dU}{dt} = C \cdot \omega \cdot U_{max} \cdot e^{i \cdot (\omega t + \frac{1}{2}\pi)}$ met $I_{max} = C \cdot \omega \cdot U_{max}$.

Dus $Z = \frac{U_{max}}{I_{max}} = \frac{U_{max}}{C \cdot \omega \cdot U_{max}} = \frac{1}{\omega C}$.

62 **a** Volgens de wet van Ohm is $U_R = I \cdot R = 6e^{i \cdot 10\pi t} \cdot 5 = 30e^{i \cdot 10\pi t}$.

b $\frac{dQ}{dt} = I = 6e^{i \cdot 10\pi t}$ geeft $Q = \int 6e^{i \cdot 10\pi t}\,dt = \frac{6}{10\pi i} e^{i \cdot 10\pi t} = -\frac{6}{10\pi} i e^{i \cdot 10\pi t}$

Uit $Q = C \cdot U$ volgt $U_C = \frac{Q}{C} = \frac{-\frac{6}{10\pi} i e^{i \cdot 10\pi t}}{0{,}01} = -\frac{60}{\pi} i e^{i \cdot 10\pi t}$.

c $U = U_R + U_C = 30e^{i \cdot 10\pi t} - \frac{60}{\pi} i e^{i \cdot 10\pi t} = \left(30 - \frac{60}{\pi} i\right) e^{i \cdot 10\pi t}$

d $Z = \frac{U_{max}}{I_{max}} = \frac{\left|30 - \frac{60}{\pi} i\right|}{6} = \frac{\sqrt{30^2 + \left(-\frac{60}{\pi}\right)^2}}{6} \approx 5{,}93\ \Omega$

e $\arg(U) - \arg(I) = \arg\left(30 - \frac{60}{\pi} i\right) + 10\pi t - 10\pi t = \arg\left(30 - \frac{60}{\pi} i\right) \approx -0{,}57$

I loopt 0,57 rad voor op U.

Bladzijde 194

63 **a** Stel $U = U_{max} e^{i \cdot \omega t}$.

Voor de condensator geldt $Q = C \cdot U = C \cdot U_{max} e^{i \cdot \omega t}$.

De stroom door de condensator is $I_1 = \frac{dQ}{dt} = C \cdot i\omega \cdot U_{max} e^{i \cdot \omega t}$.

De stroom door de weerstand is $I_2 = \frac{U}{R} = \frac{U_{max} e^{i \cdot \omega t}}{R}$.

$I = I_1 + I_2 = C \cdot i\omega \cdot U_{max} e^{i \cdot \omega t} + \frac{U_{max} e^{i \cdot \omega t}}{R}$

$$\frac{1}{Z} = \frac{I_{max}}{U_{max}} = \frac{\left|C \cdot i\omega \cdot U_{max} e^{i \cdot \omega t} + \frac{U_{max} e^{i \cdot \omega t}}{R}\right|}{\left|U_{max} e^{i \cdot \omega t}\right|} = \frac{\left|U_{max} e^{i \cdot \omega t}\right| \cdot \left|C \cdot i\omega + \frac{1}{R}\right|}{\left|U_{max} e^{i \cdot \omega t}\right|} = \left|C \cdot i\omega + \frac{1}{R}\right| = \sqrt{(\omega C)^2 + \left(\frac{1}{R}\right)^2}$$

b Stel $I = I_{max} e^{i \cdot \omega t}$.

Voor de spanning tussen de platen van de condensator geldt $\frac{dQ}{dt} = I = I_{max} e^{i \cdot \omega t}$, dus

$Q = \frac{I_{max} e^{i \cdot \omega t}}{i\omega}$ en $U = \frac{I_{max} e^{i \cdot \omega t}}{i\omega C}$.

De spanning tussen de uiteinden van de weerstand is $U = I \cdot R = I_{max} e^{i \cdot \omega t} \cdot R = I_{max} R e^{i \cdot \omega t}$.

$U = \frac{I_{max} e^{i \cdot \omega t}}{i\omega C} + I_{max} R e^{i \cdot \omega t}$

$$Z = \frac{U_{max}}{I_{max}} = \frac{\left|\frac{I_{max} e^{i \cdot \omega t}}{i\omega C} + I_{max} R e^{i \cdot \omega t}\right|}{\left|I_{max} e^{i \cdot \omega t}\right|} = \frac{\left|I_{max} e^{i \cdot \omega t}\right| \cdot \left|\frac{1}{i\omega C} + R\right|}{\left|I_{max} e^{i \cdot \omega t}\right|} = \left|\frac{1}{i\omega C} + R\right| = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$$

64 **a** $\frac{dI}{dt} = 5$ en $L = 0{,}012$ geeft $U_{spoel} = L \cdot \frac{dI}{dt} = 0{,}012 \cdot 5 = 0{,}06$ volt.

b $\omega = 2\pi f = 2\pi \cdot 50 = 100\pi$ rad/s en $U_{max} = 220$ V, dus $U = 220e^{i \cdot 100\pi t}$.

$U = L \cdot \frac{dI}{dt}$ ofwel $\frac{dI}{dt} = \frac{U}{L}$ met $U = 220e^{i \cdot 100\pi t}$ en $L = 0{,}005$ geeft $\frac{dI}{dt} = \frac{220e^{i \cdot 100\pi t}}{0{,}005} = 44\,000e^{i \cdot 100\pi t}$.

Dus $I = \frac{44\,000}{100\pi i} e^{i \cdot 100\pi t} = -\frac{440}{\pi} i e^{i \cdot 100\pi t}$.

$Z = \frac{U_{max}}{I_{max}} = \frac{220}{\left|-\frac{440}{\pi} i\right|} = \frac{220}{\left(\frac{440}{\pi}\right)} = \frac{1}{2}\pi\ \Omega$

c $\arg(I) - \arg(U) = \arg\left(-\frac{440}{\pi}\mathrm{i}\right) + 100\pi t - 100\pi t = \arg\left(-\frac{440}{\pi}\mathrm{i}\right) = -\frac{1}{2}\pi$ rad
U loopt $\frac{1}{2}\pi$ rad voor op I.

Bladzijde 195

65 **a** Uit $U = U_{max} \cdot \mathrm{e}^{\mathrm{i}\omega t}$ en $I = \frac{U_{max}}{\omega L} \cdot \mathrm{e}^{\mathrm{i} \cdot (\omega t - \frac{1}{2}\pi)}$ volgt $\arg(U) - \arg(I) = \omega t - \left(\omega t - \frac{1}{2}\pi\right) = \frac{1}{2}\pi$ rad.
U loopt $\frac{1}{2}\pi$ rad voor op I.

b $Z = \frac{U_{max}}{I_{max}} = \frac{U_{max}}{\left(\frac{U_{max}}{\omega L}\right)} = \omega L$

Bladzijde 196

66 De stroom op $t = 0$ is $I = \frac{U_{max}}{\omega L} \cdot \mathrm{e}^{\mathrm{i} \cdot (0 - \frac{1}{2}\pi)} = \frac{U_{max}}{\omega L} \cdot \mathrm{e}^{-\frac{1}{2}\pi\mathrm{i}} = -\frac{U_{max}}{\omega L}\mathrm{i}$.
Het reële deel hiervan is 0, dus is er geen stroom op $t = 0$.

67 **a** Voor de weerstand geldt $U_R = I \cdot R = 2\mathrm{e}^{\mathrm{i} \cdot 100\pi t} \cdot 20 = 40\mathrm{e}^{\mathrm{i} \cdot 100\pi t}$.
Voor de spoel geldt $U_{spoel} = L \cdot \frac{\mathrm{d}I}{\mathrm{d}t} = 0{,}1 \cdot 200\pi\mathrm{i}\,\mathrm{e}^{\mathrm{i} \cdot 100\pi t} = 20\pi\mathrm{i}\,\mathrm{e}^{\mathrm{i} \cdot 100\pi t}$.
$U = U_R + U_{spoel} = 40\mathrm{e}^{\mathrm{i} \cdot 100\pi t} + 20\pi\mathrm{i}\,\mathrm{e}^{\mathrm{i} \cdot 100\pi t} = (40 + 20\pi\mathrm{i})\mathrm{e}^{\mathrm{i} \cdot 100\pi t}$

b $Z = \frac{U_{max}}{I_{max}} = \frac{|40 + 20\pi\mathrm{i}|}{2} = \frac{\sqrt{40^2 + (20\pi)^2}}{2} \approx 37{,}2\ \Omega$

c $\arg(U) - \arg(I) = \arg(40 + 20\pi\mathrm{i}) + 100\pi t - 100\pi t = \arg(40 + 20\pi\mathrm{i}) \approx 1{,}00$ rad
U loopt 1,00 rad voor op I.

