12 Functies met e-machten en natuurlijke logaritmen

Voorkennis Machten, logaritmen en differentiequotiënten

Bladzijde 122

1 **a** $32 \cdot 2^x = 2^5 \cdot 2^x = 2^{5+x} = 2^{x+5}$

b $\frac{1}{8} \cdot 2^x = \frac{1}{2^3} \cdot 2^x = 2^{-3} \cdot 2^x = 2^{-3+x} = 2^{x-3}$

c $3 \cdot 3^x = 3^1 \cdot 3^x = 3^{1+x} = 3^{x+1}$

d $\frac{3^x}{27} = \frac{3^x}{3^3} = 3^{x-3}$

e $4\sqrt{2} \cdot 2^x = 2^2 \cdot 2^{\frac{1}{2}} \cdot 2^x = 2^{2+\frac{1}{2}+x} = 2^{2\frac{1}{2}+x} = 2^{x+2\frac{1}{2}}$

f $\frac{1}{3}\sqrt{3} \cdot 3^x = 3^{-1} \cdot 3^{\frac{1}{2}} \cdot 3^x = 3^{-1+\frac{1}{2}+x} = 3^{-\frac{1}{2}+x} = 3^{x-\frac{1}{2}}$

2 **a** $2^{x+5} = 2^x \cdot 2^5 = 2^x \cdot 32 = 32 \cdot 2^x$

b $2^{x-4} = 2^x \cdot 2^{-4} = 2^x \cdot \frac{1}{2^4} = 2^x \cdot \frac{1}{16} = \frac{1}{16} \cdot 2^x$

c $3^{x+2} = 3^x \cdot 3^2 = 3^x \cdot 9 = 9 \cdot 3^x$

d $3^{x-1} = 3^x \cdot 3^{-1} = 3^x \cdot \frac{1}{3} = \frac{1}{3} \cdot 3^x$

e $2^{x+\frac{1}{2}} = 2^x \cdot 2^{\frac{1}{2}} = 2^x \cdot \sqrt{2} = \sqrt{2} \cdot 2^x$

f $3^{x+1\frac{1}{2}} = 3^x \cdot 3^{1\frac{1}{2}} = 3^x \cdot 3\sqrt{3} = 3\sqrt{3} \cdot 3^x$

Bladzijde 123

3 **a** ${}^{2}\log(6) + {}^{2}\log(10) = {}^{2}\log(6 \cdot 10) = {}^{2}\log(60)$

b ${}^{3}\log(30) - {}^{3}\log(6) = {}^{3}\log\left(\frac{30}{6}\right) = {}^{3}\log(5)$

c $2 \cdot {}^{5}\log(3) + {}^{5}\log\left(\frac{1}{2}\right) = {}^{5}\log(3^2) + {}^{5}\log\left(\frac{1}{2}\right) = {}^{5}\log(9) + {}^{5}\log\left(\frac{1}{2}\right) = {}^{5}\log\left(9 \cdot \frac{1}{2}\right) = {}^{5}\log\left(4\frac{1}{2}\right)$

d ${}^{\frac{1}{2}}\log(15) - 4 \cdot {}^{\frac{1}{2}}\log(3) = {}^{\frac{1}{2}}\log(15) - {}^{\frac{1}{2}}\log(3^4) = {}^{\frac{1}{2}}\log(15) - {}^{\frac{1}{2}}\log(81) = {}^{\frac{1}{2}}\log\left(\frac{15}{81}\right) = {}^{\frac{1}{2}}\log\left(\frac{5}{27}\right)$

e $-2 \cdot {}^{4}\log(6) + {}^{4}\log(12) = {}^{4}\log(6^{-2}) + {}^{4}\log(12) = {}^{4}\log\left(\frac{1}{6^2}\right) + {}^{4}\log(12) = {}^{4}\log\left(\frac{1}{36}\right) + {}^{4}\log(12) =$
${}^{4}\log\left(\frac{1}{36} \cdot 12\right) = {}^{4}\log\left(\frac{12}{36}\right) = {}^{4}\log\left(\frac{1}{3}\right)$

f $\log(50) - 2 \cdot \log(5) = \log(50) - \log(5^2) = \log(50) - \log(25) = \log\left(\frac{50}{25}\right) = \log(2)$

