

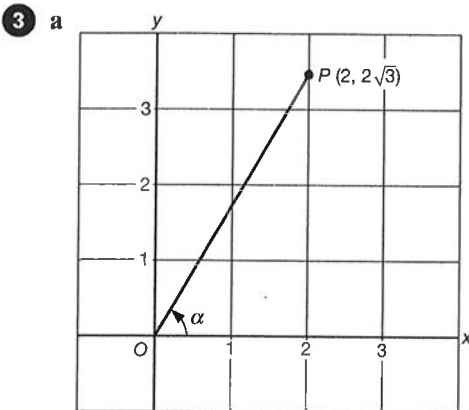
10 Complexe getallen

Voorkennis De exacte-waarden-cirkel

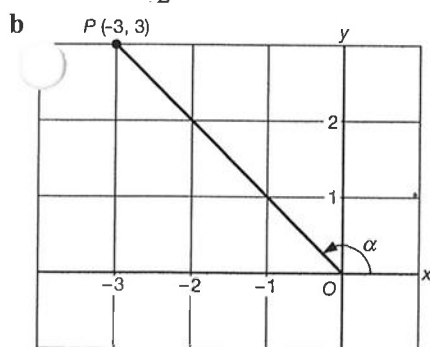
Bladzijde 40

- 1 a $\sin\left(\frac{3}{4}\pi\right) = \frac{1}{2}\sqrt{2}$ d $\cos\left(-\frac{2}{3}\pi\right) = -\frac{1}{2}$
 b $\cos\left(-\frac{3}{4}\pi\right) = -\frac{1}{2}\sqrt{2}$ e $\sin\left(-\frac{1}{3}\pi\right) = -\frac{1}{2}\sqrt{3}$
 c $\sin\left(-\frac{2}{3}\pi\right) = -\frac{1}{2}\sqrt{3}$ f $\cos\left(-\frac{5}{6}\pi\right) = -\frac{1}{2}\sqrt{3}$
- 2 a $\sin(\alpha) = \frac{1}{2}\sqrt{3}$ geeft $\alpha = \frac{1}{3}\pi \vee \alpha = \frac{2}{3}\pi$
 b $\cos(\alpha) = -\frac{1}{2}$ geeft $\alpha = -\frac{2}{3}\pi \vee \alpha = \frac{2}{3}\pi$
 c $\sin(\alpha) = -\frac{1}{2}\sqrt{2}$ geeft $\alpha = -\frac{1}{4}\pi \vee \alpha = -\frac{3}{4}\pi$
 d $\cos(\alpha) = 0$ geeft $\alpha = -\frac{1}{2}\pi \vee \alpha = \frac{1}{2}\pi$
 e $\sin(\alpha) = -\frac{1}{2}\sqrt{3}$ geeft $\alpha = -\frac{1}{3}\pi \vee \alpha = -\frac{2}{3}\pi$
 f $\cos(\alpha) = \frac{1}{2}\sqrt{2}$ geeft $\alpha = -\frac{1}{4}\pi \vee \alpha = \frac{1}{4}\pi$

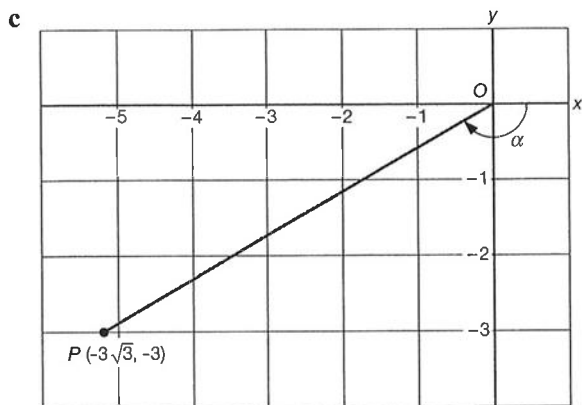
Bladzijde 41



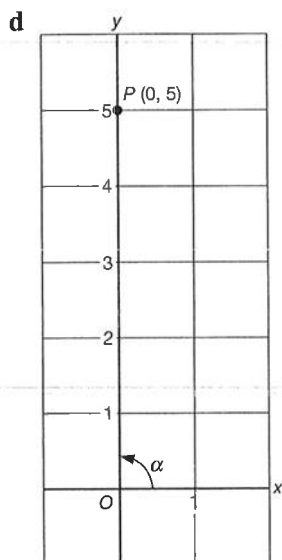
$$\tan(\alpha) = \frac{2\sqrt{3}}{2} = \sqrt{3} \text{ geeft } \alpha = \frac{1}{3}\pi$$



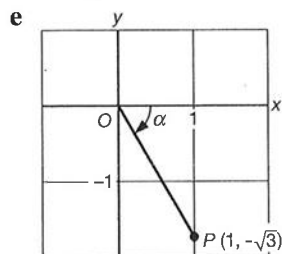
$$\tan(\alpha) = \frac{3}{-3} = -1 \text{ geeft } \alpha = \frac{3}{4}\pi$$



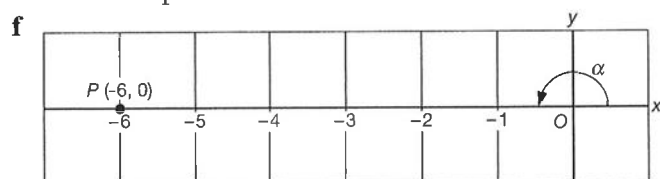
$$\tan(\alpha) = \frac{-3}{-3\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ geeft } \alpha = \frac{\pi}{6}$$



$$\alpha = \frac{\pi}{2}$$

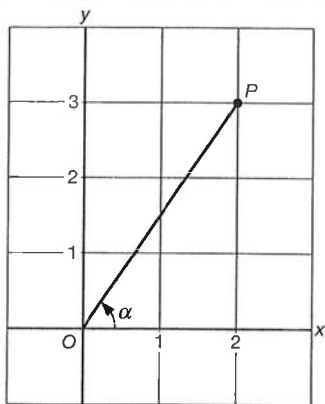


$$\tan(\alpha) = \frac{-\sqrt{3}}{1} = -\sqrt{3} \text{ geeft } \alpha = \frac{5\pi}{6}$$



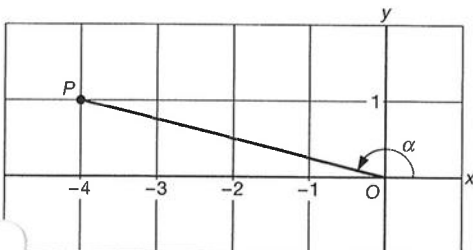
$$\alpha = \pi$$

4 a



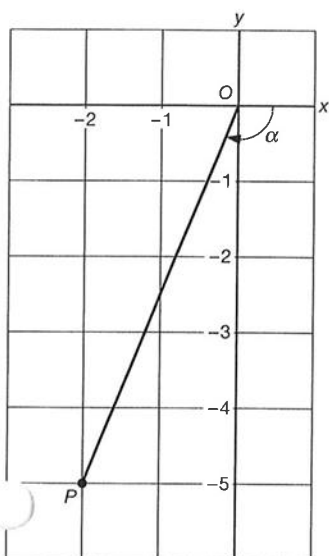
$$\tan(\alpha) = \frac{3}{2} = 1\frac{1}{2} \text{ geeft } \alpha \approx 0,983$$

b



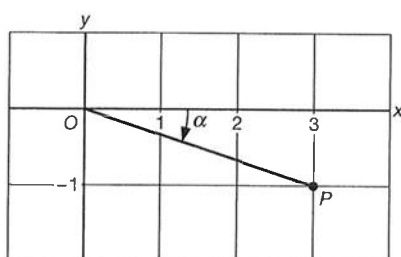
$$\tan(\alpha) = \frac{1}{-4} = -\frac{1}{4} \text{ geeft } \alpha \approx 2,897$$

c



$$\tan(\alpha) = \frac{-5}{-2} = 2\frac{1}{2} \text{ geeft } \alpha \approx -1,951$$

d



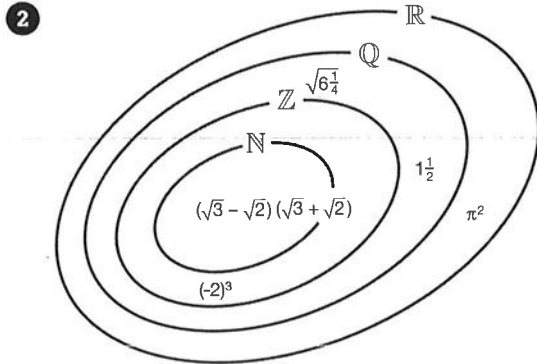
$$\tan(\alpha) = \frac{-1}{3} = -\frac{1}{3} \text{ geeft } \alpha \approx -0,322$$

10.1 Rekenen met complexe getallen

Bladzijde 42

- 1 a I $2x + 5 = 13$
 $2x = 8$
 $x = 4$
 II $2x + 5 = 3$
 $2x = -2$
 $x = -1$
 b Bij III en IV.
- III $2x + 5 = 8$
 $2x = 3$
 $x = 1\frac{1}{2}$
 IV $x^2 + 5 = 8$
 $x^2 = 3$
 $x = \sqrt{3} \vee x = -\sqrt{3}$

Bladzijde 43



- 3 a $x^2 + x = 6$
 $x^2 + x - 6 = 0$
 $(x - 2)(x + 3) = 0$
 $x = 2 \vee x = -3$
 b $x^2 + x = 4$
 $x^2 + x - 4 = 0$
 $D = 1^2 - 4 \cdot 1 \cdot -4 = 17$
 geen oplossing in $\mathbb{Q}$
 c $x^3 - x = 0$
 $x(x^2 - 1) = 0$
 $x = 0 \vee x^2 = 1$
 $x = 0 \vee x = 1 \vee x = -1$
 d $x^3 - 2x = 0$
 $x(x^2 - 2) = 0$
 $x = 0 \vee x^2 = 2$
 $x = 0$
 e $x^4 - 9x = 0$
 $x(x^3 - 9) = 0$
 $x = 0 \vee x^3 = 9$
 $x = 0$
 f $x^4 - 9x^2 = 0$
 $x^2(x^2 - 9) = 0$
 $x^2 = 0 \vee x^2 = 9$
 $x = 0 \vee x = 3 \vee x = -3$
- 4 a $(x + 3)^2 = 7$
 $x + 3 = \sqrt{7} \vee x + 3 = -\sqrt{7}$
 $x = -3 + \sqrt{7} \vee x = -3 - \sqrt{7}$
 b $(x + 3)^2 = -7$ heeft geen oplossing in $\mathbb{R}$ omdat $(x + 3)^2$ niet negatief kan zijn.

Bladzijde 45

- 5 a $3x + 5i + 3 = 2i - x$
 $4x = -3 - 3i$
 $x = -\frac{3}{4} - \frac{3}{4}i$
 b $2x^2 + 10 = 0$
 $2x^2 = -10$
 $x^2 = -5$
 $x^2 = 5i^2$
 $x = i\sqrt{5} \vee x = -i\sqrt{5}$
 c $(x + 2)^2 + 10 = 0$
 $(x + 2)^2 = -10$
 $(x + 2)^2 = 10i^2$
 $x + 2 = i\sqrt{10} \vee x + 2 = -i\sqrt{10}$
 $x = -2 + i\sqrt{10} \vee x = -2 - i\sqrt{10}$
 d $x^2 - 10x + 40 = 0$
 $(x - 5)^2 - 25 + 40 = 0$
 $(x - 5)^2 = -15$
 $(x - 5)^2 = 15i^2$
 $x - 5 = i\sqrt{15} \vee x - 5 = -i\sqrt{15}$
 $x = 5 + i\sqrt{15} \vee x = 5 - i\sqrt{15}$
 e $x^2 + 8x + 14 = 0$
 $(x + 4)^2 - 16 + 14 = 0$
 $(x + 4)^2 = 2$
 $x + 4 = \sqrt{2} \vee x + 4 = -\sqrt{2}$
 $x = -4 + \sqrt{2} \vee x = -4 - \sqrt{2}$
 f $(x + 3)^2 = -16$
 $(x + 3)^2 = 16i^2$
 $x + 3 = 4i \vee x + 3 = -4i$
 $x = -3 + 4i \vee x = -3 - 4i$

6 a $\frac{1}{3}x + 10 + 2i = \frac{1}{4}x + 12 - 5i$

$$\frac{1}{12}x = 2 - 7i$$

$$x = 24 - 84i$$

b $(2x + 3)^2 + 10 = 0$

$$(2x + 3)^2 = -10$$

$$(2x + 3)^2 = 10i^2$$

$$2x + 3 = i\sqrt{10} \vee 2x + 3 = -i\sqrt{10}$$

$$2x = -3 + i\sqrt{10} \vee 2x = -3 - i\sqrt{10}$$

$$x = -1\frac{1}{2} + \frac{1}{2}i\sqrt{10} \vee x = -1\frac{1}{2} - \frac{1}{2}i\sqrt{10}$$

c $(x - 3)^2 + 2x = 0$

$$x^2 - 6x + 9 + 2x = 0$$

$$x^2 - 4x + 9 = 0$$

$$(x - 2)^2 - 4 + 9 = 0$$

$$(x - 2)^2 = -5$$

$$(x - 2)^2 = 5i^2$$

$$x - 2 = i\sqrt{5} \vee x - 2 = -i\sqrt{5}$$

$$x = 2 + i\sqrt{5} \vee x = 2 - i\sqrt{5}$$

d $4x^2 + 4x + 7 = 0$

$$(2x + 1)^2 - 1 + 7 = 0$$

$$(2x + 1)^2 = -6$$

$$(2x + 1)^2 = 6i^2$$

$$2x + 1 = i\sqrt{6} \vee 2x + 1 = -i\sqrt{6}$$

$$2x = -1 + i\sqrt{6} \vee 2x = -1 - i\sqrt{6}$$

$$x = -\frac{1}{2} + \frac{1}{2}i\sqrt{6} \vee x = -\frac{1}{2} - \frac{1}{2}i\sqrt{6}$$

7 a $(2 + i) \cdot (10 - 5i) = 20 - 10i + 10i - 5i^2 = 20 + 5 = 25$

b $(2 + i) \cdot (2 - i) = 4 - i^2 = 4 + 1 = 5$

c $(3 + 2i) \cdot (3 - 2i) = 9 - 4i^2 = 9 + 4 = 13$

d $(a + bi) \cdot (c + di) = ac + adi + bci + bdi^2 = ac + adi + bci - bd = (ac - bd) + (ad + bc)i$

Panelside 46

8 a $(2 + 5i) + (4 - 6i) = 6 - i$

b $(5 - i)(5 + i) = 25 - i^2 = 25 + 1 = 26$

c $(2 + i)^2 = 4 + 4i + i^2 = 4 + 4i - 1 = 3 + 4i$

d $i(6 + 7i) = 6i + 7i^2 = 6i - 7 = -7 + 6i$

e $i^5 = i^2 \cdot i^2 \cdot i = -1 \cdot -1 \cdot i = i$

f $(1 + i)(6 - i) + (3 - i)(3 + 2i) = 6 - i + 6i - i^2 + 9 + 6i - 3i - 2i^2 = 6 - i + 6i + 1 + 9 + 6i - 3i + 2 = 18 + 8i$

9 a $\frac{2}{1+i} = \frac{2}{1+i} \cdot \frac{1-i}{1-i} = \frac{2-2i}{1-i^2} = \frac{2-2i}{1+1} = \frac{2-2i}{2} = 1-i$

b $\frac{2+3i}{i} = \frac{2+3i}{i} \cdot \frac{i}{i} = \frac{2i+3i^2}{i^2} = \frac{2i-3}{-1} = -2i+3 = 3-2i$

c $\frac{2+i}{2-i} = \frac{2+i}{2-i} \cdot \frac{2+i}{2+i} = \frac{4+4i+i^2}{4-i^2} = \frac{4+4i-1}{4+1} = \frac{3+4i}{5} = \frac{3}{5} + \frac{4}{5}i$

d $\frac{3+5i}{12+5i} = \frac{3+5i}{12+5i} \cdot \frac{12-5i}{12-5i} = \frac{36-15i+60i-25i^2}{144-25i^2} = \frac{36+45i+25}{144+25} = \frac{61+45i}{169} = \frac{61}{169} + \frac{45}{169}i$

e $\frac{2i}{1+3i} = \frac{2i}{1+3i} \cdot \frac{1-3i}{1-3i} = \frac{2i-6i^2}{1-9i^2} = \frac{2i+6}{1+9} = \frac{2i+6}{10} = \frac{1}{5}i + \frac{3}{5} = \frac{3}{5} + \frac{1}{5}i$

f $(2+3i) \cdot \frac{3}{2+i} = \frac{6+9i}{2+i} \cdot \frac{2-i}{2-i} = \frac{12-6i+18i-9i^2}{4-i^2} = \frac{12+12i+9}{4+1} = \frac{21+12i}{5} = 4\frac{1}{5} + 2\frac{2}{5}i$

10 a $(3+4i)^2 = 9 + 24i + 16i^2 = 9 + 24i - 16 = -7 + 24i$

b $\frac{3+i}{3+4i} = \frac{3+i}{3+4i} \cdot \frac{3-4i}{3-4i} = \frac{9-12i+3i-4i^2}{9-16i^2} = \frac{9-9i+4}{9+16} = \frac{13-9i}{25} = \frac{13}{25} - \frac{9}{25}i$

c $(2+i)^2 - (2-i)^2 = 4 + 4i + i^2 - (4 - 4i + i^2) = 4 + 4i + i^2 - 4 + 4i - i^2 = 8i$

d $i^2(i^3 - i^4 - i^5) = -1 \cdot (i^2 \cdot i - i^2 \cdot i^2 - i^2 \cdot i^2 \cdot i) = -1 \cdot (-i - 1 - i) = -1 \cdot (-1 - 2i) = 1 + 2i$

e $\frac{5}{2+3i} = \frac{5}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{10-15i}{4-9i^2} = \frac{10-15i}{4+9} = \frac{10-15i}{13} = \frac{10}{13} - 1\frac{2}{13}i$

f $i + i^2 + i^3 + \dots + i^{10} = i - 1 - i + 1 + i - 1 - i + 1 + i - 1 = -1 + i$

Bladzijde 47

$$11 \quad \frac{a+bi}{c+d\bar{i}} = \frac{a+bi}{c+d\bar{i}} \cdot \frac{c-d\bar{i}}{c-d\bar{i}} = \frac{ac - a\bar{d}i + bci - b\bar{d}i^2}{c^2 - d^2i^2} = \frac{ac - a\bar{d}i + bci + bd}{c^2 + d^2} = \frac{ac + bd - (ad - bc)i}{c^2 + d^2}$$

$$12 \quad a \quad (a+bi)(a-bi) = a^2 - b^2i^2 = a^2 + b^2 \text{ is een reëel getal.}$$

$$b \quad (3+4i)(3-4i) = 3^2 + 4^2 = 25$$

$$(100-200i)(100+200i) = 100^2 + 200^2 = 50\,000$$

$$13 \quad a \quad \frac{z+\bar{z}}{2} = \frac{a+bi+a-bi}{2} = \frac{2a}{2} = a = \operatorname{Re}(z)$$

$$b \quad \frac{z-\bar{z}}{2i} = \frac{a+bi-(a-bi)}{2i} = \frac{a+bi-a+bi}{2i} = \frac{2bi}{2i} = b = \operatorname{Im}(z)$$

$$c \quad \bar{\bar{z}} = \overline{a+bi} = a-bi = z$$

$$14 \quad a \quad \overline{z_1 + z_2} = \overline{a+bi+c+d\bar{i}} = \overline{a+c+(b+d\bar{i})i} = a+c-(b+d\bar{i})i = a-bi+c-d\bar{i} = \bar{z}_1 + \bar{z}_2$$

$$b \quad \overline{z_1 \cdot z_2} = \overline{(a+bi) \cdot (c+d\bar{i})} = \overline{ac + a\bar{d}i + bci + b\bar{d}i^2} = \overline{ac - bd + (ad + bc\bar{i})i} = \overline{ac - bd - (ad + bc)i} \dots (1)$$

$$\overline{z_1 \cdot z_2} = \overline{a+bi} \cdot \overline{c+d\bar{i}} = (a-bi) \cdot (c-d\bar{i}) = ac - a\bar{d}i - bci + b\bar{d}i^2 = ac - bd - (ad + bc)i \dots (2)$$

Uit (1) en (2) volgt dat $\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$.

$$c \quad \overline{\left(\frac{z_1}{z_2}\right)} = \overline{\left(\frac{a+bi}{c+d\bar{i}}\right)} = \overline{\left(\frac{ac + bd - (ad - bc)i}{c^2 + d^2}\right)} = \overline{\left(\frac{ac + bd}{c^2 + d^2} - \frac{ad - bc}{c^2 + d^2}i\right)} = \frac{ac + bd}{c^2 + d^2} + \frac{ad - bc}{c^2 + d^2}i \dots (1)$$

$$\frac{\bar{z}_1}{\bar{z}_2} = \frac{\overline{a+bi}}{\overline{c+d\bar{i}}} = \frac{a-bi}{c-d\bar{i}} \cdot \frac{c+d\bar{i}}{c+d\bar{i}} = \frac{ac + a\bar{d}i - bci - b\bar{d}i^2}{c^2 + d^2} = \frac{ac + bd + (ad - bc)i}{c^2 + d^2} = \frac{ac + bd}{c^2 + d^2} + \frac{ad - bc}{c^2 + d^2}i \dots (2)$$

Uit (1) en (2) volgt dat $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$.