68 **a** Voor de condensator geldt $Q = C \cdot U = 0{,}001 \cdot 50\mathrm{e}^{\mathrm{i} \cdot 1000\pi t} = 0{,}05\mathrm{e}^{\mathrm{i} \cdot 1000\pi t}$ en $I_C = \frac{\mathrm{d}Q}{\mathrm{d}t} = 50\pi\mathrm{i}\,\mathrm{e}^{\mathrm{i} \cdot 1000\pi t}$.
Voor de spoel geldt $I_{spoel} = -\mathrm{i} \cdot \frac{U_{max}}{\omega L} \cdot \mathrm{e}^{\mathrm{i}\omega t} = -\mathrm{i} \cdot \frac{50}{1000\pi \cdot 0{,}0005} \cdot \mathrm{e}^{\mathrm{i}\omega t} = -\frac{100}{\pi}\mathrm{i}\,\mathrm{e}^{\mathrm{i}\omega t}$.
Dus $I = I_C + I_{spoel} = 50\pi\mathrm{i}\,\mathrm{e}^{\mathrm{i} \cdot 1000\pi t} - \frac{100}{\pi}\mathrm{i}\,\mathrm{e}^{\mathrm{i} \cdot 1000\pi t} = \left(50\pi - \frac{100}{\pi}\right)\mathrm{i}\,\mathrm{e}^{\mathrm{i} \cdot 1000\pi t}$.

b $Z = \frac{U_{max}}{I_{max}} = \frac{50}{50\pi - \frac{100}{\pi}} \approx 0{,}399\ \Omega$

c $\arg(I) - \arg(U) = \arg\left(\left(50\pi - \frac{100}{\pi}\right)\mathrm{i}\right) + 1000\pi t - 1000\pi t = \frac{1}{2}\pi + 1000\pi t - 1000\pi t = \frac{1}{2}\pi$ rad
I loopt $\frac{1}{2}\pi$ rad voor op U.

69 **a** Voor de stroom I_1 geldt $I_1 = \frac{\mathrm{d}Q}{\mathrm{d}t} = C \cdot \frac{\mathrm{d}U}{\mathrm{d}t} = C \cdot U_{max} \cdot \mathrm{i}\omega\,\mathrm{e}^{\mathrm{i}\omega t}$.
Voor de stroom I_2 geldt $\frac{\mathrm{d}I_2}{\mathrm{d}t} = \frac{U}{L} = \frac{U_{max} \cdot \mathrm{e}^{\mathrm{i}\omega t}}{L} = \frac{U_{max}}{L}\mathrm{e}^{\mathrm{i}\omega t}$, dus $I_2 = \frac{U_{max}}{\mathrm{i}\omega L}\mathrm{e}^{\mathrm{i}\omega t} = -\frac{U_{max}}{\omega L}\mathrm{i}\,\mathrm{e}^{\mathrm{i}\omega t}$.
Hieruit volgt $I = I_1 + I_2 = C \cdot U_{max} \cdot \mathrm{i}\omega\,\mathrm{e}^{\mathrm{i}\omega t} - \frac{U_{max}}{\omega L}\mathrm{i}\,\mathrm{e}^{\mathrm{i}\omega t} = U_{max}\left(\omega C - \frac{1}{\omega L}\right)\mathrm{i}\,\mathrm{e}^{\mathrm{i}\omega t} =$
$U_{max}\left(\omega C - \frac{1}{\omega L}\right)\mathrm{e}^{\frac{1}{2}\pi\mathrm{i}} \cdot \mathrm{e}^{\mathrm{i}\omega t} = U_{max}\left(\omega C - \frac{1}{\omega L}\right)\mathrm{e}^{\mathrm{i} \cdot (\omega t + \frac{1}{2}\pi)}$.

b $I = 0$ geeft $U_{max}\left(\omega C - \frac{1}{\omega L}\right)\mathrm{e}^{\mathrm{i} \cdot (\omega t + \frac{1}{2}\pi)} = 0$

$$\omega C - \frac{1}{\omega L} = 0$$
$$\omega C = \frac{1}{\omega L}$$
$$\omega^2 = \frac{1}{LC}$$
$$\omega = \frac{1}{\sqrt{LC}}$$

Bladzijde 197

70 Voor de weerstand geldt $I_R = \dfrac{U_{max} \cdot e^{i\omega t}}{R}$.

Voor de condensator geldt $I_C = C \cdot U_{max} \cdot i\omega\, e^{i\omega t}$ (zie opgave 69).

Voor de spoel geldt $I_{spoel} = -\dfrac{U_{max}}{\omega L} i\, e^{i\omega t}$ (zie opgave 69).

Dus $I = I_R + I_C + I_{spoel} = \dfrac{U_{max} \cdot e^{i\omega t}}{R} + C \cdot U_{max} \cdot i\omega\, e^{i\omega t} - \dfrac{U_{max}}{\omega L} i\, e^{i\omega t} = U_{max}\left(\dfrac{1}{R} + \left(\omega C - \dfrac{1}{\omega L}\right)i\right)e^{i\omega t}$.

$$Z = \frac{U_{max}}{I_{max}} = \frac{U_{max}}{\left|U_{max}\left(\frac{1}{R} + \left(\omega C - \frac{1}{\omega L}\right)i\right)\right|} = \frac{U_{max}}{|U_{max}| \cdot \left|\frac{1}{R} + \left(\omega C - \frac{1}{\omega L}\right)i\right|} = \frac{U_{max}}{U_{max} \cdot \left|\frac{1}{R} + \left(\omega C - \frac{1}{\omega L}\right)i\right|} =$$

$$\frac{1}{\left|\frac{1}{R} + \left(\omega C - \frac{1}{\omega L}\right)i\right|} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

De impedantie is maximaal als $\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$ minimaal is, dus als $\left(\omega C - \frac{1}{\omega L}\right)^2 = 0$ ofwel

$\omega C - \dfrac{1}{\omega L} = 0$. Hieruit volgt $\omega = \dfrac{1}{\sqrt{LC}}$.

Dus voor $\omega = \dfrac{1}{\sqrt{LC}}$ is de impedantie maximaal.

71 96,8 MHz = $96{,}8 \cdot 10^6$ Hz

$\omega = 2\pi f = 2\pi \cdot 96{,}8 \cdot 10^6 = 193{,}6\pi \cdot 10^6$ rad/s en $L = 6{,}76 \cdot 10^{-9}$

Uit $\omega = \dfrac{1}{\sqrt{LC}}$ volgt $\sqrt{LC} = \dfrac{1}{\omega}$

$$LC = \frac{1}{\omega^2}$$

$$C = \frac{1}{L\omega^2}$$

Dit geeft $C = \dfrac{1}{6{,}76 \cdot 10^{-9} \cdot (193{,}6\pi \cdot 10^6)^2} \approx 4 \cdot 10^{-10}\, F$.

De condensator moet worden ingesteld op 400 pF (1 pF = 10^{-12} F).

Diagnostische toets

Bladzijde 200

1 **a** $f(2 + i) = 0$ geeft $2(2 + i) + a + bi = 0$

$4 + 2i + a + bi = 0$

$a + bi = -4 - 2i$

Dus $a = -4$ en $b = -2$.