4 **a** $4 + {}^{2}\log(3) = {}^{2}\log(2^4) + {}^{2}\log(3) = {}^{2}\log(16) + {}^{2}\log(3) = {}^{2}\log(16 \cdot 3) = {}^{2}\log(48)$

b $3 + {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log\left(\left(\frac{1}{2}\right)^3\right) + {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log\left(\frac{1}{8}\right) + {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log\left(\frac{1}{8} \cdot 10\right) = {}^{\frac{1}{2}}\log\left(\frac{10}{8}\right) = {}^{\frac{1}{2}}\log\left(1\frac{1}{4}\right)$

c $2 - {}^{3}\log(5) = {}^{3}\log(3^2) - {}^{3}\log(5) = {}^{3}\log(9) - {}^{3}\log(5) = {}^{3}\log\left(\frac{9}{5}\right) = {}^{3}\log\left(1\frac{4}{5}\right)$

d ${}^{2}\log(12) - {}^{3}\log(9) = {}^{2}\log(12) - {}^{3}\log(3^2) = {}^{2}\log(12) - 2 = {}^{2}\log(12) - {}^{2}\log(2^2) = {}^{2}\log(12) - {}^{2}\log(4) =$
${}^{2}\log\left(\frac{12}{4}\right) = {}^{2}\log(3)$

e $\frac{1}{2} \cdot {}^{3}\log(16) + {}^{\frac{1}{2}}\log(8) = {}^{3}\log\left(16^{\frac{1}{2}}\right) + {}^{\frac{1}{2}}\log(2^3) = {}^{3}\log\left(\sqrt{16}\right) + {}^{\frac{1}{2}}\log\left((2^{-1})^{-3}\right) = {}^{3}\log(4) + {}^{\frac{1}{2}}\log\left(\left(\frac{1}{2}\right)^{-3}\right) =$
${}^{3}\log(4) + -3 = {}^{3}\log(4) - 3 = {}^{3}\log(4) - {}^{3}\log(3^3) = {}^{3}\log(4) - {}^{3}\log(27) = {}^{3}\log\left(\frac{4}{27}\right)$

f $\log(500) - {}^{5}\log(125) = \log(500) - {}^{5}\log(5^3) = \log(500) - 3 = \log(500) - \log(10^3) =$
$\log(500) - \log(1000) = \log\left(\frac{500}{1000}\right) = \log\left(\frac{1}{2}\right)$

5 **a** $\log(900) = \log(100 \cdot 9) = \log(100) + \log(9) = 2 + \log(9)$

b ${}^{3}\log(900) = {}^{3}\log(9 \cdot 100) = {}^{3}\log(9) + {}^{3}\log(100) = 2 + {}^{3}\log(100)$

c ${}^{2}\log(48) = {}^{2}\log(16 \cdot 3) = {}^{2}\log(16) + {}^{2}\log(3) = 4 + {}^{2}\log(3)$

d ${}^{3}\log(405) = {}^{3}\log(81 \cdot 5) = {}^{3}\log(81) + {}^{3}\log(5) = 4 + {}^{3}\log(5)$

e ${}^{5}\log(75) = {}^{5}\log(25 \cdot 3) = {}^{5}\log(25) + {}^{5}\log(3) = 2 + {}^{5}\log(3)$

f ${}^{2}\log\left(\frac{3}{16}\right) = {}^{2}\log\left(\frac{1}{16} \cdot 3\right) = {}^{2}\log\left(\frac{1}{16}\right) + {}^{2}\log(3) = {}^{2}\log\left(\frac{1}{2^4}\right) + {}^{2}\log(3) = {}^{2}\log(2^{-4}) + {}^{2}\log(3) = -4 + {}^{2}\log(3)$

Bladzijde 124

6 Op $[1, 1+h]$ is $\frac{\Delta y}{\Delta x} = \frac{f(1+h) - f(1)}{h} = \frac{3^{1+h} - 3^1}{h} = \frac{3^{1+h} - 3}{h}$.

Dus $f'(1) \approx \frac{3^{1{,}001} - 3}{0{,}001} \approx 3{,}298$.

7 Op $[1, 1+h]$ is $\frac{\Delta y}{\Delta x} = \frac{f(1+h)-f(1)}{h} = \frac{\log(1+h)-\log(1)}{h} = \frac{\log(1+h)-0}{h} = \frac{\log(1+h)}{h}$.

Dus $f'(1) \approx \frac{\log(1{,}001)}{0{,}001} \approx 0{,}434$.