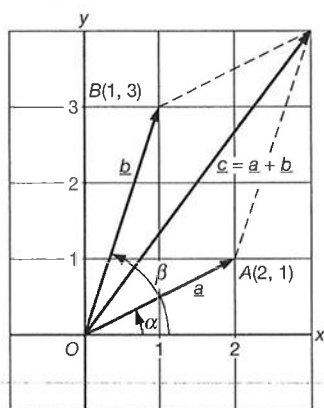
$$15 \quad a \quad \overline{(4+3i)^2} = \overline{16+24i+9i^2} = \overline{16+24i-9} = \overline{7+24i} = 7-24i$$

$$b \quad \overline{\left(\frac{4+3i}{2+i}\right)} = \frac{\overline{4+3i}}{\overline{2+i}} = \frac{4-3i}{2-i} \cdot \frac{2+i}{2+i} = \frac{8+4i-6i-3i^2}{4-i^2} = \frac{8-2i+3}{4+1} = \frac{11-2i}{5} = 2\frac{1}{5} - \frac{2}{5}i$$

$$c \quad \frac{\overline{3+4i}}{\overline{3-4i}} = \frac{3-4i}{3+4i} = 1$$

10.2 Het complexe vlak**Bladzijde 49**

16 a



$$\vec{c} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$b \quad |\vec{a}| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$|\vec{b}| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$|\vec{c}| = \sqrt{3^2 + 4^2} = 5$$

$$|\vec{a}| + |\vec{b}| = \sqrt{5} + \sqrt{10} \text{ en } |\vec{c}| = |\vec{a} + \vec{b}| = 5, \text{ dus er geldt niet } |\vec{a}| + |\vec{b}| = |\vec{a} + \vec{b}|.$$

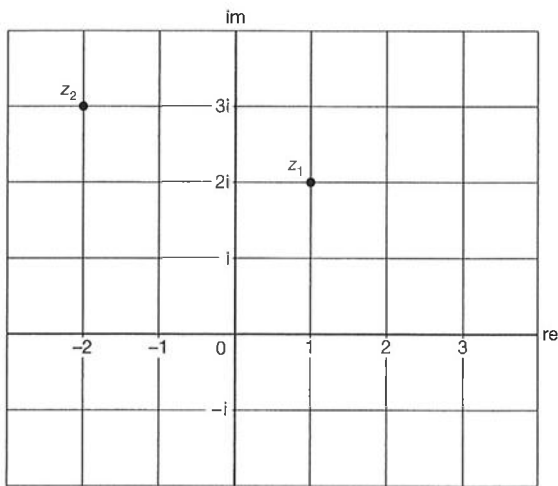
c $\tan(\alpha) = \frac{1}{2}$ geeft $\alpha \approx 0,46$

$\tan(\beta) = \frac{3}{1} = 3$ geeft $\beta \approx 1,11$

$\tan(\angle(\vec{c}, \text{positieve } x\text{-as})) = \frac{4}{3}$ geeft $\angle(\vec{c}, \text{positieve } x\text{-as}) \approx 0,93$

Bladzijde 51

17 a

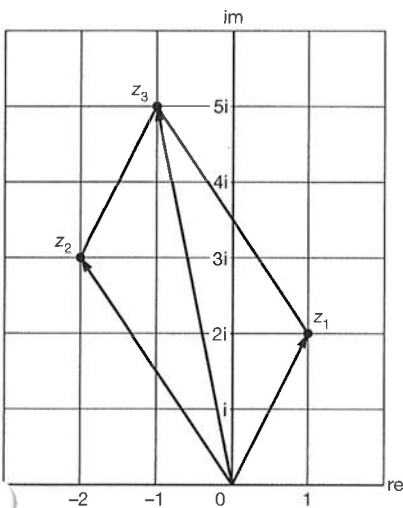


$|z_1| = \sqrt{1^2 + 2^2} = \sqrt{5}$

$|z_2| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$

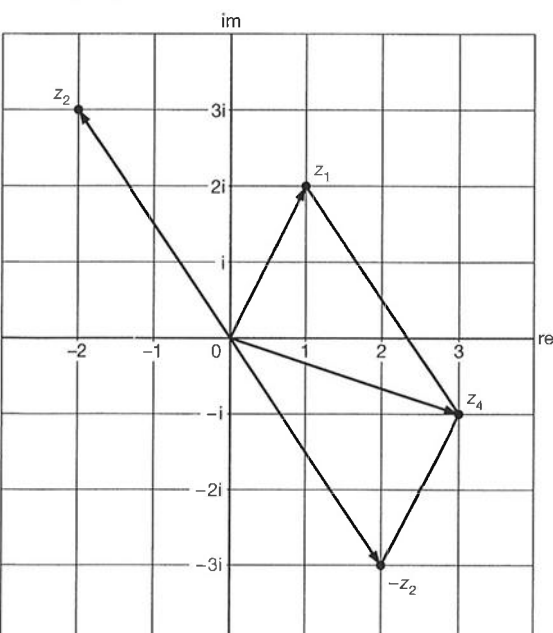
b $z_3 = z_1 + z_2 = 1 + 2i - 2 + 3i = -1 + 5i$

c



d $z_4 = z_1 - z_2 = 1 + 2i - (-2 + 3i) = 1 + 2i + 2 - 3i = 3 - i$

e



f $z_5 = z_1 \cdot z_2 = (1 + 2i) \cdot (-2 + 3i) = -2 + 3i - 4i + 6i^2 = -2 + 3i - 4i - 6 = -8 - i$

g $|z_1 \cdot z_2| = |-8 - i| = \sqrt{(-8)^2 + (-1)^2} = \sqrt{65}$

$|z_1| = \sqrt{5}$ en $|z_2| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$ geeft $|z_1| \cdot |z_2| = \sqrt{5} \cdot \sqrt{13} = \sqrt{65}$.

Dus $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$.

18 a,b,c,d

$\text{Re}(z) = \text{Im}(z)$

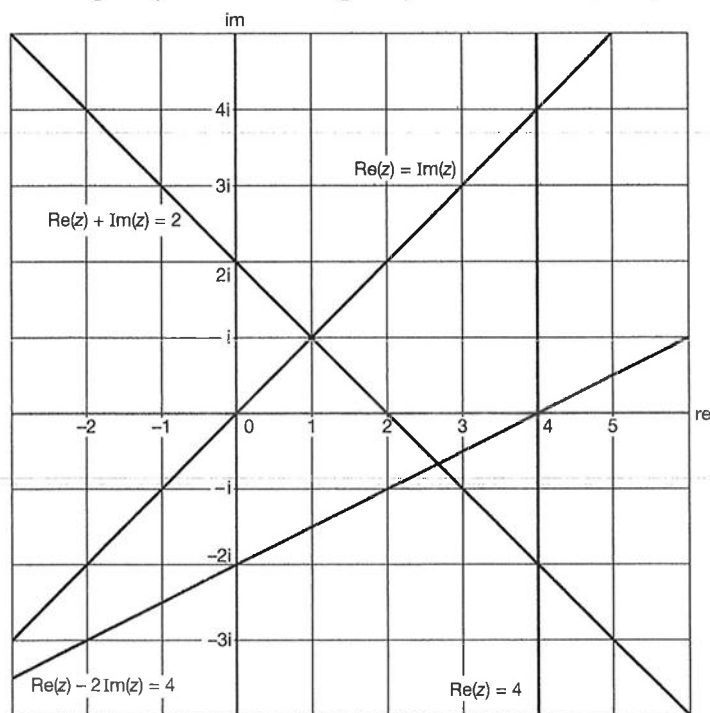
$\text{Re}(z) + \text{Im}(z) = 2$

$\text{Re}(z) - 2\text{Im}(z) = 4$

$\text{Re}(z)$	0	1
$\text{Im}(z)$	0	1

$\text{Re}(z)$	0	2
$\text{Im}(z)$	2	0

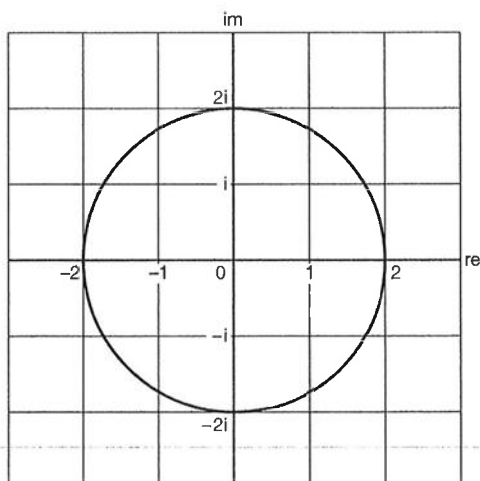
$\text{Re}(z)$	0	4
$\text{Im}(z)$	-2	0



19 a $(\text{Re}(z))^2 + (\text{Im}(z))^2 = 4$ dus $|z| = 2$.

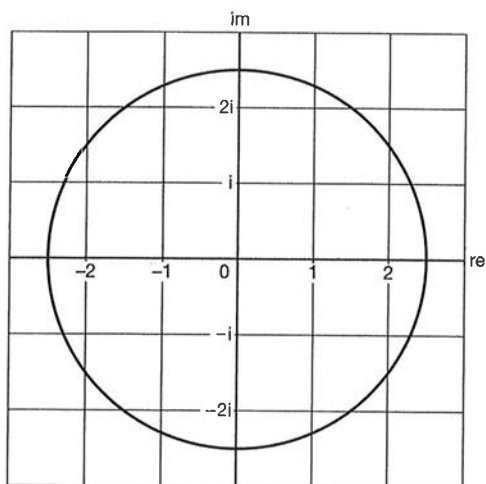
Dit is de cirkel met middelpunt 0 en straal 2.

10



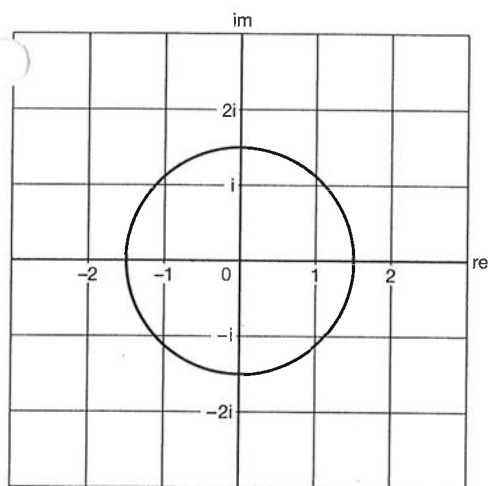
b $|z| = 2,5$

Dit is de cirkel met middelpunt 0 en straal 2,5.



c $z \cdot \bar{z} = 2\frac{1}{4}$ geeft $|z| = \sqrt{2\frac{1}{4}} = 1\frac{1}{2}$

Dit is de cirkel met middelpunt 0 en straal $1\frac{1}{2}$.



d $z = \bar{z}$

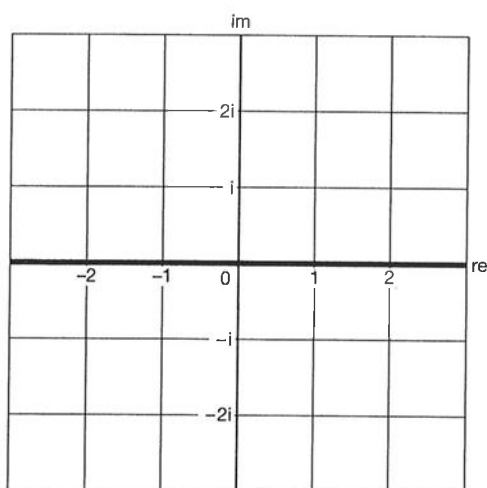
$$a + bi = a - bi$$

$$2bi = 0$$

$$b = 0$$

$$\text{Im}(z) = 0$$

It is de reële as.



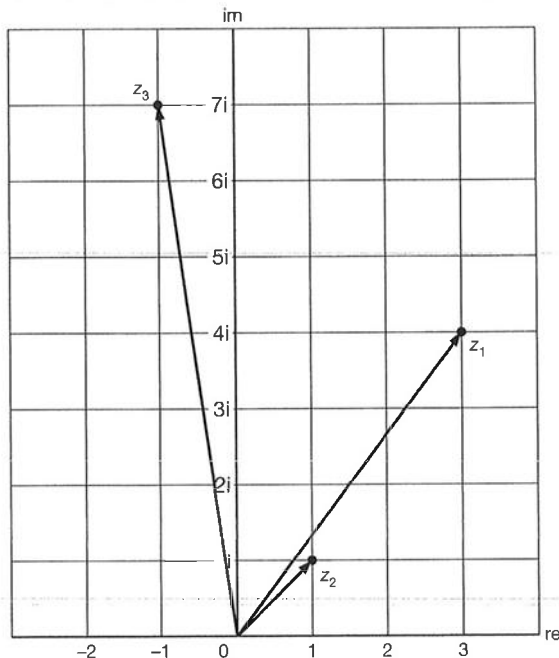
20 $z \cdot \bar{z} = (a + bi) \cdot (a - bi) = a^2 - b^2 i^2 = a^2 + b^2$
 $|z| = \sqrt{a^2 + b^2}$, dus $|z|^2 = a^2 + b^2$.
 Dus $z \cdot \bar{z} = |z|^2$.

- 21 $|z_1 \cdot z_2| = |(a + bi) \cdot (c + di)| = |ac + adi + bci + bdi^2| = |ac - bd + (bc + ad)i| = \sqrt{(ac - bd)^2 + (bc + ad)^2} = \sqrt{a^2c^2 - 2abcd + b^2d^2 + b^2c^2 + 2abcd + a^2d^2} = \sqrt{a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2} \dots (1)$
- $|z_1| \cdot |z_2| = |a + bi| \cdot |c + di| = \sqrt{a^2 + b^2} \cdot \sqrt{c^2 + d^2} = \sqrt{(a^2 + b^2)(c^2 + d^2)} = \sqrt{a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2} \dots (2)$
- Uit (1) en (2) volgt $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$.

Bladzijde 52

- 22 a $z_3 = z_1 \cdot z_2 = (3 + 4i)(1 + i) = 3 + 3i + 4i + 4i^2 = 3 + 3i + 4i - 4 = -1 + 7i$

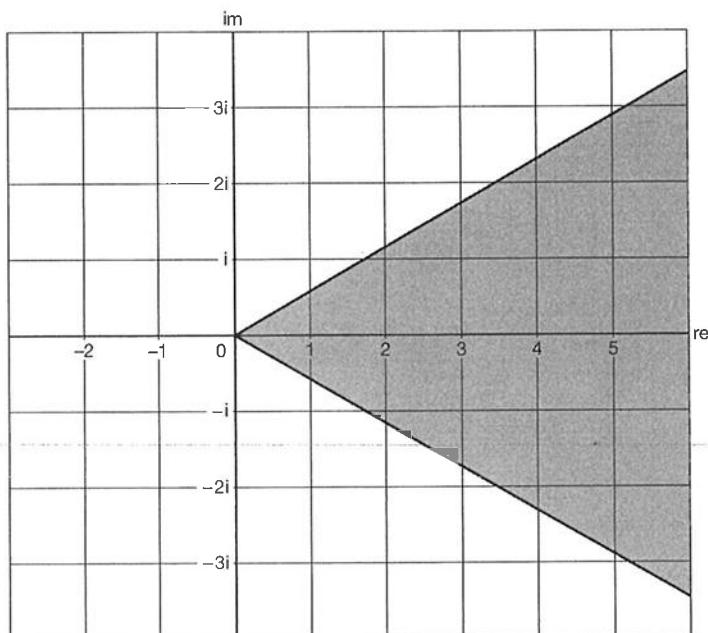
b

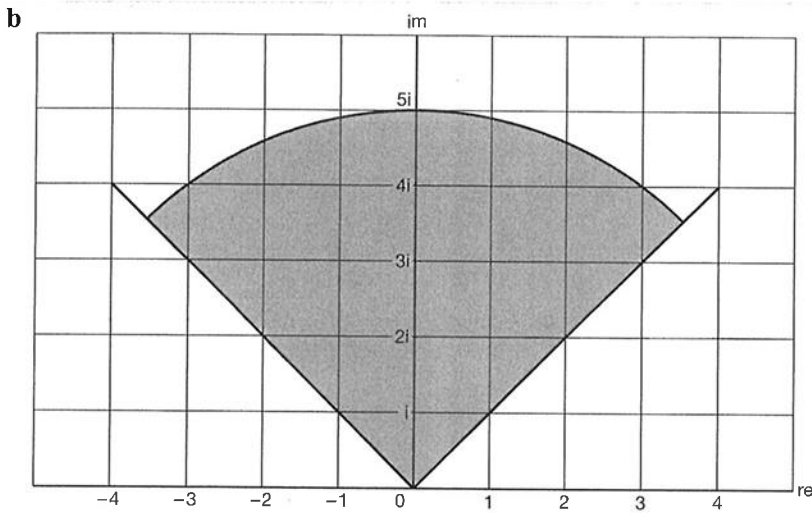


- c Voor z_1 geldt: $\tan(\varphi_1) = \frac{4}{3}$ geeft $\varphi_1 \approx 0,927$.
 Voor z_2 geldt: $\tan(\varphi_2) = \frac{1}{1} = 1$ geeft $\varphi_2 \approx 0,785$.
 Voor z_3 geldt: $\tan(\varphi_3) = \frac{7}{-1} = -7$ geeft $\varphi_3 \approx 1,713$.
 Voor de draaiingshoeken geldt $\varphi_1 + \varphi_2 = \varphi_3$.

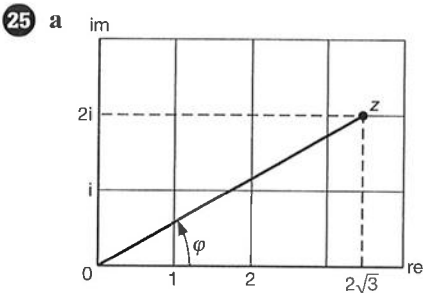
Bladzijde 54

23 a





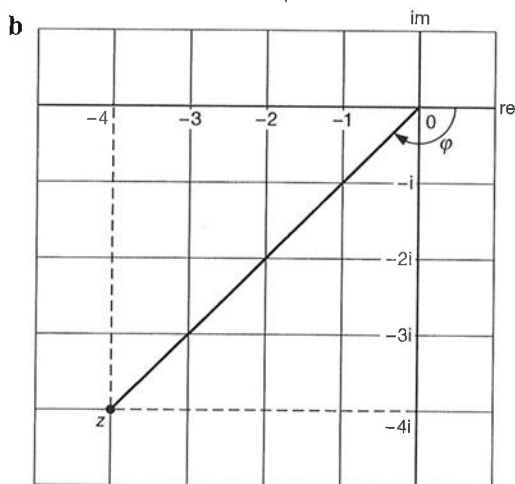
- 24 a** $|z| = \sqrt{3^2 + 2^2} = \sqrt{13}$
 $\text{angle}(3 + 2i)$ (TI) of $\text{Arg}(3 + 2i)$ (Casio) geeft $\varphi \approx 0,59$
- b** $|z| = \sqrt{(-2)^2 + 5^2} = \sqrt{29}$
 $\text{angle}(-2 + 5i)$ (TI) of $\text{Arg}(-2 + 5i)$ (Casio) geeft $\varphi \approx 1,95$
- c** $|z| = \sqrt{(-3)^2 + (-4)^2} = \sqrt{25} = 5$
 $\text{angle}(-3 - 4i)$ (TI) of $\text{Arg}(-3 - 4i)$ (Casio) geeft $\varphi \approx -2,21$
- d** $|z| = \sqrt{4^2 + (-1)^2} = \sqrt{17}$
 $\text{angle}(4 - i)$ (TI) of $\text{Arg}(4 - i)$ (Casio) geeft $\varphi \approx -0,24$



$$\tan(\varphi) = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ geeft } \varphi = \frac{1}{6}\pi$$

Dus $\text{Arg}(2\sqrt{3} + 2i) = \frac{1}{6}\pi$.