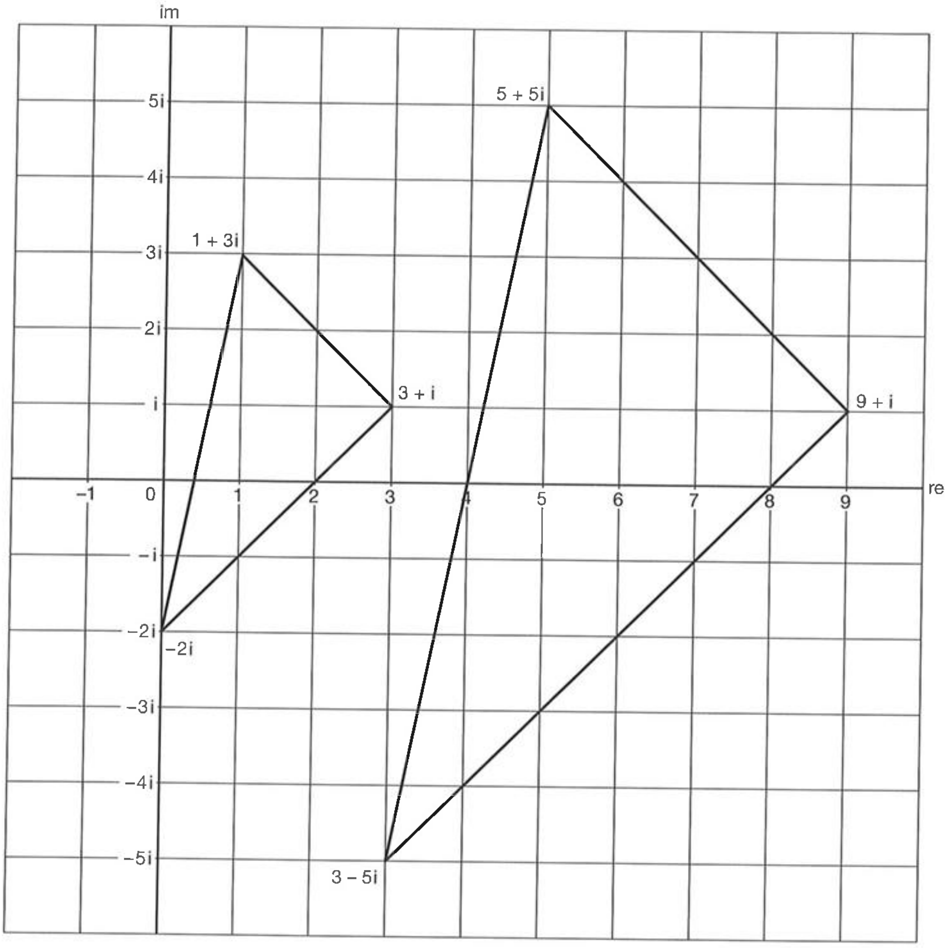
b $f(2 + i) = 2 + i$ geeft $2(2 + i) + a + bi = 2 + i$

$4 + 2i + a + bi = 2 + i$

$a + bi = -2 - i$

Dus $a = -2$ en $b = -1$.

c $a = 3$ en $b = -1$ geeft $f(z) = 2z + 3 - i$
$f(-2i) = 3 - 5i$
$f(3 + i) = 9 + i$
$f(1 + 3i) = 5 + 5i$



2 **a** $f(z) = 0$ geeft $2iz + 3 - i = 0$

$$2iz = -3 + i$$

$$z = \tfrac{1}{2} - \frac{3}{2i} = \tfrac{1}{2} + 1\tfrac{1}{2}i$$

Het nulpunt is $\frac{1}{2} + 1\frac{1}{2}i$.

$f(z) = z$ geeft $2iz + 3 - i = z$

$$2iz - z = -3 + i$$

$$(2i - 1)z = -3 + i$$

$$z = \frac{-3 + i}{2i - 1} \cdot \frac{2i + 1}{2i + 1} = \frac{-6i - 3 + 2i^2 + i}{4i^2 - 1} = \frac{-5 - 5i}{-5} = 1 + i$$

Het dekpunt is $1 + i$.

b $g(z) = 0$ geeft $(1 + i)z + 2 - 3i = 0$

$$(1 + i)z = -2 + 3i$$

$$z = \frac{-2 + 3i}{1 + i} \cdot \frac{1 - i}{1 - i} = \frac{-2 + 2i + 3i - 3i^2}{1 - i^2} = \frac{1 + 5i}{2} = \tfrac{1}{2} + 2\tfrac{1}{2}i$$

Het nulpunt is $\frac{1}{2} + 2\frac{1}{2}i$.

$g(z) = z$ geeft $(1 + i)z + 2 - 3i = z$

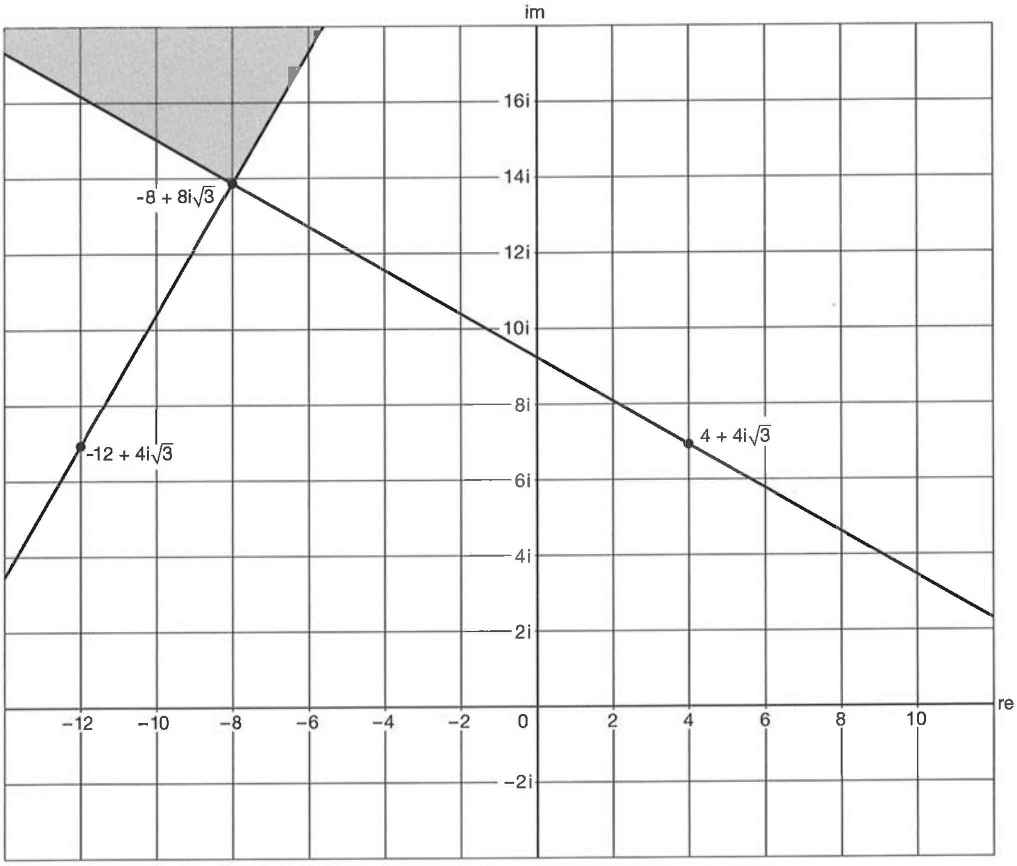
$$iz = -2 + 3i$$

$$z = 3 - \frac{2}{i} = 3 + 2i$$

Het dekpunt is $3 + 2i$.

3 **a** $|2+2i\sqrt{3}| = 4$ en $\text{Arg}(2+2i\sqrt{3}) = \frac{1}{3}\pi$, dus rotatie om 0 over $\frac{1}{3}\pi$ en vermenigvuldiging ten opzichte van 0 met 4.
Het bereik is $z \geq 8 \wedge \frac{5}{6}\pi \leq \text{Arg}(z) \leq 1\frac{1}{3}\pi$.

b De lijnen $\text{Re}(z) = 2$ en $\text{Im}(z) = 2\sqrt{3}$ snijden elkaar in $2 + 2i\sqrt{3}$.
$f(2+2i\sqrt{3}) = (2+2i\sqrt{3})(2+2i\sqrt{3}) = 4 + 8i\sqrt{3} - 12 = -8 + 8i\sqrt{3}$
2 ligt op $\text{Re}(z) = 2$.
$f(2) = (4+4i\sqrt{3}) \cdot 2 = 4 + 4i\sqrt{3}$
Het beeld van $\text{Re}(z) = 2$ is de lijn door $-8 + 8i\sqrt{3}$ en $4 + 4i\sqrt{3}$.
$2i\sqrt{3}$ ligt op $\text{Im}(z) = 2\sqrt{3}$.
$f(2i\sqrt{3}) = (2+2i\sqrt{3}) \cdot 2i\sqrt{3} = -12 + 4i\sqrt{3}$
Het beeld van $\text{Im}(z) = 2\sqrt{3}$ is de lijn door $-8 + 8i\sqrt{3}$ en $-12 + 4i\sqrt{3}$.