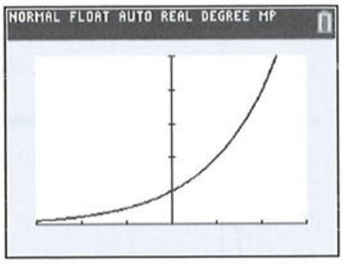
8 Op $[2, 2+h]$ is $\frac{\Delta y}{\Delta x} = \frac{f(2+h)-f(2)}{h} = \frac{(2+h)^{2+h}-2^2}{h} = \frac{(2+h)^{2+h}-4}{h}$.

Dus $f'(2) \approx \frac{(2{,}0001)^{2{,}0001}-4}{0{,}0001} \approx 6{,}773$.

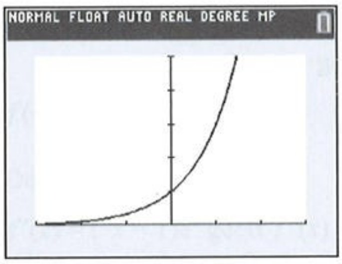
12.1 Het grondtal e

Bladzijde 125

1 a $2^{x+h} - 2^x = 2^x \cdot 2^h - 2^x = 2^x(2^h - 1)$
b $a^{x+h} - a^x = a^x \cdot a^h - a^x = a^x(a^h - 1)$

2 a


b De optie dy/dx (TI) of d/dx (Casio) met $y_1 = 2^x$ geeft $\left[\frac{dy}{dx}\right]_{x=0} \approx 0{,}693$.

c


De optie dy/dx (TI) of d/dx (Casio) met $y_1 = 3^x$ geeft $\left[\frac{dy}{dx}\right]_{x=0} \approx 1{,}099$.

d Uitproberen geeft $a \approx 2{,}72$.

Bladzijde 126

3 a

h	0,1	0,01	0,001	0,0001	0,00001	0,000001
a	2,5937	2,7048	2,7169	2,7181	2,7183	2,7183

b Voor $a \approx 2{,}7183$.

Bladzijde 127

4 a $2e^2 - e^2 = e^2$
b $4\sqrt{e} - \sqrt{e} = 3\sqrt{e}$
c $5e^2 \cdot 3e^3 = 15e^5$
d $\frac{12e^6}{4e^2} = 3e^4$
e $e^x \cdot e^2 = e^{x+2}$
f $e^{5x} \cdot e^x = e^{6x}$
g $(e^x + 1)^2 = e^{2x} + 2e^x + 1$
h $\frac{6e^{2x} - e^x}{e^x} = \frac{6e^{2x}}{e^x} - \frac{e^x}{e^x} = 6e^x - 1$

5 a $(2x-4)e^x = 0$
$2x - 4 = 0 \vee e^x = 0$
$2x = 4$ geen opl.
$x = 2$

b $(x^2 - 3x)e^x = 0$
$x^2 - 3x = 0 \vee e^x = 0$
$x(x-3) = 0$ geen opl.
$x = 0 \vee x = 3$

c $e^x \cdot e^x = e^6$
$e^{2x} = e^6$
$2x = 6$
$x = 3$

d $e^x + e^x = 2e^6$
$2e^x = 2e^6$
$e^x = e^6$
$x = 6$

e $x^2 e^x = e^x$
$x^2 e^x - e^x = 0$
$(x^2 - 1)e^x = 0$
$x^2 - 1 = 0 \vee e^x = 0$
$x^2 = 1$ geen opl.
$x = 1 \vee x = -1$

f $\frac{e^{5x}}{e^x} = e$
$e^{4x} = e^1$
$4x = 1$
$x = \frac{1}{4}$

6 **a** $e^{3x} - e^x = 0$
$(e^{2x} - 1)e^x = 0$
$e^{2x} - 1 = 0 \vee e^x = 0$
$e^{2x} = 1$ geen opl.
$2x = 0$
$x = 0$

b $2x e^x + e^x = 0$
$(2x + 1)e^x = 0$
$2x + 1 = 0 \vee e^x = 0$
$2x = -1$ geen opl.
$x = -\frac{1}{2}$

c $x^2 e^x - x e^x = 0$
$(x^2 - x)e^x = 0$
$x^2 - x = 0 \vee e^x = 0$
$x(x - 1) = 0$ geen opl.
$x = 0 \vee x = 1$

d $e^{x+2} - \sqrt{e} = 0$
$e^{x+2} = \sqrt{e}$
$e^{x+2} = e^{\frac{1}{2}}$
$x + 2 = \frac{1}{2}$
$x = -1\frac{1}{2}$

e $e^{4x} - 1 = 0$
$e^{4x} = 1$
$4x = 0$
$x = 0$

f $e^{2x-1} - e\sqrt{e} = 0$
$e^{2x-1} = e\sqrt{e}$
$e^{2x-1} = e^1 \cdot e^{\frac{1}{2}}$
$e^{2x-1} = e^{1\frac{1}{2}}$
$2x - 1 = 1\frac{1}{2}$
$2x = 2\frac{1}{2}$
$x = 1\frac{1}{4}$