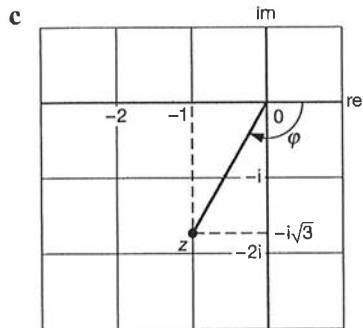
De modulus is $|z| = \sqrt{(2\sqrt{3})^2 + 2^2} = \sqrt{16} = 4$.



$$\tan(\varphi) = \frac{-4}{-4} = 1 \text{ geeft } \varphi = -\frac{3}{4}\pi$$

Dus $\text{Arg}(-4 - 4i) = -\frac{3}{4}\pi$.

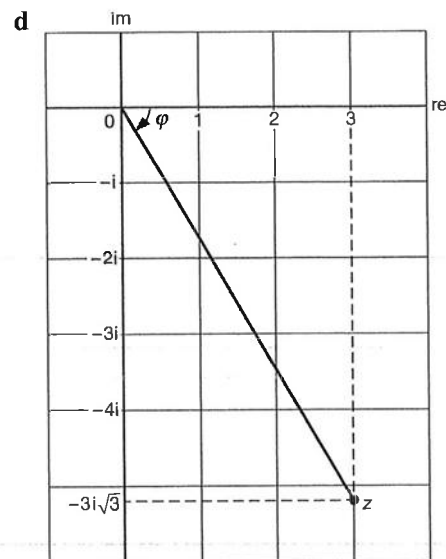
De modulus is $|z| = \sqrt{(-4)^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$.



$$\tan(\varphi) = \frac{-\sqrt{3}}{-1} = \sqrt{3} \text{ geeft } \varphi = -\frac{2}{3}\pi$$

$$\text{Dus } \text{Arg}(-1 - i\sqrt{3}) = -\frac{2}{3}\pi.$$

$$\text{De modulus is } |z| = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{4} = 2.$$

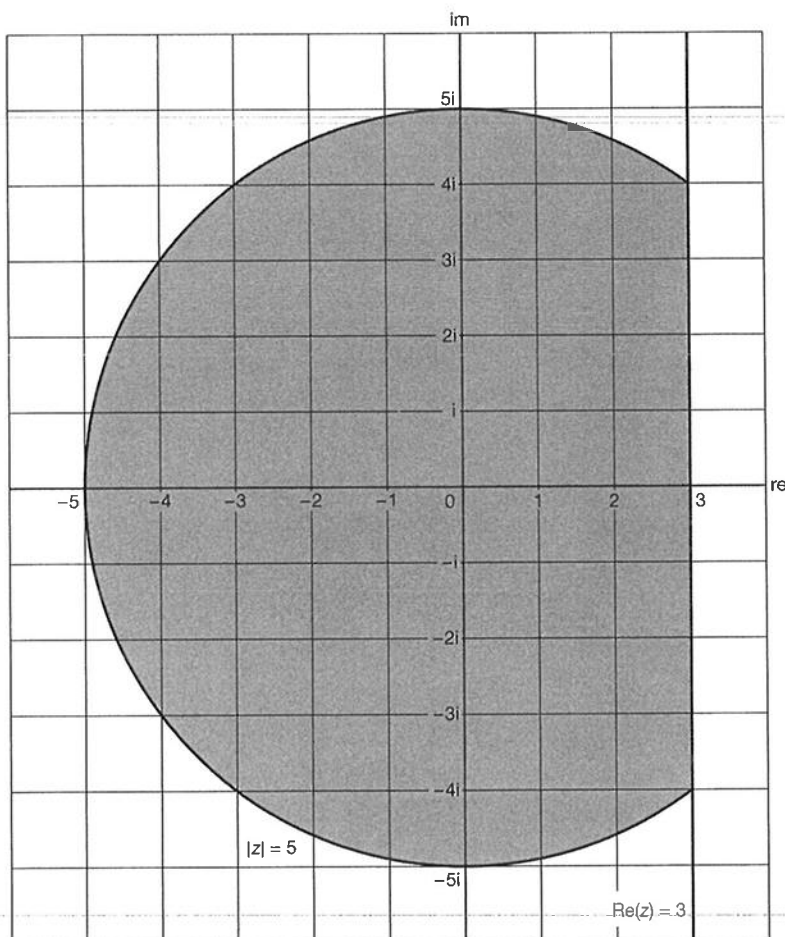


$$\tan(\varphi) = \frac{-3\sqrt{3}}{3} = -\sqrt{3} \text{ geeft } \varphi = -\frac{1}{3}\pi$$

$$\text{Dus } \text{Arg}(3 - 3i\sqrt{3}) = -\frac{1}{3}\pi.$$

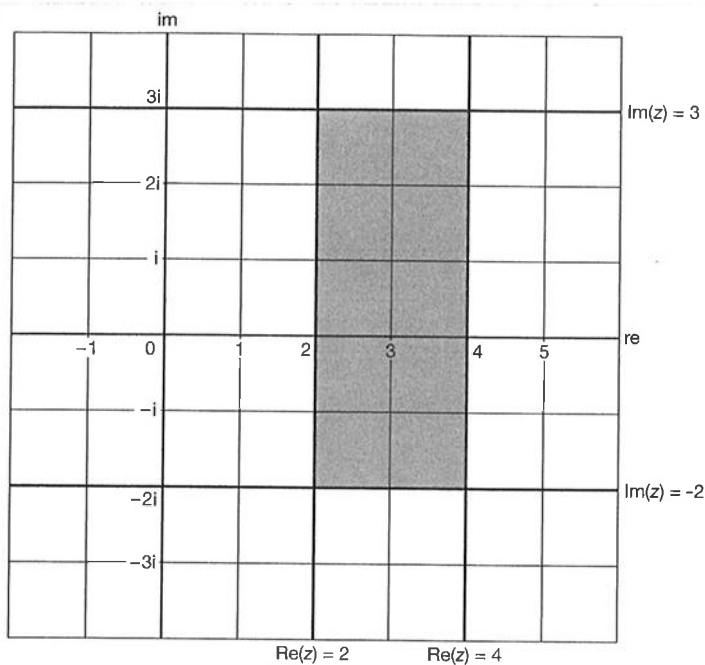
$$\text{De modulus is } |z| = \sqrt{3^2 + (-3\sqrt{3})^2} = \sqrt{36} = 6.$$

26 a



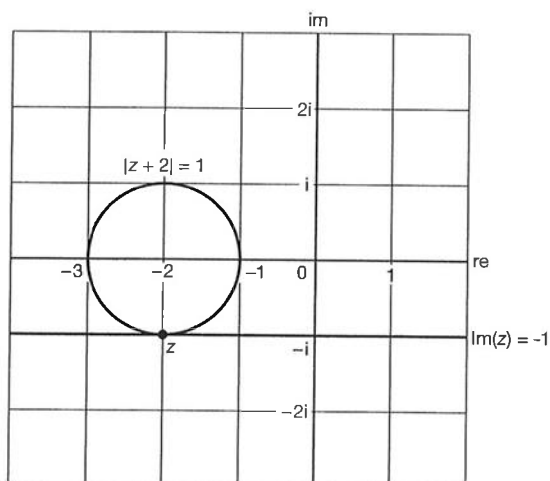
10

b

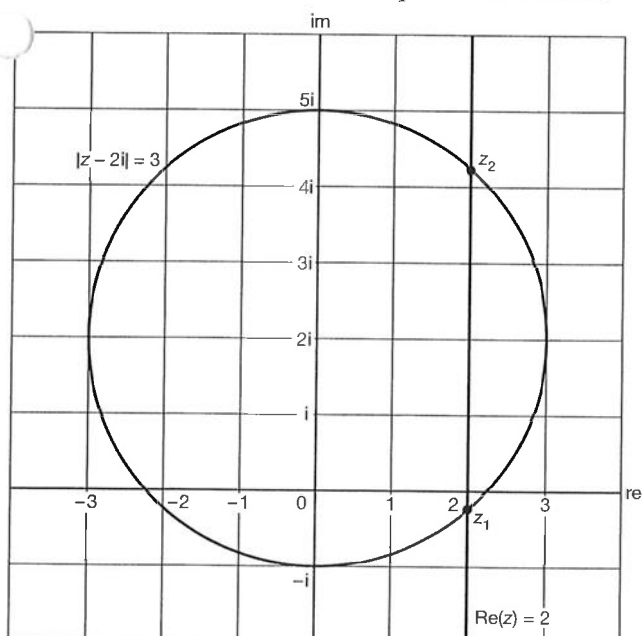


27

a $|z + 2| = 1$ is de cirkel met middelpunt -2 en straal 1.

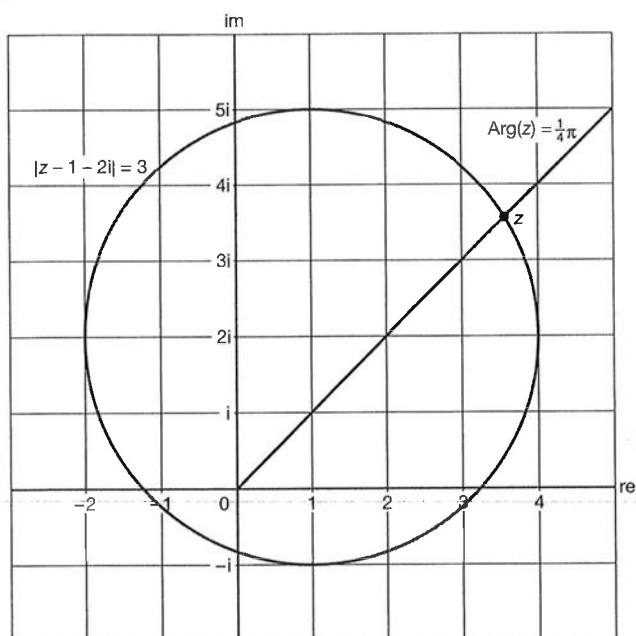


b $|z - 2i| = 3$ is de cirkel met middelpunt $2i$ and straal 3.

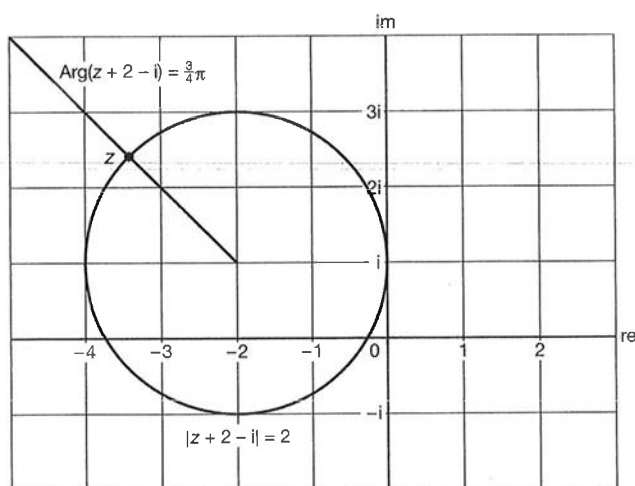


10

c $|z - 1 - 2i| = 3$ is de cirkel met middelpunt $1 + 2i$ en straal 3.

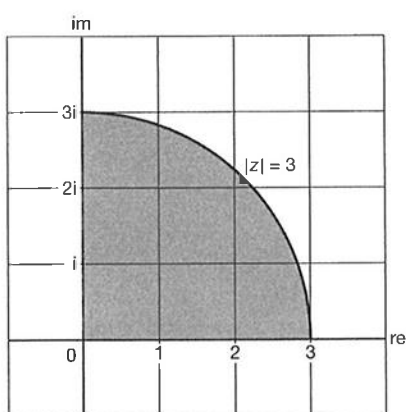


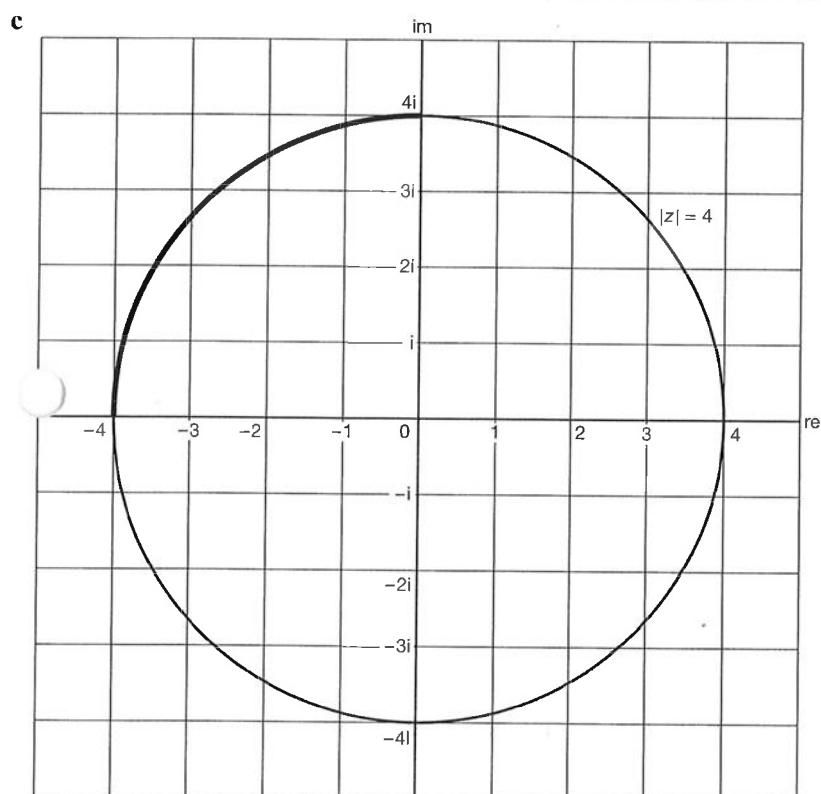
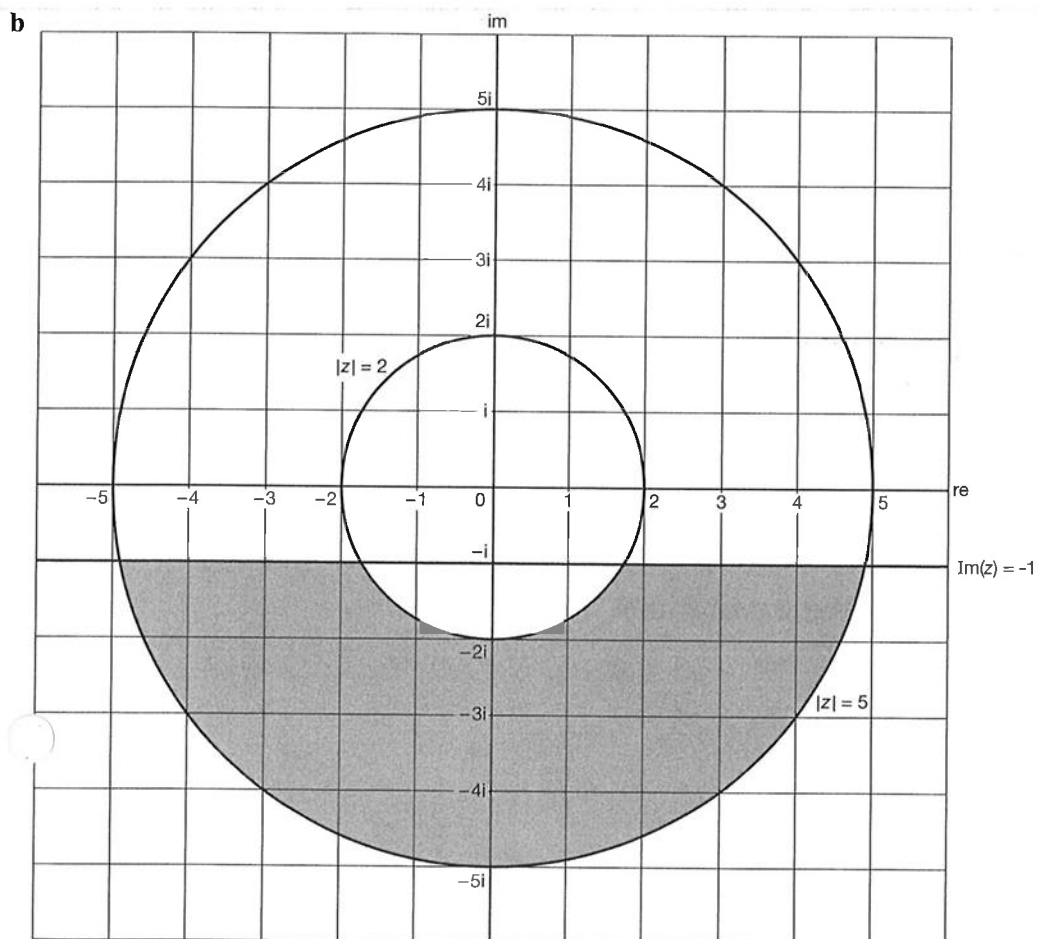
d $|z + 2 - i| = 2$ is de cirkel met middelpunt $-2 + i$ en straal 2.

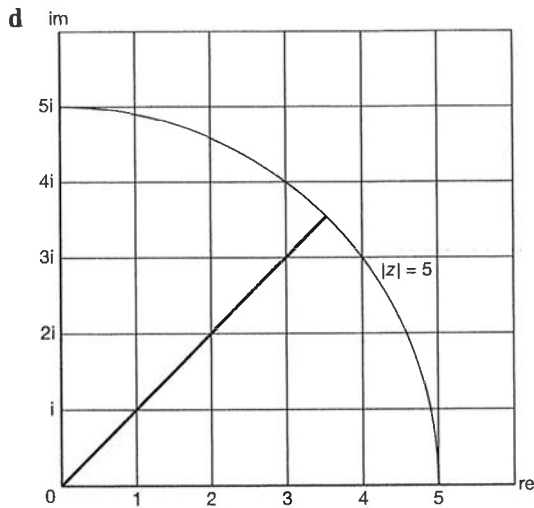


10

29 a







10.3 Poolcoördinaten en de formule van Euler

Bladzijde 56

30 $OP = \sqrt{2^2 + 3^2} = \sqrt{13}$

$\tan(\alpha) = \frac{3}{2}$ geeft $\alpha \approx 0,983$

Bladzijde 57

31 $x_A = r \cos(\theta) = 2 \cos\left(\frac{1}{6}\pi\right) = 2 \cdot \frac{1}{2} \sqrt{3} = \sqrt{3}$

$y_A = r \sin(\theta) = 2 \sin\left(\frac{1}{6}\pi\right) = 2 \cdot \frac{1}{2} = 1$

Dus $A(\sqrt{3}, 1)$.

$x_B = r \cos(\theta) = 2 \cos\left(\frac{2}{3}\pi\right) = 2 \cdot -\frac{1}{2} = -1$

$y_B = r \sin(\theta) = 2 \sin\left(\frac{2}{3}\pi\right) = 2 \cdot \frac{1}{2} \sqrt{3} = \sqrt{3}$

Dus $B(-1, \sqrt{3})$.

$x_C = r \cos(\theta) = 4 \cos\left(-\frac{3}{4}\pi\right) = 4 \cdot -\frac{1}{2} \sqrt{2} = -2\sqrt{2}$

$y_C = r \sin(\theta) = 4 \sin\left(-\frac{3}{4}\pi\right) = 4 \cdot -\frac{1}{2} \sqrt{2} = -2\sqrt{2}$

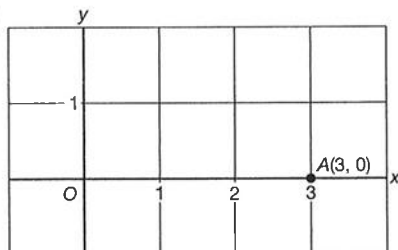
Dus $C(-2\sqrt{2}, -2\sqrt{2})$.