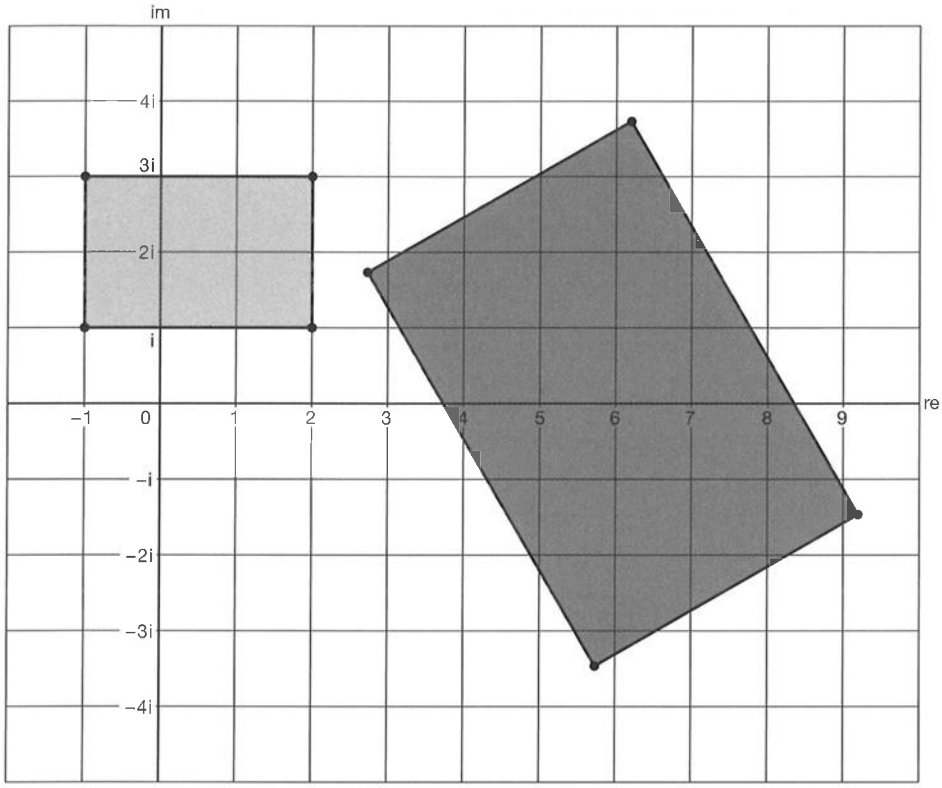
4 **a** $f(z) = 2 + 2i$ geeft $(1 - i\sqrt{3})z + 2 - i = 2 + 2i$

$$(1 - i\sqrt{3})z = 3i$$

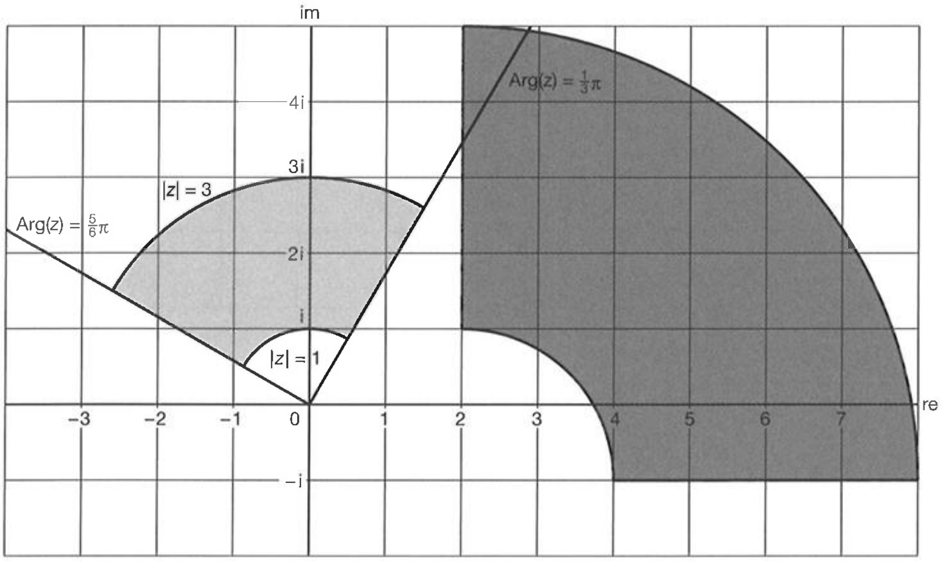
$$z = \frac{3i}{1 - i\sqrt{3}} \cdot \frac{1 + i\sqrt{3}}{1 + i\sqrt{3}} = \frac{-3\sqrt{3} + 3i}{4} = -\frac{3}{4}\sqrt{3} + \frac{3}{4}i$$

Her origineel is $-\frac{3}{4}\sqrt{3} + \frac{3}{4}i$.

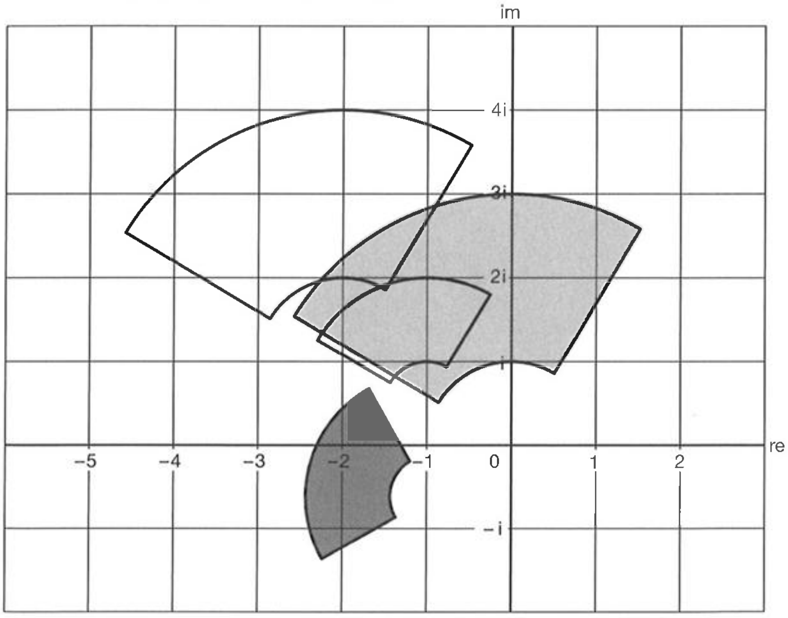
b $|1 - i\sqrt{3}| = 2$ en $\text{Arg}(1 - i\sqrt{3}) = -\frac{1}{3}\pi$, dus rotatie om 0 over $-\frac{1}{3}\pi$, vermenigvuldiging ten opzichte van 0 met 2 en translatie $(2, -1)$.