7 $e^{2x} - (e^2 + e)e^x + e^3 = 0$
$(e^x - e^2)(e^x - e^1) = 0$
$e^x - e^2 = 0 \vee e^x - e^1 = 0$
$e^x = e^2 \vee e^x = e^1$
$x = 2 \vee x = 1$

Bladzijde 129

8 **a** $f(x) = 4e^{\frac{1}{2}x^2}$ geeft $f'(x) = 4e^{\frac{1}{2}x^2} \cdot x = 4xe^{\frac{1}{2}x^2}$
b $f(x) = 5x^3 - 3e^{x^2+1}$ geeft $f'(x) = 15x^2 - 3e^{x^2+1} \cdot 2x = 15x^2 - 6xe^{x^2+1}$
c $f(x) = \frac{e^x}{x+1}$ geeft $f'(x) = \frac{(x+1) \cdot e^x - e^x \cdot 1}{(x+1)^2} = \frac{xe^x + e^x - e^x}{(x+1)^2} = \frac{xe^x}{(x+1)^2}$
d $f(x) = 3x^2 e^{2x}$ geeft $f'(x) = 6x \cdot e^{2x} + 3x^2 \cdot e^{2x} \cdot 2 = 6x e^{2x} + 6x^2 e^{2x} = (6x + 6x^2)e^{2x}$

9 **a** $f(x) = e^x + 2$ geeft $f'(x) = e^x$
b $f(x) = 2e^x + x^2$ geeft $f'(x) = 2e^x + 2x$
c $f(x) = xe^x + 4$ geeft $f'(x) = 1 \cdot e^x + x \cdot e^x = e^x + xe^x = (1 + x)e^x$
d $f(x) = (2x - 4)e^x$ geeft $f'(x) = 2 \cdot e^x + (2x - 4) \cdot e^x = (2x - 2)e^x$

10 **a** $f(x) = \frac{e^x}{x^2}$ geeft $f'(x) = \frac{x^2 \cdot e^x - e^x \cdot 2x}{(x^2)^2} = \frac{x^2e^x - 2xe^x}{x^4} = \frac{xe^x - 2e^x}{x^3}$

b $f(x) = \frac{x^2 + 1}{e^x}$ geeft $f'(x) = \frac{e^x \cdot 2x - (x^2 + 1) \cdot e^x}{(e^x)^2} = \frac{2x - (x^2 + 1)}{e^x} = \frac{2x - x^2 - 1}{e^x}$

c $f(x) = \frac{e^x + 1}{e^x + 2}$ geeft $f'(x) = \frac{(e^x + 2) \cdot e^x - (e^x + 1) \cdot e^x}{(e^x + 2)^2} = \frac{e^{2x} + 2e^x - e^{2x} - e^x}{(e^x + 2)^2} = \frac{e^x}{(e^x + 2)^2}$

d $f(x) = \frac{3x - 2}{e^x}$ geeft $f'(x) = \frac{e^x \cdot 3 - (3x - 2) \cdot e^x}{(e^x)^2} = \frac{3e^x - 3xe^x + 2e^x}{(e^x)^2} = \frac{5e^x - 3xe^x}{(e^x)^2} = \frac{5 - 3x}{e^x}$

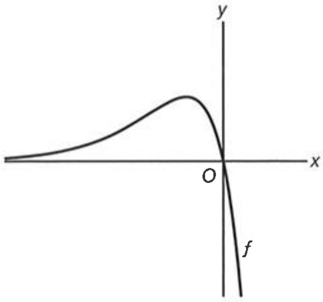
12

11 **a** $f(x) = \mathrm{e}^{2x^2-3x}$ geeft $f'(x) = \mathrm{e}^{2x^2-3x} \cdot (4x-3) = (4x-3)\mathrm{e}^{2x^2-3x}$
b $f(x) = x^2 + \mathrm{e}^{x^2}$ geeft $f'(x) = 2x + \mathrm{e}^{x^2} \cdot 2x = 2x + 2x\,\mathrm{e}^{x^2}$
c $f(x) = 2\,\mathrm{e}^{3x} + 3\mathrm{e}^{2x}$ geeft $f'(x) = 2\,\mathrm{e}^{3x} \cdot 3 + 3\,\mathrm{e}^{2x} \cdot 2 = 6\,\mathrm{e}^{3x} + 6\,\mathrm{e}^{2x}$
d $f(x) = 4\,\mathrm{e}^{\sqrt{x}}$ geeft $f'(x) = 4\,\mathrm{e}^{\sqrt{x}} \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{2\,\mathrm{e}^{\sqrt{x}}}{\sqrt{x}}$