$x_D = r \cos(\theta) = 4 \cos\left(-\frac{5}{6}\pi\right) = 4 \cdot -\frac{1}{2} \sqrt{3} = -2\sqrt{3}$

$y_D = r \sin(\theta) = 4 \sin\left(-\frac{5}{6}\pi\right) = 4 \cdot -\frac{1}{2} = -2$

Dus $D(-2\sqrt{3}, -2)$.

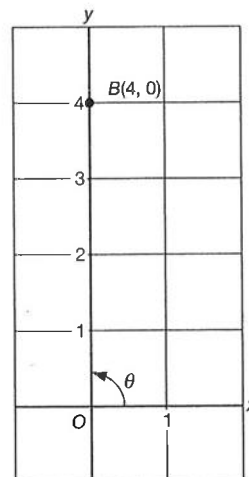
32



$r = 3$

$\tan(\theta) = \frac{0}{3} = 0$ geeft $\theta = 0$

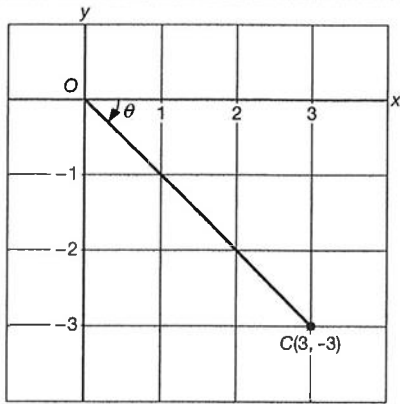
Dus $A(3, 0)$.



$r = 4$

$\theta = \frac{1}{2}\pi$

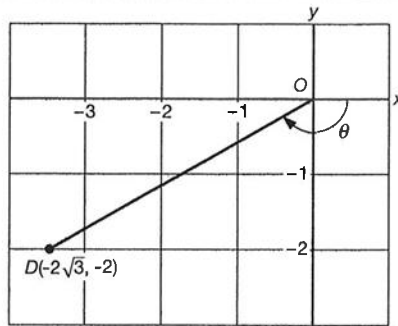
Dus $B(4, \frac{1}{2}\pi)$.



$$r = \sqrt{3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$$

$$\tan(\theta) = \frac{-3}{3} = -1 \text{ geeft } \theta = -\frac{1}{4}\pi$$

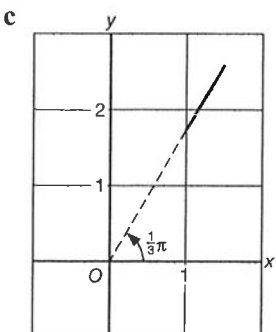
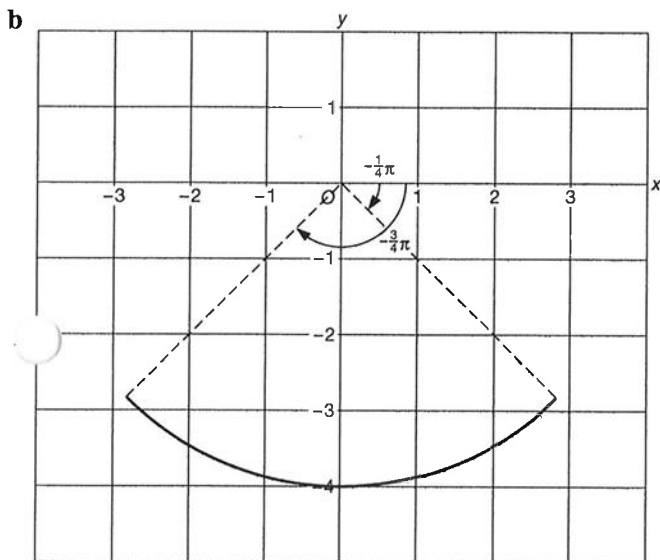
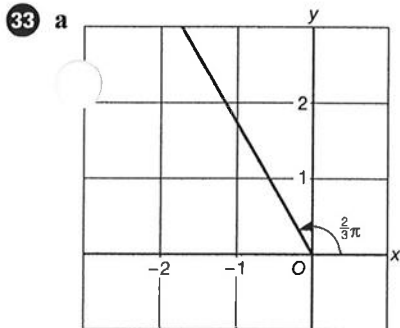
Dus $C(3\sqrt{2}, -\frac{1}{4}\pi)$.

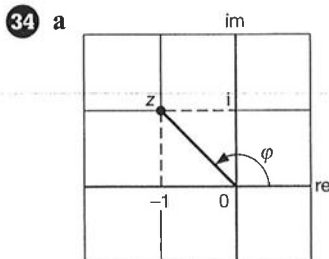
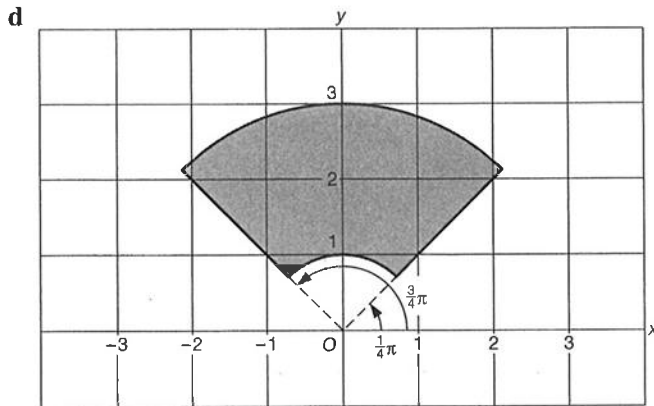


$$r = \sqrt{(-2\sqrt{3})^2 + (-2)^2} = \sqrt{16} = 4$$

$$\tan(\theta) = \frac{-2}{-2\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ geeft } \theta = -\frac{5}{6}\pi$$

Dus $D(4, -\frac{5}{6}\pi)$.





$$\tan(\varphi) = \frac{1}{-1} = -1 \text{ geeft } \varphi = \frac{3}{4}\pi$$

$$\text{Dus } \text{Arg}(z) = \text{Arg}(-1 + i) = \frac{3}{4}\pi.$$

$$\text{De modulus is } |z| = |-1 + i| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}.$$

b De poolcoördinaten (r, θ) van het punt P zijn

- de afstand r van P tot O
- de draaiingshoek θ van het lijnstuk OP met $-\pi < \theta \leq \pi$.

De afstand van z tot O is $\sqrt{2}$ en de draaiingshoek van z is $\frac{3}{4}\pi$.

Dus je hebt in vraag a de poolcoördinaten van z berekend.

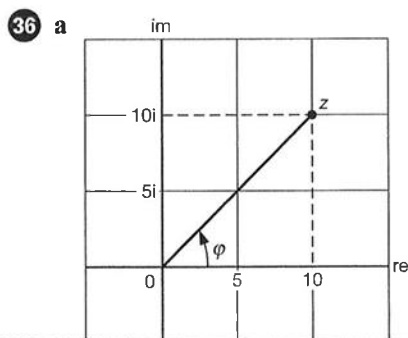
Bladzijde 58

35 a $|z| = \sqrt{4^2 + (-3)^2} = 5$ en $\arg(z) \approx -0,64$
Dus $z = 5(\cos(-0,64) + i \sin(-0,64))$.

b $|z| = \sqrt{12^2 + 5^2} = 13$ en $\arg(z) \approx 0,39$
Dus $z = 13(\cos(0,39) + i \sin(0,39))$.

c $|z| = \sqrt{(-\sqrt{3})^2 + (-2)^2} = \sqrt{7}$ en $\arg(z) \approx -2,28$
Dus $z = \sqrt{7}(\cos(-2,28) + i \sin(-2,28))$.

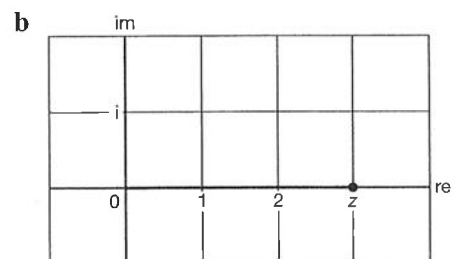
d $|z| = \sqrt{2^2 + (-\sqrt{5})^2} = 3$ en $\arg(z) \approx -0,84$
Dus $z = 3(\cos(-0,84) + i \sin(-0,84))$.



$$\tan(\varphi) = \frac{10}{10} = 1 \text{ geeft } \varphi = \frac{1}{4}\pi$$

$$|z| = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$$

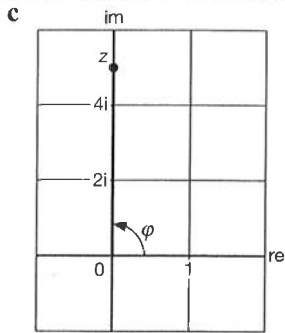
$$\text{Dus } z = 10\sqrt{2}(\cos(\frac{1}{4}\pi) + i \sin(\frac{1}{4}\pi)).$$



$$\varphi = 0$$

$$|z| = 3$$

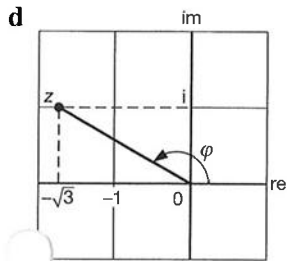
$$\text{Dus } z = 3(\cos(0) + i \sin(0)).$$



$$\varphi = \frac{1}{2}\pi$$

$$|z| = 5$$

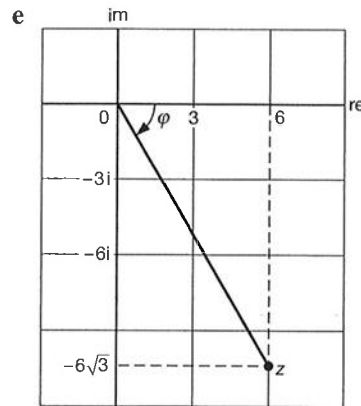
$$\text{Dus } z = 5(\cos(\frac{1}{2}\pi) + i\sin(\frac{1}{2}\pi)).$$



$$\tan(\varphi) = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}} \text{ geeft } \varphi = \frac{5}{6}\pi$$

$$|z| = \sqrt{(-\sqrt{3})^2 + 1^2} = 2$$

$$\text{Dus } z = 2(\cos(\frac{5}{6}\pi) + i\sin(\frac{5}{6}\pi)).$$



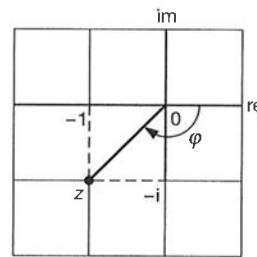
$$\tan(\varphi) = \frac{-6\sqrt{3}}{6} = -\sqrt{3} \text{ geeft } \varphi = -\frac{1}{3}\pi$$

$$|z| = \sqrt{6^2 + (-6\sqrt{3})^2} = 12$$

$$\text{Dus } z = 12(\cos(-\frac{1}{3}\pi) + i\sin(-\frac{1}{3}\pi)).$$

f

$$z = \frac{1-i}{i} = \frac{1-i}{i} \cdot \frac{i}{i} = \frac{i-i^2}{i^2} = \frac{i+1}{-1} = -1-i$$



$$\tan(\varphi) = \frac{-1}{-1} = 1 \text{ geeft } \varphi = -\frac{3}{4}\pi$$

$$|z| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$$

$$\text{Dus } z = \sqrt{2}(\cos(-\frac{3}{4}\pi) + i\sin(-\frac{3}{4}\pi)).$$

Bladzijde 59

37 a $\operatorname{Re}(z) = 3\cos(\frac{1}{6}\pi) = 3 \cdot \frac{1}{2}\sqrt{3} = 1\frac{1}{2}\sqrt{3}$

$$\operatorname{Im}(z) = 3\sin(\frac{1}{6}\pi) = 3 \cdot \frac{1}{2} = 1\frac{1}{2}$$

b $\operatorname{Re}(z) = 1\frac{1}{2}\sqrt{3}$ en $b = \operatorname{Im}(z) = 1\frac{1}{2}$, dus $z = 1\frac{1}{2}\sqrt{3} + 1\frac{1}{2}i$.

38 a $z = 4(\cos(\frac{1}{3}\pi) + i\sin(\frac{1}{3}\pi)) = 4(\frac{1}{2} + \frac{1}{2}i\sqrt{3}) = 2 + 2i\sqrt{3}$

b $z = 5(\cos(\frac{3}{4}\pi) + i\sin(\frac{3}{4}\pi)) = 5(-\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}) = -2\frac{1}{2}\sqrt{2} + 2\frac{1}{2}i\sqrt{2}$

c $z = 6(\cos(-\frac{5}{6}\pi) + i\sin(-\frac{5}{6}\pi)) = 6(-\frac{1}{2}\sqrt{3} - \frac{1}{2}i) = -3\sqrt{3} - 3i$

d $z = 8(\cos(-\frac{1}{6}\pi) + i\sin(-\frac{1}{6}\pi)) = 8(\frac{1}{2}\sqrt{3} - \frac{1}{2}i) = 4\sqrt{3} - 4i$

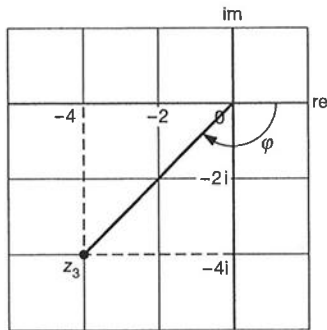
39 a $|z_1| = \sqrt{5^2 + 3^2} = \sqrt{34}$ en $\arg(z) \approx 0,54$

$$\text{Dus } z_1 = \sqrt{34}(\cos(0,54) + i\sin(0,54)).$$

$$|z_2| = \sqrt{(-2)^2 + 3^2} = \sqrt{13} \text{ en } \arg(z) \approx 2,16$$

$$\text{Dus } z_2 = \sqrt{13}(\cos(2,16) + i\sin(2,16)).$$

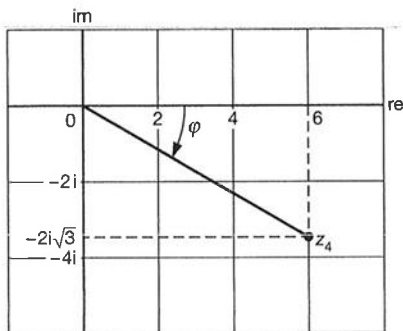
b



$$\tan(\varphi) = \frac{-4}{-4} = 1 \text{ geeft } \varphi = -\frac{3}{4}\pi$$

$$|z_3| = \sqrt{(-4)^2 + (-4)^2} = 4\sqrt{2}$$

$$\text{Dus } z_3 = 4\sqrt{2}(\cos(-\frac{3}{4}\pi) + i\sin(-\frac{3}{4}\pi)).$$



$$\tan(\varphi) = \frac{-2\sqrt{3}}{6} = -\frac{1}{3}\sqrt{3} \text{ geeft } \varphi = -\frac{1}{6}\pi$$

$$|z_4| = \sqrt{6^2 + (-2\sqrt{3})^2} = 4\sqrt{3}$$

$$\text{Dus } z_4 = 4\sqrt{3}(\cos(-\frac{1}{6}\pi) + i\sin(-\frac{1}{6}\pi)).$$

$$\text{c } z_5 = 3(\cos(\frac{1}{4}\pi) + i\sin(\frac{1}{4}\pi)) = 3(\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}) = \frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}$$

$$z_6 = 4(\cos(-1\frac{1}{3}\pi) + i\sin(-1\frac{1}{3}\pi)) = 4(-\frac{1}{2} + \frac{1}{2}i\sqrt{3}) = -2 + 2i\sqrt{3}$$

$$\text{d } z_7 = 2(\cos(-\frac{1}{3}\pi) + i\sin(-\frac{1}{3}\pi)) = 2(\frac{1}{2} - \frac{1}{2}i\sqrt{3}) = 1 - i\sqrt{3}$$

$$40 \text{ a } \left. \begin{array}{l} f(0) = e^0 = 1 \\ f(0) = a_0 + 0 + 0 + 0 + \dots = a_0 \end{array} \right\} a_0 = 1$$

$$\text{b } \left. \begin{array}{l} f'(0) = e^0 = 1 \\ f'(0) = a_1 + 0 + 0 + \dots = a_1 \end{array} \right\} a_1 = 1$$

$$\text{c } f''(x) = e^x \text{ en } f''(x) = 2a_2 + 3 \cdot 2 \cdot a_3x + 4 \cdot 3 \cdot a_4x^2 + 5 \cdot 4 \cdot a_5x^3 + \dots$$

$$f''(0) = 1 \text{ en } f''(0) = 2a_2 \text{ geeft } 2a_2 = 1, \text{ dus } a_2 = \frac{1}{2}.$$

$$f^3(x) = e^x \text{ en } f^3(x) = 3 \cdot 2 \cdot a_3 + 4 \cdot 3 \cdot 2 \cdot a_4x + 5 \cdot 4 \cdot 3 \cdot 2 \cdot a_5x^2 + \dots$$

$$f^3(0) = 1 \text{ en } f^3(0) = 6a_3 \text{ geeft } 6a_3 = 1, \text{ dus } a_3 = \frac{1}{6}.$$

$$f^4(x) = e^x \text{ en } f^4(x) = 4 \cdot 3 \cdot 2 \cdot a_4 + 5 \cdot 4 \cdot 3 \cdot 2 \cdot a_5x + \dots$$

$$f^4(0) = 1 \text{ en } f^4(0) = 24a_4 \text{ geeft } 24a_4 = 1, \text{ dus } a_4 = \frac{1}{24}.$$

$$f^5(x) = e^x \text{ en } f^5(x) = 5 \cdot 4 \cdot 3 \cdot 2 \cdot a_5 + 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot a_6x + \dots$$

$$f^5(0) = 1 \text{ en } f^5(0) = 120a_5 \text{ geeft } 120a_5 = 1, \text{ dus } a_5 = \frac{1}{120}.$$

$$\text{d } \text{Er geldt steeds } n! \cdot a_n = 1 \text{ ofwel } a_n = \frac{1}{n!}.$$

$$\text{Dus } a_6 = \frac{1}{6!}.$$

- 41 a** $f(0) = \sin(0) = 0$
 $f(0) = a_0 + 0 + 0 + 0 + \dots = a_0 \} a_0 = 0$
- b** $f'(0) = \cos(0) = 1$
 $f'(0) = a_1 + 0 + 0 + \dots = a_1 \} a_1 = 1$
- c** $f''(x) = -\sin(x)$ en $f''(x) = 2a_2 + 3 \cdot 2 \cdot a_3x + 4 \cdot 3 \cdot a_4x^2 + 5 \cdot 4 \cdot a_5x^3 + \dots$
 $f''(0) = 0$ en $f''(0) = 2a_2$ geeft $2a_2 = 0$, dus $a_2 = 0$.
 $f^3(x) = -\cos(x)$ en $f^3(x) = 3 \cdot 2 \cdot a_3 + 4 \cdot 3 \cdot 2 \cdot a_4x + 5 \cdot 4 \cdot 3 \cdot 2 \cdot a_5x^2 + \dots$
 $f^3(0) = -1$ en $f^3(0) = 6a_3$ geeft $6a_3 = -1$, dus $a_3 = -\frac{1}{6}$.
 $f^4(x) = \sin(x)$ en $f^4(x) = 4 \cdot 3 \cdot 2 \cdot a_4 + 5 \cdot 4 \cdot 3 \cdot 2 \cdot a_5x + \dots$
 $f^4(0) = 0$ en $f^4(0) = 24a_4$ geeft $24a_4 = 0$, dus $a_4 = 0$.
 $f^5(0) = f'(0) = 1$ geeft $a_5 = \frac{1}{5!} = \frac{1}{120}$.
 $f^6(0) = f''(0) = 0$ geeft $a_6 = \frac{0}{6!} = 0$.
 $f^7(0) = f^3(0) = -1$ geeft $a_7 = \frac{-1}{7!} = -\frac{1}{5040}$.
d $f^{10}(0) = f''(0) = 0$ geeft $a_{10} = \frac{0}{10!} = 0$.
 $f^{100}(0) = f(0) = 0$ geeft $a_{100} = \frac{0}{100!} = 0$.
 $f^{15}(0) = f^3(0) = -1$ geeft $a_{15} = \frac{-1}{15!}$.
 $f^{27}(0) = f^3(0) = -1$ geeft $a_{27} = \frac{-1}{27!}$.