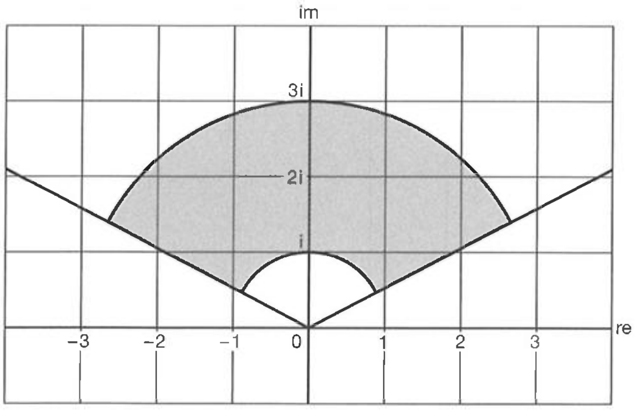
c



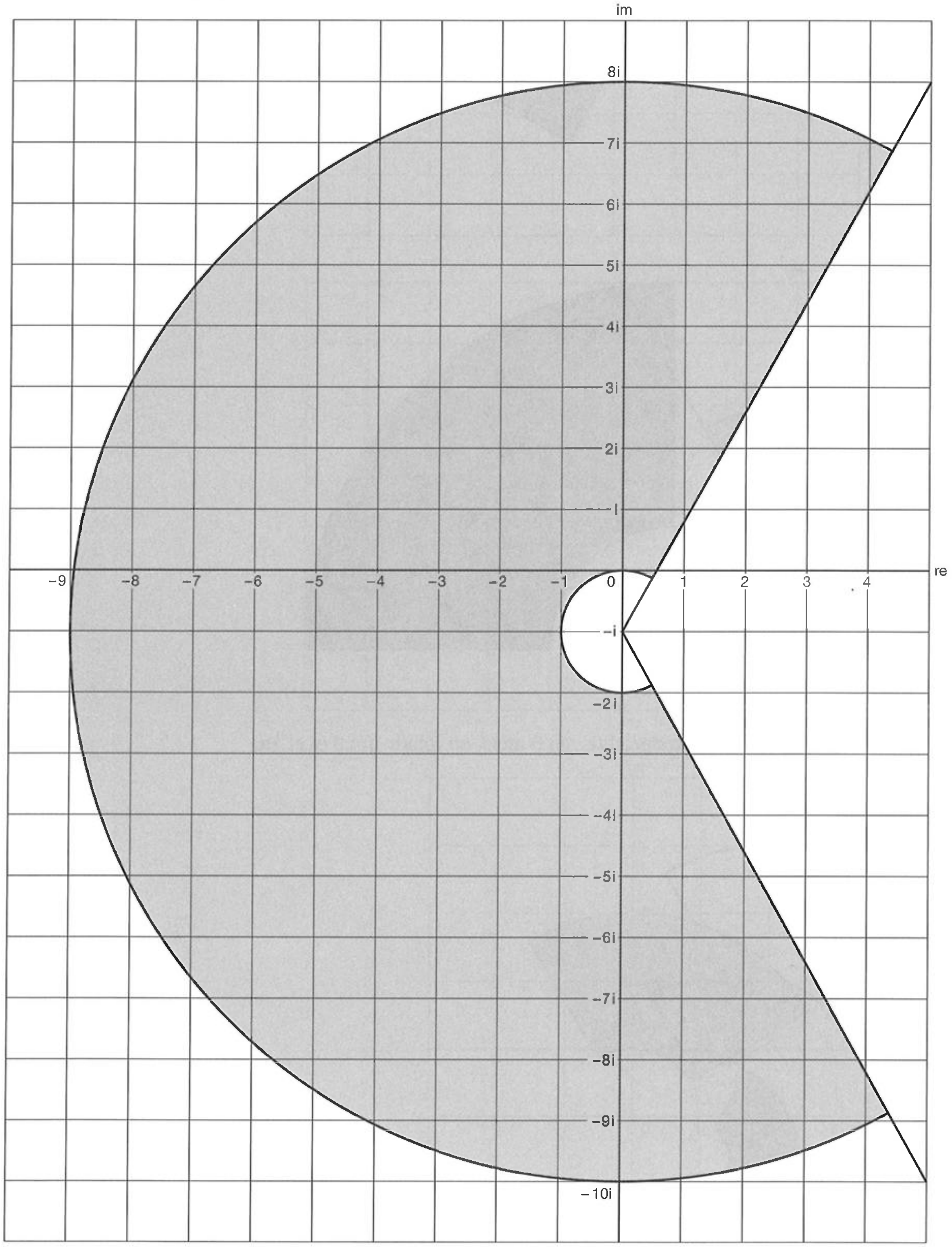
d Translatie (−2, 1), vermenigvuldiging ten opzichte van 0 met $\frac{1}{2}$ en rotatie om 0 over $\frac{1}{3}\pi$.



5

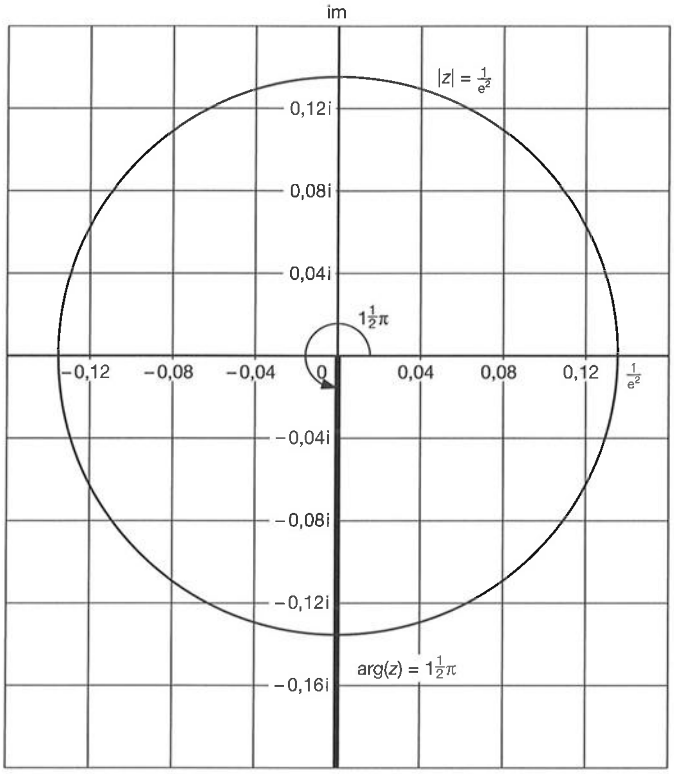


Eerst z^2: $|z^2| = |z|^2$, dus $1 \leq |z^2| \leq 9$ en $\arg(z^2) = 2 \cdot \arg(z)$, dus $\frac{1}{3}\pi \leq \arg(z^2) \leq 1\frac{2}{3}\pi$.
Dan de translatie $(0, -1)$.

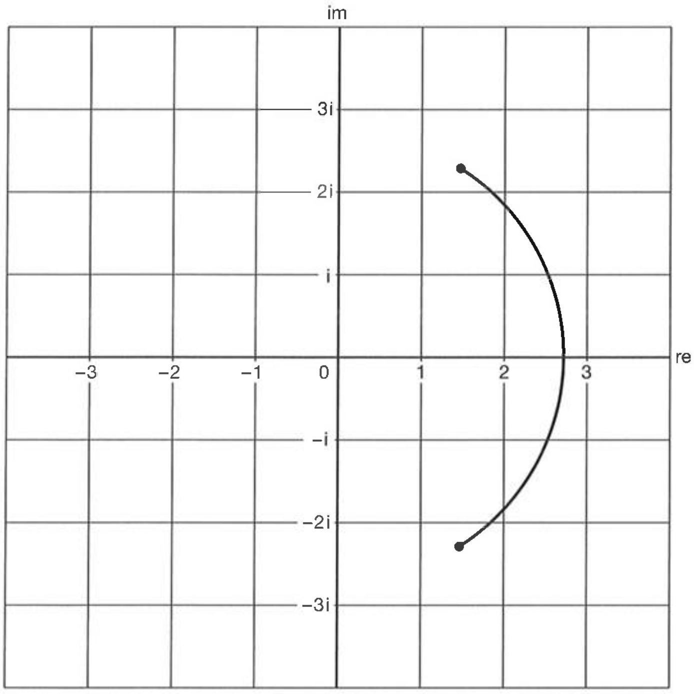


6 **a** Het beeld van $\text{Re}(z) = -2$ is de cirkel met middelpunt 0 en straal e^{-2}, ofwel de cirkel met vergelijking $|z| = e^{-2} = \frac{1}{e^2}$.

Het beeld van $\text{Im}(z) = 1\frac{1}{2}\pi$ is de halve lijn met beginpunt 0 die een hoek van $1\frac{1}{2}\pi$ radialen maakt met de positieve reële as, ofwel de halve lijn met vergelijking $\arg(z) = 1\frac{1}{2}\pi$.



b De eindpunten van het lijnstuk zijn $1 - i$ en $1 + i$, dus $\text{Re}(z) = 1$ is vast.
Het beeld van het lijnstuk is een deel van een cirkel met middelpunt 0 en straal e waarbij een hoek wordt doorlopen van –1 tot 1 radialen.