12 **a** $f(x) = 4x^2\,\mathrm{e}^{2x}$ geeft $f'(x) = 8x \cdot \mathrm{e}^{2x} + 4x^2 \cdot \mathrm{e}^{2x} \cdot 2 = (8x + 8x^2)\mathrm{e}^{2x}$
b $f(x) = \dfrac{x^2-1}{\mathrm{e}^{2x}}$ geeft $f'(x) = \dfrac{\mathrm{e}^{2x} \cdot 2x - (x^2-1) \cdot \mathrm{e}^{2x} \cdot 2}{(\mathrm{e}^{2x})^2} = \dfrac{2x\mathrm{e}^{2x} - 2x^2\mathrm{e}^{2x} + 2\,\mathrm{e}^{2x}}{(\mathrm{e}^{2x})^2} = \dfrac{2x - 2x^2 + 2}{\mathrm{e}^{2x}}$
c $f(x) = 2x\,\mathrm{e}^{x^2} - \mathrm{e}^2$ geeft $f'(x) = 2 \cdot \mathrm{e}^{x^2} + 2x \cdot \mathrm{e}^{x^2} \cdot 2x = 2\,\mathrm{e}^{x^2} + 4x^2\,\mathrm{e}^{x^2}$
d $f(x) = \dfrac{\mathrm{e}^{3x+1}}{\mathrm{e}^{2x+1}} = \mathrm{e}^{3x+1-(2x+1)} = \mathrm{e}^{3x+1-2x-1} = \mathrm{e}^x$ geeft $f'(x) = \mathrm{e}^x$

Bladzijde 130

13 **a**



b $f(x) = -x\,\mathrm{e}^x$ geeft $f'(x) = -1 \cdot \mathrm{e}^x + -x \cdot \mathrm{e}^x = (-x-1)\mathrm{e}^x$
$f'(x) = 0$ geeft $(-x-1)\mathrm{e}^x = 0$
$-x - 1 = 0 \vee \mathrm{e}^x = 0$
$x = -1$ geen opl.
$f(-1) = \mathrm{e}^{-1} = \dfrac{1}{\mathrm{e}}$
De top is het punt $\left(-1, \dfrac{1}{\mathrm{e}}\right)$.

c $f'(x) = (-x-1)\mathrm{e}^x$ geeft $f''(x) = -1 \cdot \mathrm{e}^x + (-x-1) \cdot \mathrm{e}^x = (-x-2)\mathrm{e}^x$
$f''(x) = 0$ geeft $(-x-2)\mathrm{e}^x = 0$, dus $x = -2$.
Stel k: $y = ax + b$ met $a = f'(-2) = (2-1)\mathrm{e}^{-2} = \dfrac{1}{\mathrm{e}^2}$.

$$\left.\begin{aligned} &y = \frac{1}{\mathrm{e}^2}x + b \\ &f(-2) = 2\,\mathrm{e}^{-2} = \frac{2}{\mathrm{e}^2} \end{aligned}\right\} \begin{aligned} &\frac{1}{\mathrm{e}^2} \cdot -2 + b = \frac{2}{\mathrm{e}^2} \\ &\frac{-2}{\mathrm{e}^2} + b = \frac{2}{\mathrm{e}^2} \end{aligned}$$

$$b = \frac{4}{\mathrm{e}^2}$$

Dus k: $y = \dfrac{1}{\mathrm{e}^2}x + \dfrac{4}{\mathrm{e}^2}$.