Bladzijde 61

- 42 a** $e^{ix} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} + \frac{(ix)^7}{7!} + \dots$
 $= 1 + ix + \frac{i^2x^2}{2!} + \frac{i^3x^3}{3!} + \frac{i^4x^4}{4!} + \frac{i^5x^5}{5!} + \frac{i^6x^6}{6!} + \frac{i^7x^7}{7!} + \dots$
 $= 1 + ix + \frac{i^2x^2}{2!} + \frac{i^2 \cdot x^3}{3!}i + \frac{i^2 \cdot i^2 \cdot x^4}{4!} + \frac{i^2 \cdot i^2 \cdot x^5}{5!}i + \frac{i^2 \cdot i^2 \cdot i^2 \cdot x^6}{6!} + \frac{i^2 \cdot i^2 \cdot i^2 \cdot x^7}{7!}i + \dots$
 $= 1 + ix + \frac{-x^2}{2!} + \frac{-x^3}{3!}i + \frac{x^4}{4!} + \frac{x^5}{5!}i + \frac{-x^6}{6!} + \frac{-x^7}{7!}i + \dots$
 $= 1 + ix - \frac{x^2}{2!} - \frac{x^3}{3!}i + \frac{x^4}{4!} + \frac{x^5}{5!}i - \frac{x^6}{6!} - \frac{x^7}{7!}i + \dots$
- b** $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$ en $i \sin(x) = i \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \right) = ix - \frac{x^3}{3!}i + \frac{x^5}{5!}i - \frac{x^7}{7!}i + \dots$ geeft
 $\cos(x) + i \sin(x) = 1 + ix - \frac{x^2}{2!} - \frac{x^3}{3!}i + \frac{x^4}{4!} + \frac{x^5}{5!}i - \frac{x^6}{6!} - \frac{x^7}{7!}i + \dots$
Dus $e^{ix} = \cos(x) + i \sin(x)$.

- 43 a** $z = 4e^{\frac{1}{4}\pi i} = 4 \left(\cos\left(\frac{1}{4}\pi\right) + i \sin\left(\frac{1}{4}\pi\right) \right) = 4 \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2} \right) = 2\sqrt{2} + 2i\sqrt{2}$
b $z = \sqrt{3}e^{\frac{1}{6}\pi i} = \sqrt{3} \left(\cos\left(\frac{1}{6}\pi\right) + i \sin\left(\frac{1}{6}\pi\right) \right) = \sqrt{3} \left(\frac{1}{2}\sqrt{3} + \frac{1}{2}i \right) = 1\frac{1}{2} + \frac{1}{2}i\sqrt{3}$
c $z = e^{\pi i} = \cos(\pi) + i \sin(\pi) = -1$

- 44 a** $|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$ en $\arg(z) = \frac{1}{4}\pi$ geeft $z = \sqrt{2}e^{\frac{1}{4}\pi i}$
b $|z| = 5$ en $\arg(z) = \pi$ geeft $z = 5e^{\pi i}$
c $|z| = \sqrt{(\sqrt{3})^2 + 1^2} = 2$ en $\arg(z) = \frac{1}{6}\pi$ geeft $z = 2e^{\frac{1}{6}\pi i}$

- 45 a** $z = 5e^i = 5(\cos(1) + i \sin(1)) = 5\cos(1) + 5i \sin(1) \approx 2,70 + 4,21i$
b $z = -4e^{3i} = -4(\cos(3) + i \sin(3)) = -4\cos(3) - 4i \sin(3) \approx 3,96 - 0,56i$
c $z = \pi e^{-i} = \pi(\cos(-1) + i \sin(-1)) = \pi \cos(-1) + \pi i \sin(-1) \approx 1,70 - 2,64i$

- 46 a $|z| = \sqrt{5^2 + 2^2} = \sqrt{29}$ en $\arg(z) \approx 0,38$ geeft $z \approx \sqrt{29}e^{0,38i}$
 b $|z| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$ en $\arg(z) \approx 2,03$ geeft $z \approx \sqrt{5}e^{2,03i}$
 c $|z| = \sqrt{(\sqrt{2})^2 + (\sqrt{3})^2} = \sqrt{5}$ en $\arg(z) \approx 0,89$ geeft $z \approx \sqrt{5}e^{0,89i}$

Bladzijde 62

- 47 a $e^{\pi i} + 1 = \cos(\pi) + i\sin(\pi) + 1 = -1 + i \cdot 0 + 1 = 0$ klopt.
 b $e^{\frac{1}{2}\pi i} = \cos(\frac{1}{2}\pi) + i\sin(\frac{1}{2}\pi) = 0 + i \cdot 1 = i$, dus $e^{\frac{1}{2}\pi i} - i = 0$.
 $e^{2\pi i} = \cos(2\pi) + i\sin(2\pi) = 1 + i \cdot 0 = 1$, dus $e^{2\pi i} - 1 = 0$.
- 48 a $z = 5e^{\frac{1}{3}\pi i} = 5(\cos(\frac{1}{3}\pi) + i\sin(\frac{1}{3}\pi)) = 5(\frac{1}{2} + \frac{1}{2}i\sqrt{3}) = 2\frac{1}{2} + 2\frac{1}{2}i\sqrt{3}$
 b $|z| = \sqrt{(2\sqrt{3})^2 + (-2)^2} = 4$ en $\arg(z) = -\frac{1}{6}\pi$ geeft $z = 4e^{-\frac{1}{6}\pi i}$
 c $z = 6e^{4i} = 6(\cos(4) + i\sin(4)) = 6\cos(4) + 6i\sin(4) \approx -3,92 - 4,54i$
 d $|z| = \sqrt{(-3)^2 + 4^2} = 5$ en $\arg(z) \approx 2,21$ geeft $z \approx 5e^{2,21i}$
- 49 a $e^{ix} = \cos(x) + i\sin(x)$ en $e^{-ix} = \cos(-x) + i\sin(-x) = \cos(x) - i\sin(x)$ geeft
 $e^{ix} - e^{-ix} = \cos(x) + i\sin(x) - (\cos(x) - i\sin(x)) = \cos(x) + i\sin(x) - \cos(x) + i\sin(x) = 2i\sin(x)$,
 dus $\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$.
- b $e^{ix} + e^{-ix} = \cos(x) + i\sin(x) + \cos(x) - i\sin(x) = 2\cos(x)$, dus $\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$.

- 50 a $\sin(x) = 2$ geeft
 $\frac{e^{ix} - e^{-ix}}{2i} = 2$
 $e^{ix} - e^{-ix} = 4i$
 Stel $e^{ix} = u$.
 $u - \frac{1}{u} = 4i$
 $u^2 - 1 = 4iu$
 $u^2 - 4iu - 1 = 0$
 $D = (-4i)^2 - 4 \cdot 1 \cdot -1 = 16i^2 + 4 = -16 + 4 = -12 = 12i^2$
 $u = \frac{4i + \sqrt{12i^2}}{2} \vee u = \frac{4i - \sqrt{12i^2}}{2}$
 $u = \frac{4i + 2i\sqrt{3}}{2} \vee u = \frac{4i - 2i\sqrt{3}}{2}$
 $u = 2i + i\sqrt{3} \vee u = 2i - i\sqrt{3}$
 $e^{ix} = 2i + i\sqrt{3} \vee e^{ix} = 2i - i\sqrt{3}$
 $ix = \ln(2i + i\sqrt{3}) \vee ix = \ln(2i - i\sqrt{3})$
 $i^2x = i\ln(2i + i\sqrt{3}) \vee i^2x = i\ln(2i - i\sqrt{3})$
 $-x = i\ln(2i + i\sqrt{3}) \vee -x = i\ln(2i - i\sqrt{3})$
 $x = -i\ln(2i + i\sqrt{3}) \vee x = -i\ln(2i - i\sqrt{3})$

10.4 Complexe e-machten en complexe wortels**Bladzijde 64**

- 51 a $(1 + i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$
 $(1 + i)^4 = (2i)^2 = 4i^2 = -4$
 b $|1 + i| = \sqrt{1^2 + 1^2} = \sqrt{2}$ en $\arg(1 + i) = \frac{1}{4}\pi$, dus $1 + i = \sqrt{2}e^{\frac{1}{4}\pi i}$.
 c $(\sqrt{2}e^{\frac{1}{4}\pi i})^4 = (\sqrt{2})^4 \cdot (e^{\frac{1}{4}\pi i})^4 = 4e^{\pi i} = 4(\cos(\pi) + i\sin(\pi)) = 4\cos(\pi) + 4i\sin(\pi) = 4 \cdot -1 + 4i \cdot 0 = -4$

Bladzijde 65

52 a $1 - i = \sqrt{2}e^{-\frac{1}{4}\pi i}$, dus $z = (1 - i)^6 = (\sqrt{2}e^{-\frac{1}{4}\pi i})^6 = 8e^{-\frac{3}{2}\pi i}$

b $1 + i = \sqrt{2}e^{\frac{1}{4}\pi i}$ en $3 - 3i\sqrt{3} = 6e^{-\frac{1}{3}\pi i}$, dus $z = (1 + i)(3 - 3i\sqrt{3}) = \sqrt{2}e^{\frac{1}{4}\pi i} \cdot 6e^{-\frac{1}{3}\pi i} = 6\sqrt{2}e^{-\frac{1}{12}\pi i}$

c $4 + 4i\sqrt{3} = 8e^{\frac{1}{3}\pi i}$ en $\sqrt{3} - i = 2e^{-\frac{1}{6}\pi i}$, dus $\frac{4 + 4i\sqrt{3}}{\sqrt{3} - i} = \frac{8e^{\frac{1}{3}\pi i}}{2e^{-\frac{1}{6}\pi i}} = 4e^{\frac{1}{2}\pi i}$

d $5 + 5i = 5\sqrt{2}e^{\frac{1}{4}\pi i}$, dus $(5 + 5i)^{-3} = (5\sqrt{2}e^{\frac{1}{4}\pi i})^{-3} = \frac{1}{(5\sqrt{2}e^{\frac{1}{4}\pi i})^3} = \frac{1}{250\sqrt{2}e^{\frac{3}{4}\pi i}} = \frac{1}{250\sqrt{2}}e^{-\frac{3}{4}\pi i}$

e $2\sqrt{3} + 2i = 4e^{\frac{1}{6}\pi i}$ en $2 - 2i\sqrt{3} = 4e^{-\frac{1}{3}\pi i}$, dus $(2\sqrt{3} + 2i)(2 - 2i\sqrt{3})^{-2} = 4e^{\frac{1}{6}\pi i} \cdot (4e^{-\frac{1}{3}\pi i})^{-2} = 4e^{\frac{1}{6}\pi i} \cdot \frac{1}{16}e^{\frac{2}{3}\pi i} = \frac{1}{4}e^{\frac{5}{6}\pi i}$

f $3\sqrt{3} - 3i = 6e^{-\frac{1}{6}\pi i}$ en $2\sqrt{3} + 2i = 4e^{\frac{1}{6}\pi i}$, dus $\frac{(3\sqrt{3} - 3i)^2}{2\sqrt{3} + 2i} = \frac{(6e^{-\frac{1}{6}\pi i})^2}{4e^{\frac{1}{6}\pi i}} = \frac{36e^{-\frac{1}{3}\pi i}}{4e^{\frac{1}{6}\pi i}} = 9e^{-\frac{1}{2}\pi i}$

53 a $|2 + 3i| = \sqrt{13}$, dus $|(2 + 3i)^5| = (\sqrt{13})^5 = 169\sqrt{13}$

Met de GR krijg je $z = (2 + 3i)^5 \approx 169\sqrt{13}e^{-1,37i}$.

b De GR geeft $z = (1 + 4i)(4 - i) \approx 17e^{1,08i}$.

c De GR geeft $z = \frac{2 - 4i}{2 + i} \approx 2e^{-1,57i}$.

d $|3 - i\sqrt{3}| = 2\sqrt{3}$, dus $|(3 - i\sqrt{3})^{-4}| = (2\sqrt{3})^{-4} = \frac{1}{(2\sqrt{3})^4} = \frac{1}{144}$

Met de GR krijg je $z = (3 - i\sqrt{3})^{-4} \approx \frac{1}{144}e^{2,09i}$.

e $|2 - i| = \sqrt{5}$ en $|3 + i| = \sqrt{10}$, dus

$$|(2 - i)^3 \cdot (3 + i)^2| = |(2 - i)^3| \cdot |(3 + i)^2| = (\sqrt{5})^3 \cdot (\sqrt{10})^2 = 50\sqrt{5}$$

Met de GR krijg je $z \approx (2 - i)^3 \cdot (3 + i)^2 = 50\sqrt{5}e^{-0,75i}$.

f De GR geeft $z = \frac{(1 + 5i)^4}{(1 - 5i)^2} \approx 135\frac{1}{5}e^{1,42i}$.

54 a $z_1 = \sqrt{3} + i = 2e^{\frac{1}{6}\pi i}$

$$z_2 = -2 + 2i\sqrt{3} = 4e^{\frac{2}{3}\pi i}$$

$$z_1 \cdot z_2 = 2e^{\frac{1}{6}\pi i} \cdot 4e^{\frac{2}{3}\pi i} = 8e^{\frac{5}{6}\pi i}$$

Dus $|z_1 \cdot z_2| = 8$ en $\arg(z_1 \cdot z_2) = \frac{5}{6}\pi$.

b $z_3 = 3\cos(-\frac{3}{4}\pi i) + i\sin(-\frac{3}{4}\pi i)$, dus $|z_3| = 3$ en $\arg(z_3) = -\frac{3}{4}\pi$, dus $z_3 = 3e^{-\frac{3}{4}\pi i}$.

$$z_1 \cdot z_3 = 2e^{\frac{1}{6}\pi i} \cdot 3e^{-\frac{3}{4}\pi i} = 6e^{-\frac{7}{12}\pi i}$$

Dus $|z_1 \cdot z_3| = 6$ en $\arg(z_1 \cdot z_3) = -\frac{7}{12}\pi$.

c $\frac{z_2}{z_3} = \frac{4e^{\frac{2}{3}\pi i}}{3e^{-\frac{3}{4}\pi i}} = 1\frac{1}{3}e^{\frac{17}{12}\pi i}$

Dus $\left|\frac{z_2}{z_3}\right| = 1\frac{1}{3}$ en $\arg\left(\frac{z_2}{z_3}\right) = 1\frac{5}{12}\pi$.

55 a $\sqrt{2} + i\sqrt{2} = 2e^{\frac{1}{4}\pi i}$, dus $z = e^{\frac{1}{4}\pi i} + \sqrt{2} + i\sqrt{2} = e^{\frac{1}{4}\pi i} + 2e^{\frac{1}{4}\pi i} = 3e^{\frac{1}{4}\pi i}$

b $1 + i\sqrt{3} = 2e^{\frac{1}{3}\pi i}$, dus $z = 1 + i\sqrt{3} - e^{\frac{1}{3}\pi i} = 2e^{\frac{1}{3}\pi i} - e^{\frac{1}{3}\pi i} = e^{\frac{1}{3}\pi i}$

c $2\sqrt{3} - 2i = 4e^{-\frac{1}{6}\pi i}$, dus $z = (2\sqrt{3} - 2i)^2 - 10e^{\frac{1}{3}\pi i} = (4e^{-\frac{1}{6}\pi i})^2 - 10e^{\frac{1}{3}\pi i} = 16e^{-\frac{1}{3}\pi i} - 10e^{\frac{1}{3}\pi i} = 6e^{-\frac{1}{3}\pi i}$

d $1 + i = \sqrt{2}e^{\frac{1}{4}\pi i}$, dus $z = \frac{(1 + i)^6}{4e^{\pi i}} = \frac{(\sqrt{2}e^{\frac{1}{4}\pi i})^6}{4e^{\pi i}} = \frac{8e^{\frac{3}{2}\pi i}}{4e^{\pi i}} = 2e^{\frac{1}{2}\pi i}$

Bladzijde 66

56 a Er bestaat een lineair verband tussen u_n en de twee voorafgaande termen u_{n-1} en u_{n-2} , dus je hebt met een lineaire differentievergelijking van de tweede orde te maken.

b Stel $u_n = g^n$.

Substitutie van $u_n = g^n$ in $u_n = 2u_{n-1} - 4u_{n-2}$ geeft $g^n = 2g^{n-1} - 4g^{n-2}$ ofwel $g^n - 2g^{n-1} + 4g^{n-2} = 0$.

Delen door g^{n-2} geeft vervolgens de karakteristieke vergelijking $g^2 - 2g + 4 = 0$.

$$c \quad g^2 - 2g + 4 = 0$$

$$(g-1)^2 - 1 + 4 = 0$$

$$(g-1)^2 = -3$$

$$(g-1)^2 = 3i^2$$

$$g-1 = i\sqrt{3} \vee g-1 = -i\sqrt{3}$$

$$g = 1 + i\sqrt{3} \vee g = 1 - i\sqrt{3}$$

$$d \quad |1 + i\sqrt{3}| = 2 \text{ en } \arg(1 + i\sqrt{3}) = \frac{1}{3}\pi$$

$$|1 - i\sqrt{3}| = 2 \text{ en } \arg(1 - i\sqrt{3}) = -\frac{1}{3}\pi$$

$$\text{Dus } g = 2(\cos(\frac{1}{3}\pi) + i\sin(\frac{1}{3}\pi)) \vee g = 2(\cos(-\frac{1}{3}\pi) + i\sin(-\frac{1}{3}\pi)).$$

Bladzijde 67

- 57 a Substitutie van $u_n = g^n$ in $u_n = 2\sqrt{3} \cdot u_{n-1} - 4u_{n-2}$ geeft

$$g^2 - 2\sqrt{3} \cdot g + 4 = 0$$

$$(g - \sqrt{3})^2 - 3 + 4 = 0$$

$$(g - \sqrt{3})^2 = -1$$

$$(g - \sqrt{3})^2 = i^2$$

$$g - \sqrt{3} = i \vee g - \sqrt{3} = -i$$

$$g = \sqrt{3} + i \vee g = \sqrt{3} - i$$

$$|\sqrt{3} + i| = 2 \text{ en } \arg(\sqrt{3} + i) = \frac{1}{6}\pi$$

$$u_n = (A \cos(\frac{1}{6}\pi n) + B \sin(\frac{1}{6}\pi n)) \cdot 2^n$$

$$u_0 = 4 \text{ geeft } A = 4$$

$$u_1 = \sqrt{3} \text{ geeft } (A \cdot \frac{1}{2}\sqrt{3} + B \cdot \frac{1}{2}) \cdot 2 = \sqrt{3} \text{ ofwel } \sqrt{3} \cdot A + B = \sqrt{3} \left. \vphantom{u_1 = \sqrt{3} \text{ geeft }} \right\} A = 4 \text{ en } B = -3\sqrt{3}$$

$$\text{Dus } u_n = (4 \cos(\frac{1}{6}\pi n) - 3\sqrt{3} \sin(\frac{1}{6}\pi n)) \cdot 2^n.$$

- b Substitutie van $v_n = g^n$ in $v_n = -4v_{n-1} - 16v_{n-2}$ geeft

$$g^2 + 4g + 16 = 0$$

$$(g+2)^2 - 4 + 16 = 0$$

$$(g+2)^2 = -12$$

$$(g+2)^2 = 12i^2$$

$$g+2 = 2i\sqrt{3} \vee g+2 = -2i\sqrt{3}$$

$$g = -2 + 2i\sqrt{3} \vee g = -2 - 2i\sqrt{3}$$

$$|-2 + 2i\sqrt{3}| = 4 \text{ en } \arg(-2 + 2i\sqrt{3}) = \frac{2}{3}\pi$$

$$v_n = (A \cos(\frac{2}{3}\pi n) + B \sin(\frac{2}{3}\pi n)) \cdot 4^n$$

$$v_0 = 1 \text{ geeft } A = 1$$

$$v_1 = 2 \text{ geeft } (A \cdot -\frac{1}{2} + B \cdot \frac{1}{2}\sqrt{3}) \cdot 4 = 2 \text{ ofwel } -2A + 2\sqrt{3} \cdot B = 2 \left. \vphantom{v_1 = 2 \text{ geeft }} \right\} A = 1 \text{ en } B = \frac{2}{3}\sqrt{3}$$

$$\text{Dus } v_n = (\cos(\frac{2}{3}\pi n) + \frac{2}{3}\sqrt{3} \sin(\frac{2}{3}\pi n)) \cdot 4^n.$$