7 **a** $f(1 + i) = \ln(1 + i) = \ln\left(\sqrt{2}\,e^{\frac{1}{4}\pi i}\right) = \ln\left(\sqrt{2}\right) + \ln\left(e^{\frac{1}{4}\pi i}\right) = \frac{1}{2}\ln(2) + \frac{1}{4}\pi i$

b $f\left(\frac{i}{e}\right) = \ln\left(\frac{i}{e}\right) = \ln(i) - \ln(e) = \ln\left(e^{\frac{1}{2}\pi i}\right) - 1 = -1 + \frac{1}{2}\pi i$

c $f(-e^3) = \ln(-e^3) = \ln(-1) + \ln(e^3) = \ln(e^{\pi i}) + 3 = 3 + \pi i$

d $f(i\sqrt{e}) = \ln(i\sqrt{e}) = \ln(i) + \ln(\sqrt{e}) = \ln\left(e^{\frac{1}{2}\pi i}\right) + \frac{1}{2} = \frac{1}{2} + \frac{1}{2}\pi i$

e $f\left(\frac{2i}{1-i}\right) = \ln\left(\frac{2i}{1-i}\right) = \ln(2) + \ln(i) - \ln(1-i) = \ln(2) + \ln\left(e^{\frac{1}{2}\pi i}\right) - \ln\left(\sqrt{2}e^{-\frac{1}{4}\pi i}\right) = \ln(2) + \frac{1}{2}\pi i - \ln\left(\sqrt{2}\right) - \ln\left(e^{-\frac{1}{4}\pi i}\right) =$
$\ln(2) + \frac{1}{2}\pi i - \frac{1}{2}\ln(2) + \frac{1}{4}\pi i = \frac{1}{2}\ln(2) + \frac{3}{4}\pi i$

f $f(e - ei\sqrt{3}) = \ln(e - ei\sqrt{3}) = \ln\left(e\left(1 - i\sqrt{3}\right)\right) = \ln(e) + \ln\left(1 - i\sqrt{3}\right) = 1 + \ln\left(2e^{-\frac{1}{3}\pi i}\right) = 1 + \ln(2) + \ln\left(e^{-\frac{1}{3}\pi i}\right) =$
$1 + \ln(2) - \frac{1}{3}\pi i$

8 **a** Bij een punt van c hoort het complexe getal $z = e^2 \cdot e^{i\varphi}$.
Dit geeft $f(e^2 \cdot e^{i\varphi}) = \ln(e^2 \cdot e^{i\varphi}) = \ln(e^2) + \ln(e^{i\varphi}) = 2 + i\varphi$.
Omdat is afgesproken dat $-\pi < \text{Im}(f(z)) \leq \pi$ is het beeld van c het lijnstuk met eindpunten $2 - \pi i$ en $2 + \pi i$.

b Voor het tekenen van het beeld van $\text{Im}(z) = \frac{1}{2}\pi$ bereken je een aantal functiewaarden.

$f\left(-3 + \frac{1}{2}\pi i\right) \approx 1{,}22 + 2{,}66i$

$f\left(-2 + \frac{1}{2}\pi i\right) \approx 0{,}93 + 2{,}48i$

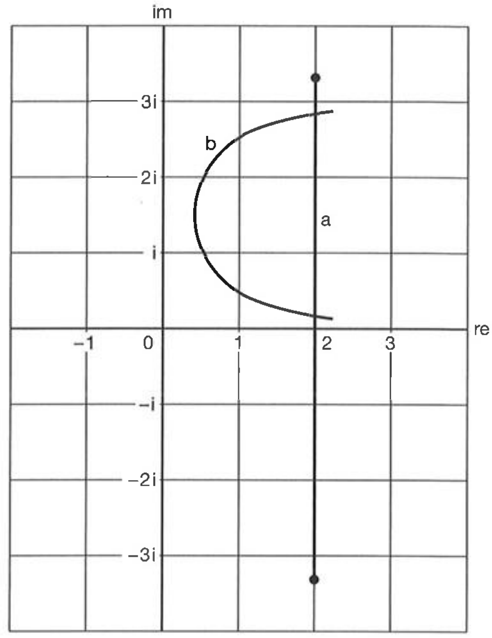
$f\left(-1 + \frac{1}{2}\pi i\right) \approx 0{,}62 + 2{,}14i$

$f\left(\frac{1}{2}\pi i\right) \approx 0{,}45 + 1{,}57i$

$f\left(1 + \frac{1}{2}\pi i\right) \approx 0{,}62 + 1{,}00i$

$f\left(2 + \frac{1}{2}\pi i\right) \approx 0{,}93 + 0{,}67i$

$f\left(3 + \frac{1}{2}\pi i\right) \approx 1{,}22 + 0{,}48i$



Bladzijde 201

9 $f\left(\frac{1}{3}\pi + 2i\right) = \sin\left(\frac{1}{3}\pi + 2i\right) = \frac{e^{i\cdot\left(\frac{1}{3}\pi + 2i\right)} - e^{-i\cdot\left(\frac{1}{3}\pi + 2i\right)}}{2i} = \frac{e^{-2+\frac{1}{3}\pi i} - e^{2-\frac{1}{3}\pi i}}{2i} = \frac{e^{-2}\cdot e^{\frac{1}{3}\pi i} - e^{2}\cdot e^{-\frac{1}{3}\pi i}}{2i} =$

$\frac{e^{-2}\left(\frac{1}{2} + \frac{1}{2}i\sqrt{3}\right) - e^{2}\left(\frac{1}{2} - \frac{1}{2}i\sqrt{3}\right)}{2i} = \frac{\frac{1}{2}e^{-2}}{2i} + \frac{\frac{1}{2}e^{-2}i\sqrt{3}}{2i} - \frac{\frac{1}{2}e^{2}}{2i} + \frac{\frac{1}{2}e^{2}i\sqrt{3}}{2i} = \frac{-i}{4e^2} + \frac{\sqrt{3}}{4e^2} + \frac{ie^2}{4} + \frac{e^2\sqrt{3}}{4} =$

$\frac{\sqrt{3}}{4e^2} + \frac{e^2\sqrt{3}}{4} + \left(\frac{e^2}{4} - \frac{1}{4e^2}\right)i$

$g\left(\frac{1}{2}\pi - i\right) = \cos\left(\frac{1}{2}\pi - i\right) = \frac{e^{i\cdot\left(\frac{1}{2}\pi - i\right)} + e^{-i\cdot\left(\frac{1}{2}\pi - i\right)}}{2} = \frac{e^{1+\frac{1}{2}\pi i} + e^{-1-\frac{1}{2}\pi i}}{2} = \frac{e^{1}\cdot e^{\frac{1}{2}\pi i} + e^{-1}\cdot e^{-\frac{1}{2}\pi i}}{2} =$

$\frac{e\cdot i + e^{-1}\cdot -i}{2} = \left(\frac{e}{2} - \frac{1}{2e}\right)i$

10 **a** $h = b + i(a - b) = ia + (1 - i)b$
b $e = b - i(c - b) = (1 + i)b - ic$
c $p = b + (h - b) + (p - h)$
$= b + (h - b) + (e - b)$
$= b + i(a - b) + i(b - c)$
$= ia + b - ic$
d $g = c - i(a - c) = -ia + (1 + i)c$
e $d = c + i(b - c) = ib + (1 - i)c$
f $q = c + (d - c) + (q - d)$
$= c + (d - c) + (g - c)$
$= c + i(b - c) + i(c - a)$
$= -ia + ib + c$
g $i(p - a) = i(ia + b - ic - a) = (-1 - i)a + ib + c$
$q - a = -ia + ib + c - a = (-1 - i)a + ib + c$
Dus $i(p - a) = q - a$.
Hieruit volgt dat $AP = AQ$ en $AP \perp AQ$.
Dus driehoek APQ is een gelijkbenige rechthoekige driehoek met $AP = AQ$ en $\angle A = 90°$.