Bladzijde 68

- 58 a Substitutie van $u_n = g^n$ in $u_n = -6u_{n-1} - 12u_{n-2}$ geeft

$$g^2 + 6g + 12 = 0$$

$$(g+3)^2 - 9 + 12 = 0$$

$$(g+3)^2 = -3$$

$$(g+3)^2 = 3i^2$$

$$g+3 = i\sqrt{3} \vee g+3 = -i\sqrt{3}$$

$$g = -3 + i\sqrt{3} \vee g = -3 - i\sqrt{3}$$

$$|-3 + i\sqrt{3}| = 2\sqrt{3} \text{ en } \arg(-3 + i\sqrt{3}) = \frac{5}{6}\pi$$

$$u_n = (A \cos(\frac{5}{6}\pi n) + B \sin(\frac{5}{6}\pi n)) \cdot (2\sqrt{3})^n$$

$$u_0 = 1 \text{ geeft } A = 1$$

$$u_1 = 1 \text{ geeft } (A \cdot -\frac{1}{2}\sqrt{3} + B \cdot \frac{1}{2}) \cdot 2\sqrt{3} = 1 \text{ ofwel } -3A + \sqrt{3} \cdot B = 1 \left. \vphantom{u_1 = 1 \text{ geeft }} \right\} A = 1 \text{ en } B = 1\frac{1}{3}\sqrt{3}$$

$$\text{Dus } u_n = (\cos(\frac{5}{6}\pi n) + 1\frac{1}{3}\sqrt{3} \sin(\frac{5}{6}\pi n)) \cdot (2\sqrt{3})^n.$$

b Substitutie van $x_n = g^n$ in $x_n = -\frac{1}{4}x_{n-2}$ geeft

$$g^2 = -\frac{1}{4}$$

$$g^2 = \frac{1}{4}i^2$$

$$g = \frac{1}{2}i \vee g = -\frac{1}{2}i$$

$$\left|\frac{1}{2}i\right| = \frac{1}{2} \text{ en } \arg\left(\frac{1}{2}i\right) = \frac{1}{2}\pi$$

$$x_n = \left(A \cos\left(\frac{1}{2}\pi n\right) + B \sin\left(\frac{1}{2}\pi n\right)\right) \cdot \left(\frac{1}{2}\right)^n$$

$$x_0 = 16 \text{ geeft } A = 16$$

$$x_1 = 12 \text{ geeft } (A \cdot 0 + B \cdot 1) \cdot \frac{1}{2} = 12 \text{ ofwel } B = 24$$

$$\text{Dus } x_n = \left(16 \cos\left(\frac{1}{2}\pi n\right) + 24 \sin\left(\frac{1}{2}\pi n\right)\right) \cdot \left(\frac{1}{2}\right)^n.$$

59 a $x_n = -2x_{n-1} + 4y_{n-1}$ geeft $x_{n+1} = -2x_n + 4y_n$

$$y_n = -x_{n-1} - 2y_{n-1}$$

$$x_{n+1} = -2x_n + 4(-x_{n-1} - 2y_{n-1})$$

$$x_{n+1} = -2x_n - 4x_{n-1} - 8y_{n-1}$$

$$x_n = -2x_{n-1} + 4y_{n-1} \text{ geeft } 4y_{n-1} = x_n + 2x_{n-1}$$

$$x_{n+1} = -2x_n - 4x_{n-1} - 2 \cdot (x_n + 2x_{n-1})$$

$$x_{n+1} = -2x_n - 4x_{n-1} - 2x_n - 4x_{n-1}$$

$$x_{n+1} = -4x_n - 8x_{n-1}$$

$$x_n = -4x_{n-1} - 8x_{n-2}$$

Substitutie van $x_n = g^n$ in $x_n = -4x_{n-1} - 8x_{n-2}$ geeft

$$g^2 + 4g + 8 = 0$$

$$(g+2)^2 - 4 + 8 = 0$$

$$(g+2)^2 = -4$$

$$(g+2)^2 = 4i^2$$

$$g+2 = 2i \vee g+2 = -2i$$

$$g = -2 + 2i \vee g = -2 - 2i$$

$$|-2 + 2i| = 2\sqrt{2} \text{ en } \arg(-2 + 2i) = \frac{3}{4}\pi$$

$$x_n = \left(A \cos\left(\frac{3}{4}\pi n\right) + B \sin\left(\frac{3}{4}\pi n\right)\right) \cdot (2\sqrt{2})^n$$

$$x_0 = 2 \text{ en } y_0 = 3 \text{ geeft } x_1 = -2 \cdot 2 + 4 \cdot 3 = 8$$

$$x_0 = 2 \text{ geeft } A = 2$$

$$x_1 = 8 \text{ geeft } \left(A \cdot -\frac{1}{2}\sqrt{2} + B \cdot \frac{1}{2}\sqrt{2}\right) \cdot 2\sqrt{2} = 8 \text{ ofwel } -2A + 2B = 8 \left. \vphantom{\begin{matrix} A = 2 \\ B = 6 \end{matrix}} \right\} A = 2 \text{ en } B = 6$$

$$\text{Dus } x_n = \left(2 \cos\left(\frac{3}{4}\pi n\right) + 6 \sin\left(\frac{3}{4}\pi n\right)\right) \cdot (2\sqrt{2})^n.$$

$$y_n = -x_{n-1} - 2y_{n-1} \text{ geeft } y_{n+1} = -x_n - 2y_n$$

$$x_n = -2x_{n-1} + 4y_{n-1}$$

$$y_{n+1} = -x_n - 2y_n = -(-2x_{n-1} + 4y_{n-1}) - 2y_n = 2x_{n-1} - 4y_{n-1} - 2y_n$$

$$y_{n+1} = 2x_{n-1} - 4y_{n-1} - 2y_n$$

$$y_{n+1} = -x_{n-1} - 2y_{n-1} \text{ geeft } x_{n-1} = -y_n - 2y_{n-1}$$

$$y_{n+1} = 2(-y_n - 2y_{n-1}) - 4y_{n-1} - 2y_n$$

$$y_{n+1} = -2y_n - 4y_{n-1} - 4y_{n-1} - 2y_n$$

$$y_{n+1} = -4y_n - 8y_{n-1}$$

$$y_n = -4y_{n-1} - 8y_{n-2}$$

$$\text{Dus } y_n = \left(A \cos\left(\frac{3}{4}\pi n\right) + B \sin\left(\frac{3}{4}\pi n\right)\right) \cdot (2\sqrt{2})^n.$$

$$x_0 = 2 \text{ en } y_0 = 3 \text{ geeft } y_1 = -2 - 2 \cdot 3 = -8$$

$$y_0 = 3 \text{ geeft } A = 3$$

$$y_1 = -8 \text{ geeft } \left(A \cdot -\frac{1}{2}\sqrt{2} + B \cdot \frac{1}{2}\sqrt{2}\right) \cdot 2\sqrt{2} = -8 \text{ ofwel } -2A + 2B = -8 \left. \vphantom{\begin{matrix} A = 3 \\ B = -1 \end{matrix}} \right\} A = 3 \text{ en } B = -1$$

$$\text{Dus } y_n = \left(3 \cos\left(\frac{3}{4}\pi n\right) - \sin\left(\frac{3}{4}\pi n\right)\right) \cdot (2\sqrt{2})^n.$$

60 a $|\sqrt{3} + i| = 2$ en $\arg(\sqrt{3} + i) = \frac{1}{6}\pi$, dus

$$(\sqrt{3} + i)^3 = (2e^{\frac{1}{6}\pi i})^3 = 8e^{\frac{1}{2}\pi i} = 8(\cos(\frac{1}{2}\pi) + i\sin(\frac{1}{2}\pi)) = 8\cos(\frac{1}{2}\pi) + 8i\sin(\frac{1}{2}\pi) = 8i$$

$$|-\sqrt{3} + i| = 2 \text{ en } \arg(-\sqrt{3} + i) = \frac{5}{6}\pi, \text{ dus}$$

$$(-\sqrt{3} + i)^3 = (2e^{\frac{5}{6}\pi i})^3 = 8e^{\frac{5}{2}\pi i} = 8(\cos(2\frac{1}{2}\pi) + i\sin(2\frac{1}{2}\pi)) = 8\cos(2\frac{1}{2}\pi) + 8i\sin(2\frac{1}{2}\pi) = 8i$$

$$(-2i)^3 = -8i^3 = -8i^2 \cdot i = 8i$$

b $z^3 = 8i$ geeft $z = \sqrt[3]{8i} = \sqrt{3} + i \vee z = -\sqrt{3} + i \vee z = -2i$

c $2e^{\frac{1}{6}\pi i} = 2(\cos(\frac{1}{6}\pi) + i\sin(\frac{1}{6}\pi)) = 2\cos(\frac{1}{6}\pi) + 2i\sin(\frac{1}{6}\pi) = 2 \cdot \frac{1}{2}\sqrt{3} + 2i \cdot \frac{1}{2} = \sqrt{3} + i$

$2e^{\frac{2}{6}\pi i} = 2(\cos(\frac{2}{6}\pi) + i\sin(\frac{2}{6}\pi)) = 2\cos(\frac{2}{6}\pi) + 2i\sin(\frac{2}{6}\pi) = 2 \cdot \frac{1}{2}\sqrt{3} + 2i \cdot \frac{1}{2} = \sqrt{3} + i$

d $2e^{\frac{5}{6}\pi i} = 2(\cos(\frac{5}{6}\pi) + i\sin(\frac{5}{6}\pi)) = 2\cos(\frac{5}{6}\pi) + 2i\sin(\frac{5}{6}\pi) = 2 \cdot -\frac{1}{2}\sqrt{3} + 2i \cdot \frac{1}{2} = -\sqrt{3} + i$

$2e^{\frac{25}{6}\pi i} = 2(\cos(\frac{25}{6}\pi) + i\sin(\frac{25}{6}\pi)) = 2\cos(\frac{25}{6}\pi) + 2i\sin(\frac{25}{6}\pi) = 2 \cdot -\frac{1}{2}\sqrt{3} + 2i \cdot \frac{1}{2} = -\sqrt{3} + i$

Dus $2e^{\frac{5}{6}\pi i} = 2e^{\frac{25}{6}\pi i}$.

e $|-2i| = 2$ en $\arg(-2i) = -\frac{1}{2}\pi$

$-2i = 2e^{-\frac{1}{2}\pi i} = 2e^{\frac{1}{2}\pi i} = 2e^{\frac{3}{2}\pi i}$

Bladzijde 70

61 a $(z-1)^2 = \frac{1}{2} + \frac{1}{2}i\sqrt{3}$

$(z-1)^2 = e^{\frac{1}{3}\pi i + k \cdot 2\pi i}$

$z-1 = e^{\frac{1}{6}\pi i + k \cdot \pi i}$

$z-1 = e^{-\frac{5}{6}\pi i} \vee z-1 = e^{\frac{1}{6}\pi i}$

$z-1 = -\frac{1}{2}\sqrt{3} - \frac{1}{2}i \vee z-1 = \frac{1}{2}\sqrt{3} + \frac{1}{2}i$

$z = 1 - \frac{1}{2}\sqrt{3} - \frac{1}{2}i \vee z = 1 + \frac{1}{2}\sqrt{3} + \frac{1}{2}i$

b $(z-1-i)^2 = -i$

$(z-1-i)^2 = e^{-\frac{1}{2}\pi i + k \cdot 2\pi i}$

$z-1-i = e^{-\frac{1}{4}\pi i + k \cdot \pi i}$

$z-1-i = \frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2} \vee z-1-i = -\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}$

$z = 1 + \frac{1}{2}\sqrt{2} + i - \frac{1}{2}i\sqrt{2} \vee z = 1 - \frac{1}{2}\sqrt{2} + i + \frac{1}{2}i\sqrt{2}$

$z = 1 + \frac{1}{2}\sqrt{2} + (1 - \frac{1}{2}\sqrt{2})i \vee z = 1 - \frac{1}{2}\sqrt{2} + (1 + \frac{1}{2}\sqrt{2})i$

c $\arg(z^3) = \arg(27) = 0$, dus één van de oplossingen heeft als argument 0.

$|z^3| = |27| = 27$ geeft $|z| = 3$, dus de straal van de cirkel is 3.

De argumenten verschillen steeds $\frac{2\pi}{3} = \frac{2}{3}\pi$ van elkaar.

De oplossing is $z = 3 \vee z = -1\frac{1}{2} + 1\frac{1}{2}i\sqrt{3} \vee z = -1\frac{1}{2} - 1\frac{1}{2}i\sqrt{3}$.

d $\arg(z^4) = \arg(-81) = \pi$, dus één van de oplossingen heeft als argument $\frac{1}{4}\pi$.

$|z^4| = |-81| = 81$ geeft $|z| = 3$, dus de straal van de cirkel is 3.

De argumenten verschillen steeds $\frac{2\pi}{4} = \frac{1}{2}\pi$ van elkaar.

De oplossing is $z = 1\frac{1}{2}\sqrt{2} + 1\frac{1}{2}i\sqrt{2} \vee z = -1\frac{1}{2}\sqrt{2} + 1\frac{1}{2}i\sqrt{2} \vee z = -1\frac{1}{2}\sqrt{2} - 1\frac{1}{2}i\sqrt{2} \vee z = 1\frac{1}{2}\sqrt{2} - 1\frac{1}{2}i\sqrt{2}$.

10

62 a $\sqrt{2+i}$, dus los op $z^2 = 2+i$

$z^2 \approx \sqrt{5}e^{0,46i + k \cdot 2\pi i}$

$z \approx 1,50e^{0,23i + k \cdot \pi i}$

$z \approx 1,50e^{-2,91i} \vee z \approx 1,50e^{0,23i}$

Dus $\sqrt{2+i} \approx 1,50e^{-2,91i} \vee \sqrt{2+i} \approx 1,50e^{0,23i}$.

b $\sqrt{-4+3i}$, dus los op $z^2 = -4+3i$

$z^2 \approx 5e^{2,50i + k \cdot 2\pi i}$

$z \approx 2,24e^{1,25i + k \cdot \pi i}$

$z \approx 2,24e^{-1,89i} \vee z \approx 2,24e^{1,25i}$

Dus $\sqrt{-4+3i} \approx 2,24e^{-1,89i} \vee \sqrt{-4+3i} \approx 2,24e^{1,25i}$.

c $\sqrt[3]{-6+3i}$, dus los op $z^3 = -6+3i$

$z^3 \approx \sqrt{45}e^{2,68i + k \cdot 2\pi i}$

$z \approx 1,89e^{0,89i + k \cdot \frac{2}{3}\pi i}$

$z \approx 1,89e^{-1,20i} \vee z \approx 1,89e^{0,89i} \vee z \approx 1,89e^{2,99i}$

Dus $\sqrt[3]{-6+3i} \approx 1,89e^{-1,20i} \vee \sqrt[3]{-6+3i} \approx 1,89e^{0,89i} \vee \sqrt[3]{-6+3i} \approx 1,89e^{2,99i}$.

d $\sqrt[4]{10}$, dus los op $z^4 = 10$

$z^4 = 10e^{k \cdot 2\pi i}$

$z \approx 1,78e^{k \cdot \frac{1}{2}\pi i}$

$z \approx 1,78e^{-1,57i} \vee z \approx 1,78 \vee z \approx 1,78e^{1,57i} \vee z \approx 1,78e^{3,14i}$

Dus $\sqrt[4]{10} \approx 1,78e^{-1,57i} \vee \sqrt[4]{10} \approx 1,78 \vee \sqrt[4]{10} \approx 1,78e^{1,57i} \vee \sqrt[4]{10} \approx 1,78e^{3,14i}$.

- 63 a** $\sqrt{-5i}$, dus los op $z^2 = -5i$.
 $\arg(z^2) = \arg(-5i) = -\frac{1}{2}\pi$, dus één van de oplossingen heeft als argument $-\frac{1}{4}\pi$.
 $|z^2| = |-5i| = 5$ geeft $|z| = \sqrt{5}$, dus de straal van de cirkel is $\sqrt{5}$.

De argumenten verschillen steeds $\frac{2\pi}{2} = \pi$.

De oplossing is $z = \sqrt{5}(\frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2}) = \frac{1}{2}\sqrt{10} - \frac{1}{2}i\sqrt{10} \vee z = \sqrt{5}(-\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}) = -\frac{1}{2}\sqrt{10} + \frac{1}{2}i\sqrt{10}$.

Dus $\sqrt{-5i} = \frac{1}{2}\sqrt{10} - \frac{1}{2}i\sqrt{10} \vee \sqrt{-5i} = -\frac{1}{2}\sqrt{10} + \frac{1}{2}i\sqrt{10}$.

b $\frac{1}{1+i\sqrt{3}} = (1+i\sqrt{3})^{-1} = (2e^{\frac{1}{3}\pi i})^{-1} = \frac{1}{2}e^{-\frac{1}{3}\pi i}$

$\sqrt{\frac{1}{1+i\sqrt{3}}}$, dus los op $z^2 = \frac{1}{1+i\sqrt{3}}$

$$z^2 = \frac{1}{2}e^{-\frac{1}{3}\pi i + k \cdot 2\pi i}$$

$$z = \frac{1}{2}\sqrt{2}e^{-\frac{1}{6}\pi i + k \cdot \pi i}$$

$$z = \frac{1}{2}\sqrt{2}e^{-\frac{1}{6}\pi i} \vee z = \frac{1}{2}\sqrt{2}e^{\frac{5}{6}\pi i}$$

$$z = \frac{1}{2}\sqrt{2}(\frac{1}{2}\sqrt{3} - \frac{1}{2}i) \vee z = \frac{1}{2}\sqrt{2}(-\frac{1}{2}\sqrt{3} + \frac{1}{2}i)$$

$$z = \frac{1}{4}\sqrt{6} - \frac{1}{4}i\sqrt{2} \vee z = -\frac{1}{4}\sqrt{6} + \frac{1}{4}i\sqrt{2}$$

Dus $\sqrt{\frac{1}{1+i\sqrt{3}}} = \frac{1}{4}\sqrt{6} - \frac{1}{4}i\sqrt{2} \vee \sqrt{\frac{1}{1+i\sqrt{3}}} = -\frac{1}{4}\sqrt{6} + \frac{1}{4}i\sqrt{2}$.

c $\sqrt[3]{-2+2i}$, dus los op $z^3 = -2+2i$

$$z^3 = 2\sqrt{2}e^{\frac{3}{4}\pi i + k \cdot 2\pi i}$$

$$z = \sqrt{2}e^{\frac{1}{4}\pi i + k \cdot \frac{2}{3}\pi i}$$

$$z = \sqrt{2}e^{-\frac{5}{12}\pi i} \vee z = \sqrt{2}e^{\frac{1}{4}\pi i} \vee z = \sqrt{2}e^{\frac{11}{12}\pi i}$$

$$z = \sqrt{2}(\cos(-\frac{5}{12}\pi) + i\sin(-\frac{5}{12}\pi)) \vee z = \sqrt{2}(\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}) \vee z = \sqrt{2}(\cos(\frac{11}{12}\pi) + i\sin(\frac{11}{12}\pi))$$

$$z = \sqrt{2}\cos(-\frac{5}{12}\pi) + i\sqrt{2}\sin(-\frac{5}{12}\pi) \vee z = 1 + i \vee z = \sqrt{2}\cos(\frac{11}{12}\pi) + i\sqrt{2}\sin(\frac{11}{12}\pi)$$

Dus $\sqrt[3]{-2+2i} = \sqrt{2}\cos(-\frac{5}{12}\pi) + i\sqrt{2}\sin(-\frac{5}{12}\pi) \vee \sqrt[3]{-2+2i} = 1 + i \vee \sqrt[3]{-2+2i} = \sqrt{2}\cos(\frac{11}{12}\pi) + i\sqrt{2}\sin(\frac{11}{12}\pi)$.

d $\sqrt[4]{-8+8i\sqrt{3}}$, dus los op $z^4 = -8+8i\sqrt{3}$

$$z^4 = 16e^{\frac{2}{3}\pi i + k \cdot 2\pi i}$$

$$z = 2e^{\frac{1}{6}\pi i + k \cdot \frac{1}{2}\pi i}$$

$$z = 2e^{-\frac{5}{6}\pi i} \vee z = 2e^{-\frac{1}{3}\pi i} \vee z = 2e^{\frac{1}{6}\pi i} \vee z = 2e^{\frac{2}{3}\pi i}$$

$$z = 2(-\frac{1}{2}\sqrt{3} - \frac{1}{2}i) \vee z = 2(\frac{1}{2} - \frac{1}{2}i\sqrt{3}) \vee z = 2(\frac{1}{2}\sqrt{3} + \frac{1}{2}i) \vee z = 2(-\frac{1}{2} + \frac{1}{2}i\sqrt{3})$$

$$z = -\sqrt{3} - i \vee z = 1 - i\sqrt{3} \vee z = \sqrt{3} + i \vee z = -1 + i\sqrt{3}$$

Dus $\sqrt[4]{-8+8i\sqrt{3}} = -\sqrt{3} - i \vee \sqrt[4]{-8+8i\sqrt{3}} = 1 - i\sqrt{3} \vee \sqrt[4]{-8+8i\sqrt{3}} = \sqrt{3} + i \vee \sqrt[4]{-8+8i\sqrt{3}} = -1 + i\sqrt{3}$.

- 64 a** $\arg(z^2) = \arg(-4i) = -\frac{1}{2}\pi$, dus één van de oplossingen heeft als argument $-\frac{1}{4}\pi$.
 $|z^2| = |-4i| = 4$ geeft $|z| = 2$, dus de straal van de cirkel is 2.

De argumenten verschillen steeds $\frac{2\pi}{2} = \pi$.

De oplossing is $z = 2(\frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2}) = \sqrt{2} - i\sqrt{2} \vee z = 2(-\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}) = -\sqrt{2} + i\sqrt{2}$.

- b** $\arg(z^6) = \arg(-64) = \pi$, dus één van de oplossingen heeft als argument $\frac{1}{6}\pi$.
 $|z^6| = |-64| = 64$ geeft $|z| = 2$, dus de straal van de cirkel is 2.

De argumenten verschillen steeds $\frac{2\pi}{6} = \frac{1}{3}\pi$.

De oplossing is $z = \sqrt{3} + i \vee z = 2i \vee z = -\sqrt{3} + i \vee z = -\sqrt{3} - i \vee z = -2i \vee z = \sqrt{3} - i$.

c $z^2 - 4z + 4 = \frac{1}{2} - \frac{1}{2}i\sqrt{3}$

$$(z-2)^2 = e^{-\frac{1}{3}\pi i + k \cdot 2\pi i}$$

$$z-2 = e^{-\frac{1}{6}\pi i + k \cdot \pi i}$$

$$z-2 = e^{-\frac{1}{6}\pi i} \vee z-2 = e^{\frac{5}{6}\pi i}$$

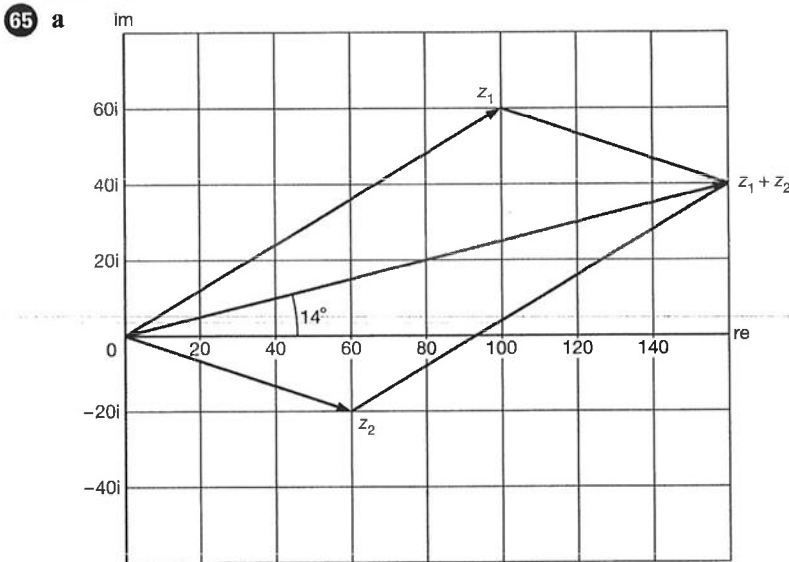
$$z-2 = \frac{1}{2}\sqrt{3} - \frac{1}{2}i \vee z-2 = -\frac{1}{2}\sqrt{3} + \frac{1}{2}i$$

$$z = 2 + \frac{1}{2}\sqrt{3} - \frac{1}{2}i \vee z = 2 - \frac{1}{2}\sqrt{3} + \frac{1}{2}i$$

$$\begin{aligned}
 \text{d } z^2 - 6z + 10 &= i\sqrt{3} \\
 (z-3)^2 - 9 + 10 &= i\sqrt{3} \\
 (z-3)^2 &= -1 + i\sqrt{3} \\
 (z-3)^2 &= 2e^{\frac{2}{3}\pi i + k \cdot 2\pi i} \\
 z-3 &= \sqrt{2}e^{\frac{1}{3}\pi i + k \cdot \pi i} \\
 z-3 &= \sqrt{2}e^{-\frac{2}{3}\pi i} \vee z-3 = \sqrt{2}e^{\frac{1}{3}\pi i} \\
 z-3 &= \sqrt{2}\left(-\frac{1}{2} - \frac{1}{2}i\sqrt{3}\right) \vee z-3 = \sqrt{2}\left(\frac{1}{2} + \frac{1}{2}i\sqrt{3}\right) \\
 z-3 &= -\frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{6} \vee z-3 = \frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{6} \\
 z &= 3 - \frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{6} \vee z = 3 + \frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{6}
 \end{aligned}$$

10.5 Krachten en snelheden met complexe getallen

Bladzijde 72



b lengte $\approx 8,25$ cm dus $|z_1 + z_2| \approx 165$.
hoek $\approx 14^\circ$

c $|z_1 + z_2| = |160 + 40i| \approx 165$
 $\arg(z_1 + z_2) = \arg(160 + 40i) \approx 14^\circ$

Bladzijde 73

66 Bij de krachten $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$ en $\vec{F}_4$ horen de complexe getallen

$$z_1 = 120$$

$$z_2 = 250(\cos(35^\circ) + i \sin(35^\circ))$$

$$z_3 = 200(\cos(100^\circ) + i \sin(100^\circ))$$

$$z_4 = 180(\cos(170^\circ) + i \sin(170^\circ))$$

De GR geeft $|z_1 + z_2 + z_3 + z_4| \approx 388$ en $\arg(z_1 + z_2 + z_3 + z_4) \approx 73^\circ$.

De resultante heeft een grootte van ongeveer 388 N en maakt een hoek van 73° met de horizontale as.

67 Bij de krachten $\vec{F}_z$, $\vec{F}_1$ en $\vec{F}_r$ horen de complexe getallen

$$z_z = -200i$$

$$z_1 = 300(\cos(25^\circ) + i \sin(25^\circ))$$

$$z_r = 300(\cos(70^\circ) + i \sin(70^\circ))$$

en z_2 .

$$z_r \approx z_1 + z_2 + z_z \text{ dus } z_2 = z_r - z_1 - z_z$$

De GR geeft $|z_r - z_1 - z_z| \approx 393$ en $\arg(z_r - z_1 - z_z) \approx 115^\circ$.

Dus de grootte van $\vec{F}_2$ is 393 N en de hoek $\alpha \approx 180^\circ - 115^\circ = 65^\circ$.

- 68 Bij de krachten $\vec{F}_z$, $\vec{F}_1$, $\vec{F}_2$ en $\vec{F}_t$ horen de complexe getallen

$$z_z = -500i$$

$$z_1 = 300(\cos(200^\circ) + i \sin(200^\circ))$$

$$z_2 = 150(\cos(-40^\circ) + i \sin(-40^\circ))$$

en z_t .

$$z_z + z_1 + z_2 + z_t = 0 \text{ geeft } z_t = -z_z - z_1 - z_2$$

De GR geeft $|-z_z - z_1 - z_2| \approx 719$ en $\arg(-z_z - z_1 - z_2) \approx 77^\circ$.

Dus de grootte van $\vec{F}_t \approx 719$ N en $\alpha \approx 90^\circ - 77^\circ = 13^\circ$.

Bladzijde 74

- 69 Bij de snelheden horen de complexe getallen

$$z_1 = 4(\cos(-155^\circ) + i \sin(-155^\circ)) \text{ en } z_2 = 3i.$$

De GR geeft $|z_1 + z_2| \approx 3,9$ en $\arg(z_1 + z_2) \approx 160^\circ$.

De resulterende snelheid is 3,9 km/uur en de koers is 290° .

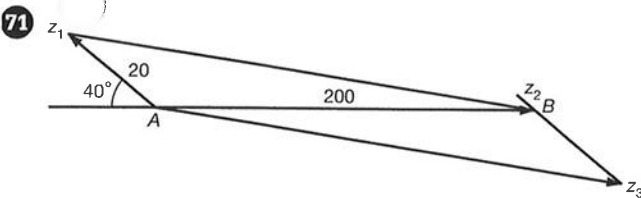
- 70 Bij de snelheden horen de complexe getallen

$$z_w = 50(\cos(45^\circ) + i \sin(45^\circ)) \text{ en}$$

$$z_v = 250(\cos(-80^\circ) + i \sin(-80^\circ)).$$

De GR geeft $|z_w + z_v| \approx 225,08$.

Het vliegtuig doet er $\frac{1000}{225,08} \approx 4,443$ uur ≈ 4 uur en 27 minuten over.



Bij de snelheden horen de complexe getallen

$$z_1 = 20(\cos(140^\circ) + i \sin(140^\circ))$$

$$z_2 = 200 \text{ en } z_3.$$

$$z_1 + z_3 = z_2 \text{ dus } z_3 = z_2 - z_1$$

De GR geeft $|z_2 - z_1| \approx 216$.

De snelheid van het vliegtuig is 216 km/uur ten opzichte van de omringende lucht.

Diagnostische toets

Bladzijde 76

1 a $3x - 1 + 2i = 4x - 2 + 4i$

$$x = -1 + 2i$$

$$x = \frac{1}{2} - i$$

b $(2x + 1)^2 + 9 = 0$

$$(2x + 1)^2 = -9$$

$$(2x + 1)^2 = 9i^2$$

$$2x + 1 = 3i \vee 2x + 1 = -3i$$

$$2x = -1 + 3i \vee 2x = -1 - 3i$$

$$x = -\frac{1}{2} + \frac{3}{2}i \vee x = -\frac{1}{2} - \frac{3}{2}i$$

c $x^2 - 4x + 10 = 0$

$$(x - 2)^2 - 4 + 10 = 0$$

$$(x - 2)^2 = -6$$

$$(x - 2)^2 = 6i^2$$

$$x - 2 = i\sqrt{6} \vee x - 2 = -i\sqrt{6}$$

$$x = 2 + i\sqrt{6} \vee x = 2 - i\sqrt{6}$$

d $4x^2 + 16x + 17 = 0$

$$(2x + 4)^2 - 16 + 17 = 0$$

$$(2x + 4)^2 = -1$$

$$(2x + 4)^2 = i^2$$

$$2x + 4 = i \vee 2x + 4 = -i$$

$$2x = -4 + i \vee 2x = -4 - i$$

$$x = -2 + \frac{1}{2}i \vee x = -2 - \frac{1}{2}i$$

2 a $z = (2 - i) - (5 - 8i) = 2 - i - 5 + 8i = -3 + 7i$

b $z = (6 - 2i)(3 + 2i) = 18 + 12i - 6i - 4i^2 = 18 + 12i - 6i + 4 = 22 + 6i$

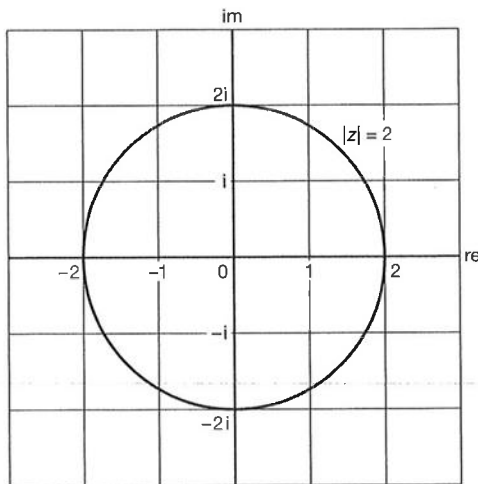
c $z = \frac{3 - i}{4 + i} = \frac{3 - i}{4 + i} \cdot \frac{4 - i}{4 - i} = \frac{12 - 3i - 4i + i^2}{16 - i^2} = \frac{12 - 3i - 4i - 1}{16 + 1} = \frac{11 - 7i}{17} = \frac{11}{17} - \frac{7}{17}i$

d $z = \frac{2 - 5i}{6i} = \frac{2 - 5i}{6i} \cdot \frac{i}{i} = \frac{2i - 5i^2}{6i^2} = \frac{2i + 5}{-6} = -\frac{5}{6} - \frac{1}{3}i$

$$\text{e } z = \overline{(2 - 3i)^2} = \overline{4 - 12i + 9i^2} = \overline{4 - 12i - 9} = \overline{-5 - 12i} = -5 + 12i$$

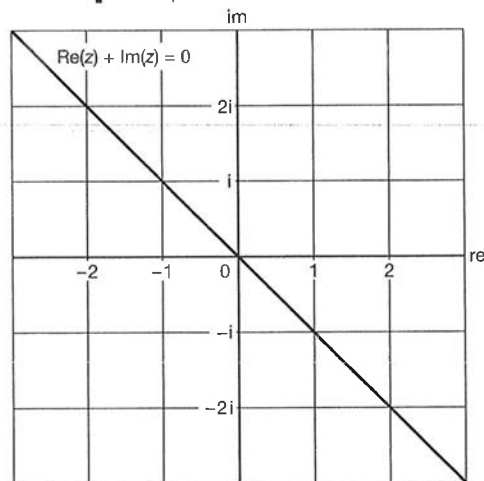
$$\text{f } z = \frac{\overline{2 - i}}{2 + 3i} = \frac{2 + i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i} = \frac{4 - 6i + 2i - 3i^2}{4 - 9i^2} = \frac{4 - 6i + 2i + 3}{4 + 9} = \frac{7 - 4i}{13} = \frac{7}{13} - \frac{4}{13}i$$

3 a



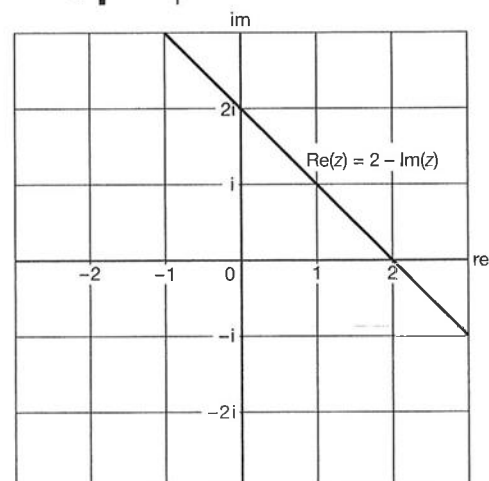
b

$$\begin{array}{c|c|c} \text{Re}(z) & 0 & 1 \\ \hline \text{Im}(z) & 0 & -1 \end{array}$$



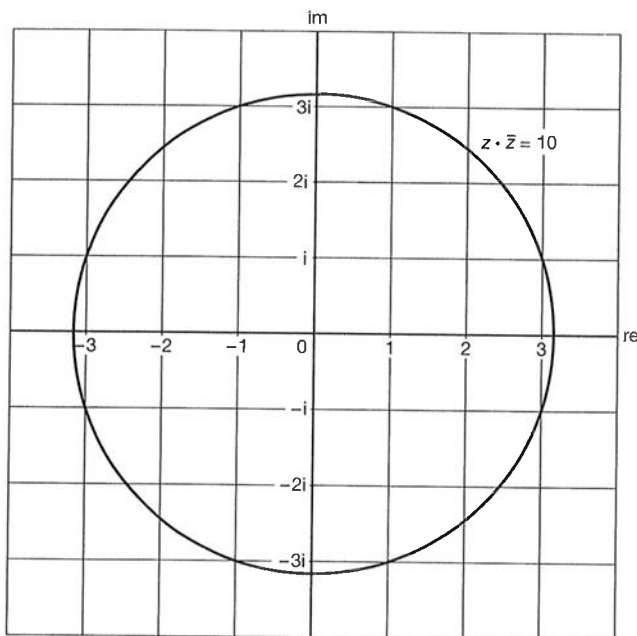
c

$$\begin{array}{c|c|c} \text{Re}(z) & 0 & 2 \\ \hline \text{Im}(z) & 2 & 0 \end{array}$$



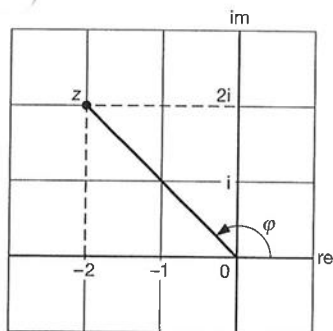
d $z \cdot \bar{z} = 10$ geeft $|z| = \sqrt{10}$

Dit is de cirkel met middelpunt 0 en straal $\sqrt{10}$.



4

a

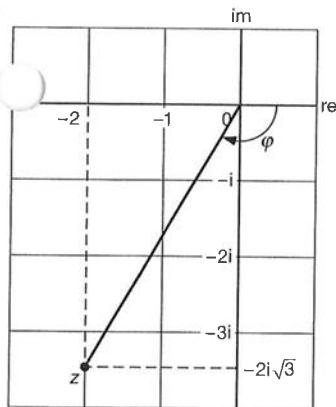


$$\tan(\varphi) = \frac{2}{-2} = -1 \text{ geeft } \varphi = \frac{3}{4}\pi$$

$$\text{Dus } \text{Arg}(-2 + 2i) = \frac{3}{4}\pi.$$

$$\text{De modulus is } |z| = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2}.$$

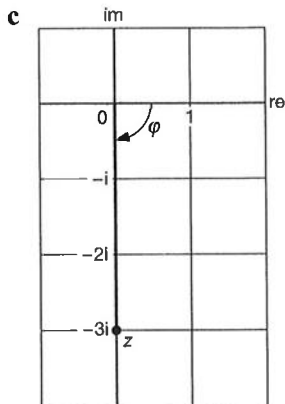
b



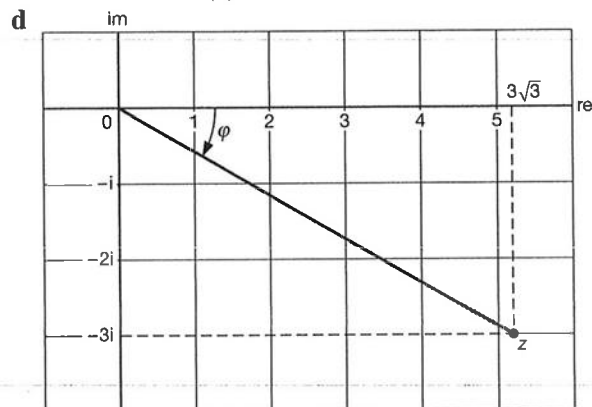
$$\tan(\varphi) = \frac{-2\sqrt{3}}{-2} = \sqrt{3} \text{ geeft } \varphi = -\frac{2}{3}\pi$$

$$\text{Dus } \text{Arg}(-2 - 2i\sqrt{3}) = -\frac{2}{3}\pi.$$

$$\text{De modulus is } |z| = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = \sqrt{16} = 4.$$



$\varphi = -\frac{1}{2}\pi$, dus $\text{Arg}(-3i) = -\frac{1}{2}\pi$.
De modulus is $|z| = 3$.