11 **a** Voer in $y_1 = x^3 + x^2 - 7x - 15$.
De optie zero (TI) of ROOT (Casio) geeft $x = 3$.

$$\begin{array}{l}
z-3\,/\,z^3 + z^2 - 7z - 15 \,\backslash\, z^2 + 4z + 5 \\
\qquad\quad \underline{z^3 - 3z^2} \;- \\
\qquad\qquad\quad 4z^2 - 7z \\
\qquad\qquad\quad \underline{4z^2 - 12z} \;- \\
\qquad\qquad\qquad\qquad 5z - 15 \\
\qquad\qquad\qquad\qquad \underline{5z - 15} \;- \\
\qquad\qquad\qquad\qquad\qquad 0
\end{array}$$

$z^2 + 4z + 5 = 0$
$(z + 2)^2 - 4 + 5 = 0$
$(z + 2)^2 = -1$
$(z + 2)^2 = \mathrm{i}^2$
$z + 2 = \mathrm{i} \vee z + 2 = -\mathrm{i}$
$z = -2 + \mathrm{i} \vee z = -2 - \mathrm{i}$
Dus $z = 3 \vee z = -2 + \mathrm{i} \vee z = -2 - \mathrm{i}$.

b $(z + 1)(z - \mathrm{i})(z + \mathrm{i}) = 15$
$(z + 1)(z^2 - \mathrm{i}^2) = 15$
$(z + 1)(z^2 + 1) = 15$
$z^3 + z^2 + z + 1 = 15$
$z^3 + z^2 + z - 14 = 0$
Voer in $y_1 = x^3 + x^2 + x - 14$.
De optie zero (TI) of ROOT (Casio) geeft $x = 2$.

$$\begin{array}{l}
z-2\,/\,z^3 + z^2 + z - 14 \,\backslash\, z^2 + 3z + 7 \\
\qquad\quad \underline{z^3 - 2z^2} \;- \\
\qquad\qquad\quad 3z^2 + z \\
\qquad\qquad\quad \underline{3z^2 - 6z} \;- \\
\qquad\qquad\qquad\quad 7z - 14 \\
\qquad\qquad\qquad\quad \underline{7z - 14} \;- \\
\qquad\qquad\qquad\qquad\quad 0
\end{array}$$

$z^2 + 3z + 7 = 0$
$\left(z + 1\tfrac{1}{2}\right)^2 - 2\tfrac{1}{4} + 7 = 0$
$\left(z + 1\tfrac{1}{2}\right)^2 = -4\tfrac{3}{4}$
$\left(z + 1\tfrac{1}{2}\right)^2 = \tfrac{19}{4}\mathrm{i}^2$
$z + 1\tfrac{1}{2} = \tfrac{1}{2}\mathrm{i}\sqrt{19} \vee z + 1\tfrac{1}{2} = -\tfrac{1}{2}\mathrm{i}\sqrt{19}$
$z = -1\tfrac{1}{2} + \tfrac{1}{2}\mathrm{i}\sqrt{19} \vee z = -1\tfrac{1}{2} - \tfrac{1}{2}\mathrm{i}\sqrt{19}$
Dus $z = 2 \vee z = -1\tfrac{1}{2} + \tfrac{1}{2}\mathrm{i}\sqrt{19} \vee z = -1\tfrac{1}{2} - \tfrac{1}{2}\mathrm{i}\sqrt{19}$.

12 **a** $z^3 + 6z - 88 = 0$ ofwel $z^3 + 6z = 88$

$$\left.\begin{array}{l} z = u + v \text{ en } 6 = -3uv \text{ geeft } u^3 + v^3 = 88 \\ \text{Uit } -3uv = 6 \text{ volgt } v = -\dfrac{2}{u}. \end{array}\right\} u^3 + \left(-\frac{2}{u}\right)^3 = 88$$

$$u^3 - \frac{8}{u^3} = 88$$

$$u^6 - 88u^3 - 8 = 0$$

$D = 7776$, dus $\sqrt{D} = 36\sqrt{6}$

$u^3 = \dfrac{88 + 36\sqrt{6}}{2} = 44 + 18\sqrt{6}$ is een oplossing

$$v^3 = 88 - u^3 = 88 - \left(44 + 18\sqrt{6}\right) = 44 - 18\sqrt{6}$$

$$z = \sqrt[3]{44 + 18\sqrt{6}} + \sqrt[3]{44 - 18\sqrt{6}} = 4$$

$$\begin{array}{l} z - 4 \,/\, z^3 \qquad + 6z - 88 \,\backslash\, z^2 + 4z + 22 \\ \quad \underline{z^3 - 4z^2} \; - \\ \qquad\quad 4z^2 + 6z \\ \qquad\quad \underline{4z^2 - 16z} \; - \\ \qquad\qquad\quad 22z - 88 \\ \qquad\qquad\quad \underline{22z - 88} \; - \\ \qquad\qquad\qquad\quad 0 \end{array}$$

$z^2 + 4z + 22 = 0$

$(z + 2)^2 - 4 + 22 = 0$

$(z + 2)^2 = -18$

$(z + 2)^2 = 18\text{i}^2$

$z + 2 = 3\text{i}\sqrt{2} \vee z + 2 = -3\text{i}\sqrt{2}$

$z = -2 + 3\text{i}\sqrt{2} \vee z = -2 - 3\text{i}\sqrt{2}$

Dus $z = 4 \vee z = -2 + 3\text{i}\sqrt{2} \vee z = -2 - 3\text{i}\sqrt{2}$.

b $z^3 - 3z^2 = 16$ ofwel $z^3 - 3z^2 - 16 = 0$

$a = -3$, $b = 0$ en $c = -16$

$p = b - \frac{1}{3}a^2 = 0 - \frac{1}{3} \cdot (-3)^2 = -3$ en $q = -\frac{2}{27}a^3 + \frac{1}{3}ab - c = -\frac{2}{27} \cdot (-3)^3 + \frac{1}{3} \cdot -3 \cdot 0 - -16 = 18$

$z = -\frac{1}{3}a + \sqrt[3]{\frac{1}{2}q + \sqrt{\left(\frac{1}{2}q\right)^2 + \left(\frac{1}{3}p\right)^3}} + \sqrt[3]{\frac{1}{2}q - \sqrt{\left(\frac{1}{2}q\right)^2 + \left(\frac{1}{3}p\right)^3}} =$

$-\frac{1}{3} \cdot -3 + \sqrt[3]{9 + \sqrt{9^2 + (-1)^3}} + \sqrt[3]{9 - \sqrt{9^2 + (-1)^3}} = 4$

$$\begin{array}{l} z - 4 \,/\, z^3 - 3z^2 \qquad - 16 \,\backslash\, z^2 + z + 4 \\ \quad \underline{z^3 - 4z^2} \; - \\ \qquad\quad z^2 \\ \qquad\quad \underline{z^2 - 4z} \; - \\ \qquad\qquad\quad 4z - 16 \\ \qquad\qquad\quad \underline{4z - 16} \; - \\ \qquad\qquad\qquad\quad 0 \end{array}$$

$z^2 + z + 4 = 0$

$\left(z + \frac{1}{2}\right)^2 - \frac{1}{4} + 4 = 0$

$\left(z + \frac{1}{2}\right)^2 = -3\frac{3}{4}$

$\left(z + \frac{1}{2}\right)^2 = \frac{15}{4}\text{i}^2$

$z + \frac{1}{2} = \frac{1}{2}\text{i}\sqrt{15} \vee z + \frac{1}{2} = -\frac{1}{2}\text{i}\sqrt{15}$

$z = -\frac{1}{2} + \frac{1}{2}\text{i}\sqrt{15} \vee z = -\frac{1}{2} - \frac{1}{2}\text{i}\sqrt{15}$

Dus $z = 4 \vee z = -\frac{1}{2} + \frac{1}{2}\text{i}\sqrt{15} \vee z = -\frac{1}{2} - \frac{1}{2}\text{i}\sqrt{15}$.