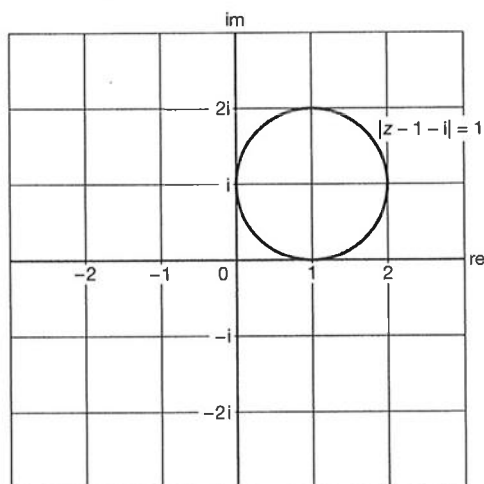


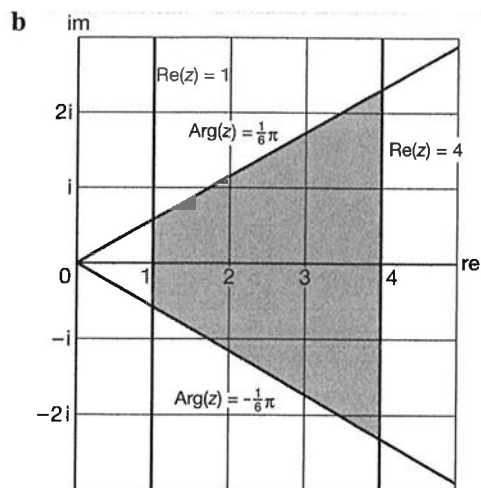
$$\tan(\varphi) = \frac{3}{-3\sqrt{3}} = -\frac{1}{3}\sqrt{3} \text{ geeft } \varphi = -\frac{1}{6}\pi$$

Dus $\text{Arg}(2\sqrt{3} - 3i) = -\frac{1}{6}\pi$.

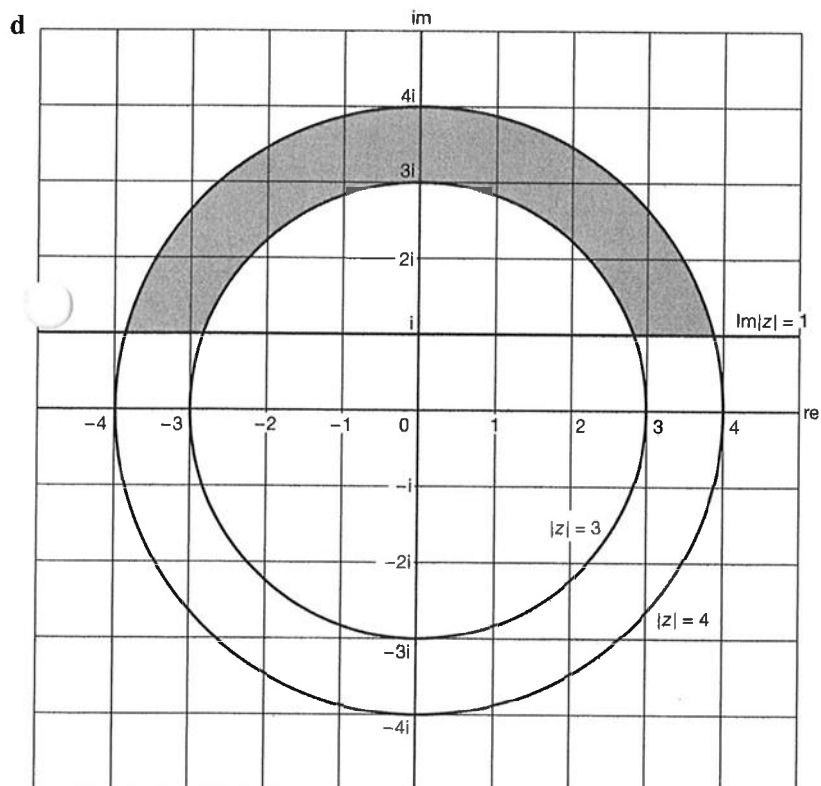
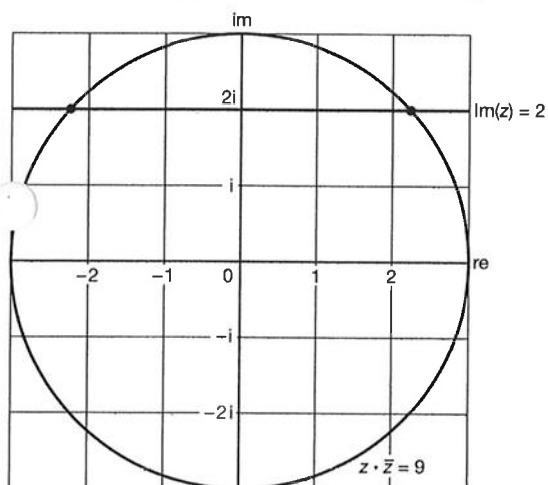
De modulus is $|z| = \sqrt{(3\sqrt{3})^2 + (-3)^2} = \sqrt{36} = 6$.

- 5 a** $|z - 1 - i| = 1$ is de cirkel met middelpunt $1 + i$ en straal 1.





c $z \cdot \bar{z} = 9 \wedge \text{Im}(z) = 2$ geeft $|z| = 3 \wedge \text{Im}(z) = 2$



- 6 a** $|z| = \sqrt{5^2 + (-12)^2} = 13$ en $\arg(z) \approx -1,18$
 Dus $z = 13(\cos(-1,18) + i \sin(-1,18))$.
- b** $|z| = \sqrt{(\sqrt{5})^2 + (-2)^2} = 3$ en $\arg(z) \approx -0,73$
 Dus $z = 3(\cos(-0,73) + i \sin(-0,73))$.
- c** $|z| = \sqrt{2^2 + 4^2} = 2\sqrt{5}$ en $\arg(z) \approx 1,11$
 Dus $z = 2\sqrt{5}(\cos(1,11) + i \sin(1,11))$.
- d** $|z| = \sqrt{(-\frac{1}{2})^2 + (-\sqrt{2})^2} = 1\frac{1}{2}$ en $\arg(z) \approx -1,91$
 Dus $z = 1\frac{1}{2}(\cos(-1,91) + i \sin(-1,91))$.
- 7 a** $z = 3(\cos(\frac{2}{3}\pi) + i \sin(\frac{2}{3}\pi)) = 3(-\frac{1}{2} + \frac{1}{2}i\sqrt{3}) = -1\frac{1}{2} + \frac{1}{2}i\sqrt{3}$
b $z = 2\sqrt{2}(\cos(\frac{1}{4}\pi) + i \sin(\frac{1}{4}\pi)) = 2\sqrt{2}(\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}) = 2 + 2i$
- 8 a** $z = 10e^{\frac{1}{2}\pi i} = 10(\cos(\frac{1}{2}\pi) + i \sin(\frac{1}{2}\pi)) = 10(\frac{1}{2} + \frac{1}{2}i\sqrt{3}) = 5 + 5i\sqrt{3}$
b $z = 6e^{\frac{1}{2}\pi i} = 6(\cos(\frac{1}{2}\pi) + i \sin(\frac{1}{2}\pi)) = 6(0 + i) = 6i$
c $z = \frac{3}{e^{-\frac{1}{6}\pi i}} = 3e^{\frac{1}{6}\pi i} = 3(\cos(\frac{1}{6}\pi) + i \sin(\frac{1}{6}\pi)) = 3(\frac{1}{2}\sqrt{3} + \frac{1}{2}i) = 1\frac{1}{2}\sqrt{3} + \frac{1}{2}i$
- 9 a** $|z| = \sqrt{3^2 + 3^2} = 3\sqrt{2}$ en $\arg(z) = \frac{1}{4}\pi$ geeft $z = 3\sqrt{2}e^{\frac{1}{4}\pi i}$
b $|z| = 2$ en $\arg(z) = \pi$ geeft $z = 2e^{\pi i}$
c $|z| = \sqrt{(\frac{1}{2}\sqrt{2})^2 + (-\frac{1}{2}\sqrt{2})^2} = 1$ en $\arg(z) = -\frac{1}{4}\pi$ geeft $z = e^{-\frac{1}{4}\pi i}$

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- 10 a** $3 - 3i\sqrt{3} = 6e^{-\frac{1}{3}\pi i}$ en $1 + i = \sqrt{2}e^{\frac{1}{4}\pi i}$, dus $z = (3 - 3i\sqrt{3})(1 + i) = 6e^{-\frac{1}{3}\pi i} \cdot \sqrt{2}e^{\frac{1}{4}\pi i} = 6\sqrt{2}e^{-\frac{1}{12}\pi i}$
b $1 + i = \sqrt{2}e^{\frac{1}{4}\pi i}$ en $1 - i = \sqrt{2}e^{-\frac{1}{4}\pi i}$, dus $z = \frac{(1 + i)^3}{(1 - i)^4} = \frac{(\sqrt{2}e^{\frac{1}{4}\pi i})^3}{(\sqrt{2}e^{-\frac{1}{4}\pi i})^4} = \frac{2\sqrt{2}e^{\frac{3}{4}\pi i}}{4e^{-\pi i}} = \frac{1}{2}\sqrt{2}e^{\frac{11}{4}\pi i} = \frac{1}{2}\sqrt{2}e^{-\frac{1}{4}\pi i}$
c $3 - i\sqrt{3} = 2\sqrt{3}e^{-\frac{1}{6}\pi i}$, dus $z = (3 - i\sqrt{3})^{-4} = (2\sqrt{3}e^{-\frac{1}{6}\pi i})^{-4} = \frac{1}{(2\sqrt{3}e^{-\frac{1}{6}\pi i})^4} = \frac{1}{144e^{-\frac{2}{3}\pi i}} = 144e^{\frac{2}{3}\pi i}$
d $e^{-\frac{1}{4}\pi i} = \cos(-\frac{1}{4}\pi) + i \sin(-\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2}$, dus $z = e^{-\frac{1}{4}\pi i} - \frac{1}{2}\sqrt{2} + i\sqrt{2} = \frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2} - \frac{1}{2}\sqrt{2} + i\sqrt{2} = \frac{1}{2}i\sqrt{2} = \frac{1}{2}\sqrt{2}e^{\frac{1}{2}\pi i}$
e $\sqrt{3} + i = 2e^{\frac{1}{6}\pi i}$, dus $z = (\sqrt{3} + i)^2 - 6e^{\frac{1}{3}\pi i} = (2e^{\frac{1}{6}\pi i})^2 - 6e^{\frac{1}{3}\pi i} = 4e^{\frac{1}{3}\pi i} - 6e^{\frac{1}{3}\pi i} = -2e^{\frac{1}{3}\pi i}$
f $2 + 2i = 2\sqrt{2}e^{\frac{1}{4}\pi i}$ en $1 - i = \sqrt{2}e^{-\frac{1}{4}\pi i}$, dus $z = \frac{(2 + 2i)^3}{1 - i} = \frac{(2\sqrt{2}e^{\frac{1}{4}\pi i})^3}{\sqrt{2}e^{-\frac{1}{4}\pi i}} = \frac{16\sqrt{2}e^{\frac{3}{4}\pi i}}{\sqrt{2}e^{-\frac{1}{4}\pi i}} = 16e^{\pi i}$
- 11** Substitutie van $u_n = g^n$ in $u_n = -2u_{n-1} - 2u_{n-2}$ geeft
 $g^2 + 2g + 2 = 0$
 $(g + 1)^2 - 1 + 2 = 0$
 $(g + 1)^2 = -1$
 $(g + 1)^2 = i^2$
 $g + 1 = i \vee g + 1 = -i$
 $g = -1 + i \vee g = -1 - i$
 $|-1 + i| = \sqrt{2}$ en $\arg(-1 + i) = \frac{3}{4}\pi$
 $u_n = (A \cos(\frac{3}{4}\pi n) + B \sin(\frac{3}{4}\pi n)) \cdot (\sqrt{2})^n$
 $u_0 = 2\sqrt{2}$ geeft $A = 2\sqrt{2}$
 $u_1 = 1$ geeft $(A \cdot \frac{1}{2}\sqrt{2} + B \cdot \frac{1}{2}\sqrt{2}) \cdot \sqrt{2} = 1$ ofwel $-A + B = 1$ } $A = 2\sqrt{2}$ en $B = 1 + 2\sqrt{2}$
 Dus $u_n = (2\sqrt{2} \cos(\frac{3}{4}\pi n) + (1 + 2\sqrt{2}) \sin(\frac{3}{4}\pi n)) \cdot (\sqrt{2})^n$.
- 12 a** $\arg(z^2) = \arg(-16i) = -\frac{1}{2}\pi$, dus één van de oplossingen heeft als argument $-\frac{1}{4}\pi$.
 $|z^2| = |-16i| = 16$ geeft $|z| = 4$, dus de straal van de cirkel is 4.
 De argumenten verschillen steeds $\frac{2\pi}{2} = \pi$ van elkaar.
 De oplossing is $z = 2\sqrt{2} - 2i\sqrt{2} \vee z = -2\sqrt{2} + 2i\sqrt{2}$.

- b** $\arg(z^3) = \arg(27i) = \frac{1}{2}\pi$, dus één van de oplossingen heeft als argument $\frac{1}{6}\pi$.
 $|z^3| = |27i| = 27$ geeft $|z| = 3$, dus de straal van de cirkel is 3.

De argumenten verschillen steeds $\frac{2\pi}{3} = \frac{2}{3}\pi$ van elkaar.

De oplossing is $z = 3\left(\frac{1}{2}\sqrt{3} + \frac{1}{2}i\right) = \frac{3}{2}\sqrt{3} + \frac{3}{2}i \vee z = 3\left(-\frac{1}{2}\sqrt{3} + \frac{1}{2}i\right) = -\frac{3}{2}\sqrt{3} + \frac{3}{2}i \vee z = 3(0 - i) = -3i$.

- c** $(z - 2i)^2 = -4i$
 $(z - 2i)^2 = 4e^{-\frac{1}{2}\pi i + k \cdot 2\pi i}$
 $z - 2i = 2e^{-\frac{1}{4}\pi i + k \cdot \pi i}$
 $z - 2i = 2e^{-\frac{1}{4}\pi i} \vee z - 2i = 2e^{\frac{3}{4}\pi i}$
 $z - 2i = 2\left(\frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2}\right) \vee z - 2i = 2\left(-\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}\right)$
 $z - 2i = \sqrt{2} - i\sqrt{2} \vee z - 2i = -\sqrt{2} + i\sqrt{2}$
 $z = \sqrt{2} + 2i - i\sqrt{2} \vee z = -\sqrt{2} + 2i + i\sqrt{2}$
 $z = \sqrt{2} + (2 - \sqrt{2})i \vee z = -\sqrt{2} + (2 + \sqrt{2})i$

- d** $z^2 + 4z + 2 = 2i\sqrt{3}$
 $(z + 2)^2 - 4 + 2 = 2i\sqrt{3}$
 $(z + 2)^2 = 2 + 2i\sqrt{3}$
 $(z + 2)^2 = 4e^{\frac{1}{3}\pi i + k \cdot 2\pi i}$
 $z + 2 = 2e^{\frac{1}{6}\pi i + k \cdot \pi i}$
 $z + 2 = 2e^{-\frac{5}{6}\pi i} \vee z + 2 = 2e^{\frac{1}{6}\pi i}$
 $z + 2 = 2\left(-\frac{1}{2}\sqrt{3} - \frac{1}{2}i\right) \vee z + 2 = 2\left(\frac{1}{2}\sqrt{3} + \frac{1}{2}i\right)$
 $z + 2 = -\sqrt{3} - i \vee z + 2 = \sqrt{3} + i$
 $z = -2 - \sqrt{3} - i \vee z = -2 + \sqrt{3} + i$

- 13 a** $\sqrt{i}$, dus los op $z^2 = i$
 $z^2 = e^{\frac{1}{2}\pi i + k \cdot 2\pi i}$
 $z = e^{\frac{1}{4}\pi i + k \cdot \pi i}$
 $z = e^{-\frac{3}{4}\pi i} \vee z = e^{\frac{1}{4}\pi i}$
Dus $\sqrt{i} = e^{-\frac{3}{4}\pi i} \vee \sqrt{i} = e^{\frac{1}{4}\pi i}$.

- b** $\sqrt[4]{-16}$, dus los op $z^4 = -16$
 $z^4 = 16e^{\pi i + k \cdot 2\pi i}$
 $z = 2e^{\frac{1}{4}\pi i + k \cdot \frac{1}{2}\pi i}$
 $z = 2e^{-\frac{3}{4}\pi i} \vee z = 2e^{-\frac{1}{4}\pi i} \vee z = 2e^{\frac{1}{4}\pi i} \vee z = 2e^{\frac{3}{4}\pi i}$
Dus $\sqrt[4]{-16} = 2e^{-\frac{3}{4}\pi i} \vee \sqrt[4]{-16} = 2e^{-\frac{1}{4}\pi i} \vee \sqrt[4]{-16} = 2e^{\frac{1}{4}\pi i} \vee \sqrt[4]{-16} = 2e^{\frac{3}{4}\pi i}$.

- c** $\sqrt[4]{-2 - 2i\sqrt{3}}$, dus los op $z^4 = -2 - 2i\sqrt{3}$
 $z^4 = 4e^{\frac{2}{3}\pi i + k \cdot 2\pi i}$
 $z = \sqrt{2}e^{\frac{1}{6}\pi i + k \cdot \frac{1}{2}\pi i}$
 $z = \sqrt{2}e^{-\frac{5}{6}\pi i} \vee z = \sqrt{2}e^{-\frac{1}{6}\pi i} \vee z = \sqrt{2}e^{\frac{1}{6}\pi i} \vee z = \sqrt{2}e^{\frac{5}{6}\pi i}$
Dus $\sqrt[4]{-2 - 2i\sqrt{3}} = \sqrt{2}e^{-\frac{5}{6}\pi i} \vee \sqrt[4]{-2 - 2i\sqrt{3}} = \sqrt{2}e^{-\frac{1}{6}\pi i} \vee \sqrt[4]{-2 - 2i\sqrt{3}} = \sqrt{2}e^{\frac{1}{6}\pi i} \vee \sqrt[4]{-2 - 2i\sqrt{3}} = \sqrt{2}e^{\frac{5}{6}\pi i}$.

- d** $\sqrt[3]{-\frac{1}{i}} = \sqrt[3]{\frac{1}{i}} = \sqrt[3]{\frac{i^2}{i}} = \sqrt[3]{i}$, dus los op $z^3 = i$
 $z^3 = e^{\frac{1}{2}\pi i + k \cdot 2\pi i}$
 $z = e^{\frac{1}{6}\pi i + k \cdot \frac{2}{3}\pi i}$
 $z = e^{\frac{1}{2}\pi i} \vee z = e^{\frac{1}{6}\pi i} \vee z = e^{\frac{5}{6}\pi i}$
Dus $\sqrt[3]{-\frac{1}{i}} = e^{-\frac{1}{2}\pi i} \vee \sqrt[3]{\frac{1}{i}} = e^{\frac{1}{6}\pi i} \vee \sqrt[3]{-\frac{1}{i}} = e^{\frac{5}{6}\pi i}$.

- 14** Bij de krachten $\vec{F}_1$, $\vec{F}_2$ en $\vec{F}_3$ horen de complexe getallen

$$z_1 = 1200(\cos(20^\circ) + i\sin(20^\circ))$$

$$z_2 = 400(\cos(-65^\circ) + i\sin(-65^\circ)) \text{ en}$$

$$z_3 = 1500(\cos(145^\circ) + i\sin(145^\circ)).$$

De GR geeft $|z_1 + z_2 + z_3| \approx 911$ en $\arg(z_1 + z_2 + z_3) \approx 86^\circ$.

De resultante is 911 N groot en maakt een hoek van 86° met de positieve reële as.

- 15 a** Bij de snelheid van het schip en de stroomsnelheid van het water horen de complexe getallen

$$z_1 = 8(\cos(50^\circ) + i \sin(50^\circ)) \text{ en}$$

$$z_2 = 3(\cos(135^\circ) + i \sin(135^\circ)).$$

De GR geeft $|z_1 + z_2| \approx 8,8$ en $\arg(z_1 + z_2) \approx 70^\circ$.

De werkelijke snelheid is ongeveer 8,8 knopen.

De koers is 20° .

- b** Stel de nieuwe snelheid ten opzichte van het water is v knopen.

$$z_1 = v(\cos(50^\circ) + i \sin(50^\circ))$$

$$z_2 = 3(\cos(135^\circ) + i \sin(135^\circ))$$

$$\operatorname{Re}(z_1 + z_2) = v \cdot \cos(50^\circ) + 3 \cos(135^\circ) = 0$$

$$v \cdot \cos(50^\circ) = -3 \cos(135^\circ)$$

$$v = \frac{-3 \cos(135^\circ)}{\cos(50^\circ)} \approx 3,3$$

Hij moet zijn snelheid met $8 - 3,3 = 4,7$ knopen verlagen.