c $z^3 + 4z^2 + 6z + 4 = 0$
$a = 4$, $b = 6$ en $c = 4$
$p = b - \frac{1}{3}a^2 = 6 - \frac{1}{3} \cdot 4^2 = \frac{2}{3}$ en $q = -\frac{2}{27}a^3 + \frac{1}{3}ab - c = -\frac{2}{27} \cdot 4^3 + \frac{1}{3} \cdot 4 \cdot 6 - 4 = -\frac{20}{27}$
$z = -\frac{1}{3}a + \sqrt[3]{\frac{1}{2}q + \sqrt{(\frac{1}{2}q)^2 + (\frac{1}{3}p)^3}} + \sqrt[3]{\frac{1}{2}q - \sqrt{(\frac{1}{2}q)^2 + (\frac{1}{3}p)^3}} =$
$-\frac{1}{3} \cdot 4 + \sqrt[3]{-\frac{10}{27} + \sqrt{(-\frac{10}{27})^2 + (\frac{2}{9})^3}} + \sqrt[3]{-\frac{10}{27} - \sqrt{(-\frac{10}{27})^2 + (\frac{2}{9})^3}} = -2$

$$\begin{array}{l}
z + 2 \,/\, z^3 + 4z^2 + 6z + 4 \,\backslash\, z^2 + 2z + 2 \\
\quad\quad \underline{z^3 + 2z^2} \; - \\
\quad\quad\quad 2z^2 + 6z \\
\quad\quad\quad \underline{2z^2 + 4z} \; - \\
\quad\quad\quad\quad 2z + 4 \\
\quad\quad\quad\quad \underline{2z + 4} \; - \\
\quad\quad\quad\quad\quad 0
\end{array}$$

$z^2 + 2z + 2 = 0$
$(z + 1)^2 - 1 + 2 = 0$
$(z + 1)^2 = -1$
$(z + 1)^2 = i^2$
$z + 1 = i \vee z + 1 = -i$
$z = -1 + i \vee z = -1 - i$
Dus $z = -2 \vee z = -1 + i \vee z = -1 - i$.

13 **a** $\omega = 2\pi f = 2\pi \cdot 500 = 1000\pi$ rad/s
$U = U_{max} \cdot e^{i \cdot \omega t} = 360 e^{i \cdot 1000\pi t}$

b $I_1 = \dfrac{360 e^{i \cdot 1000\pi t}}{0{,}4} = 900 e^{i \cdot 1000\pi t}$
$I_2 = \dfrac{360 e^{i \cdot 1000\pi t}}{0{,}8} = 450 e^{i \cdot 1000\pi t}$
$I = I_1 + I_2 = 900 e^{i \cdot 1000\pi t} + 450 e^{i \cdot 1000\pi t} = 1350 e^{i \cdot 1000\pi t}$

14 **a** Voor de stroom I_1 geldt volgens de wet van Ohm $I_1 = \dfrac{U}{R} = \dfrac{U_{max} \cdot e^{i\omega t}}{R} = \dfrac{10 e^{i \cdot 1000\pi t}}{500} = \frac{1}{50} e^{i \cdot 1000\pi t}$.
Zie opgave 69. Voor de stroom I_2 geldt $I_2 = C \cdot U_{max} \cdot i\omega\, e^{i\omega t} = 0{,}001 \cdot 10 \cdot 1000\pi i\, e^{i \cdot 1000\pi t} = 10\pi i\, e^{i \cdot 1000\pi t}$.
Zie opgave 69. Voor de stroom I_3 geldt $I_3 = -\dfrac{U_{max}}{\omega L}\, i\, e^{i\omega t} = -\dfrac{10}{1000\pi \cdot 0{,}01}\, i\, e^{i \cdot 1000\pi t} = -\dfrac{1}{\pi}\, i\, e^{i \cdot 1000\pi t}$.
Dus $I = I_1 + I_2 + I_3 = \frac{1}{50} e^{i \cdot 1000\pi t} + 10\pi i\, e^{i \cdot 1000\pi t} - \dfrac{1}{\pi}\, i\, e^{i \cdot 1000\pi t} = \left(\frac{1}{50} + \left(10\pi - \dfrac{1}{\pi}\right) i\right) e^{i \cdot 1000\pi t}$.

b $Z = \dfrac{U_{max}}{I_{max}} = \dfrac{10}{\left|\frac{1}{50} + \left(10\pi - \dfrac{1}{\pi}\right) i\right|} = \dfrac{10}{\sqrt{(\frac{1}{50})^2 + \left(10\pi - \dfrac{1}{\pi}\right)^2}} \approx 0{,}32\ \Omega$

c $\arg(I) - \arg(U) = \arg\left(\frac{1}{50} + \left(10\pi - \dfrac{1}{\pi}\right) i\right) + 1000\pi t - 1000\pi t = \arg\left(\frac{1}{50} + \left(10\pi - \dfrac{1}{\pi}\right) i\right) \approx 1{,}57$ rad
I loopt 1,57 rad voor op U.

Gemengde opgaven

13 Kegelsneden

Bladzijde 202

1 c_1: $(x-3)^2+(y-4)^2=25$ ofwel c_1: $x^2+y^2-6x-8y=0$
De poollijn van Q is $x_Qx+y_Qy-3x_Q-3x-4y_Q-4y=0$.
$Q(-22, 29)$ geeft $-22x+29y+66-3x-116-4y=0$ ofwel $x=y-2$.
Substitutie van $x=y-2$ in $x^2+y^2-6x-8y=0$ geeft
$(y-2)^2+y^2-6(y-2)-8y=0$
$y^2-4y+4+y^2-6y+12-8y=0$
$2y^2-18y+16=0$
$y^2-9y+8=0$
$(y-1)(y-8)=0$
$y=1 \vee y=8$
$y=1$ geeft $x=-1$ en $y=8$ geeft $x=6$.
Dus $A(-1, 1)$ en $B(6,8)$.
Het midden van lijnstuk AB is $\left(\frac{-1+6}{2}, \frac{1+8}{2}\right)=\left(2\frac{1}{2}, 4\frac{1}{2}\right)$.
$d(A, B)=\sqrt{(6--1)^2+(8-1)^2}=7\sqrt{2}$, dus de straal van c_2 is $3\frac{1}{2}\sqrt{2}$.
Dit geeft $\left(x-2\frac{1}{2}\right)^2+\left(y-4\frac{1}{2}\right)^2=\left(3\frac{1}{2}\sqrt{2}\right)^2$
$x^2-5x+6\frac{1}{4}+y^2-9y+20\frac{1}{4}=24\frac{1}{2}$
$x^2+y^2-5x-9y+2=0$
Dus $p=-5$, $q=-9$ en $r=2$.

2 Stel $P(\lambda, 0)$.
$\text{rc}_l=m$ en l door $A(0, 5)$ geeft l: $y=mx+5$.
P op l geeft $m\lambda+5=0$
$m\lambda=-5$
$m=-\frac{5}{\lambda}$
Dus l: $y=-\frac{5}{\lambda}x+5$.
$k \perp l$, dus $\text{rc}_k \cdot \text{rc}_l=-1$ geeft $\text{rc}_k=\frac{1}{5}\lambda$.
k door $A(0, 5)$, dus k: $y=\frac{1}{5}\lambda x+5$.
p loodrecht op de x-as en p door $P(\lambda, 0)$ geeft p: $x=\lambda$.
k en p snijden elkaar in S, dus $\left.\begin{array}{l} y=\frac{1}{5}\lambda x+5 \\ x=\lambda \end{array}\right\} y=\frac{1}{5}x^2+5$
De kromme waarop alle punten S liggen is de parabool $y=\frac{1}{5}x^2+5$.

3 **a** p_a: $x^2+ax=ay-a^2$
$\left(x+\frac{1}{2}a\right)^2-\frac{1}{4}a^2=ay-a^2$
$\left(x+\frac{1}{2}a\right)^2=ay-\frac{3}{4}a^2$
$\left(x+\frac{1}{2}a\right)^2=a\left(y-\frac{3}{4}a\right)$
$2p=a$, dus $p=\frac{1}{2}a$.
$x^2=ay \xrightarrow{\text{translatie } \left(-\frac{1}{2}a,\, \frac{3}{4}a\right)} \left(x+\frac{1}{2}a\right)^2=a\left(y-\frac{3}{4}a\right)$
top$\left(-\frac{1}{2}a, \frac{3}{4}a\right)$
brandpunt$\left(0-\frac{1}{2}a, \frac{1}{4}a+\frac{3}{4}a\right)$, dus brandpunt$\left(-\frac{1}{2}a, a\right)$
richtlijn $y=-\frac{1}{2}a+a$, dus $y=\frac{1}{2}a$