14 **a** $f(x) = x^2\,\mathrm{e}^x$ geeft $f'(x) = 2x \cdot \mathrm{e}^x + x^2 \cdot \mathrm{e}^x = (2x + x^2)\mathrm{e}^x$
$f'(x) = 0$ geeft $(2x + x^2)\mathrm{e}^x = 0$
$2x + x^2 = 0 \vee \mathrm{e}^x = 0$
$x(2+x) = 0$ geen opl.
$x = 0 \vee x = -2$
max. is $f(-2) = 4\,\mathrm{e}^{-2} = \dfrac{4}{\mathrm{e}^2}$
min. is $f(0) = 0$

b $f(x) = g(x)$ geeft $x^2 e^x = e^x$
$x^2 e^x - e^x = 0$
$(x^2 - 1)e^x = 0$
$x^2 - 1 = 0 \vee e^x = 0$
$x^2 = 1$ geen opl.
$x = 1 \vee x = -1$
$g(-1) = e^{-1} = \dfrac{1}{e}$ en $g(1) = e$
Dus $A\left(-1, \dfrac{1}{e}\right)$ en $B(1, e)$.

15 **a** $h(4) = 5$ geeft $2(e^1 + e^{-1}) + c = 5$
$2\left(e + \dfrac{1}{e}\right) + c = 5$
$2e + \dfrac{2}{e} + c = 5$
$c = 5 - 2e - \dfrac{2}{e}$

b $h_T(x) = 5(e^{0,1x} + e^{-0,1x}) - 5{,}81 = 5e^{0,1x} + 5e^{-0,1x} - 5{,}81$ geeft
$h_T'(x) = 5e^{0,1x} \cdot 0{,}1 + 5e^{-0,1x} \cdot -0{,}1 = 0{,}5e^{0,1x} - 0{,}5e^{-0,1x}$
helling $= h_T'(4) = 0{,}5e^{0,4} - 0{,}5e^{-0,4} \approx 0{,}41$

c $h(0) = 0$ geeft $\dfrac{1}{2k}(e^0 + e^0 - e^{4k} - e^{-4k}) + 5 = 0$
$\dfrac{1}{2k}(1 + 1 - e^{4k} - e^{-4k}) + 5 = 0$
$\dfrac{1}{2k}(2 - e^{4k} - e^{-4k}) + 5 = 0$
Voer in $y_1 = \dfrac{1}{2x}(2 - e^{4x} - e^{-4x}) + 5$.
De optie zero (TI) of ROOT (Casio) geeft $x \approx 0{,}47$.
Dus $k = 0{,}47$.

Bladzijde 131

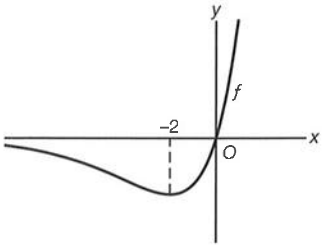
16 **a** $f(x) = 2x e^{-x^2}$ geeft $f'(x) = 2 \cdot e^{-x^2} + 2x \cdot e^{-x^2} \cdot -2x = 2e^{-x^2} - 4x^2 e^{-x^2} = (-4x^2 + 2)e^{-x^2}$

b $f'(x) = 0$ geeft $(-4x^2 + 2)e^{-x^2} = 0$
$-4x^2 + 2 = 0 \vee e^{-x^2} = 0$
$-4x^2 = -2$ geen opl.
$x^2 = \frac{1}{2}$
$x = \sqrt{\tfrac{1}{2}} \vee x = -\sqrt{\tfrac{1}{2}}$
Dus max. is $f\left(\sqrt{\tfrac{1}{2}}\right) = 2\sqrt{\tfrac{1}{2}} \cdot e^{-\frac{1}{2}}$.

c $2\sqrt{\tfrac{1}{2}} \cdot e^{-\frac{1}{2}} = \sqrt{4} \cdot \sqrt{\tfrac{1}{2}} \cdot \dfrac{1}{e^{\frac{1}{2}}} = \sqrt{2} \cdot \dfrac{1}{\sqrt{e}} = \dfrac{\sqrt{2}}{\sqrt{e}} = \sqrt{\dfrac{2}{e}}$

d min. is $f\left(-\sqrt{\tfrac{1}{2}}\right) = 2 \cdot -\sqrt{\tfrac{1}{2}} \cdot e^{-\frac{1}{2}} = -\sqrt{4} \cdot \sqrt{\tfrac{1}{2}} \cdot \dfrac{1}{e^{\frac{1}{2}}} = -\sqrt{2} \cdot \dfrac{1}{\sqrt{e}} = -\dfrac{\sqrt{2}}{\sqrt{e}} = -\sqrt{\dfrac{2}{e}}$

17 **a** $f(x) = 8x e^{\frac{1}{2}x}$ geeft $f'(x) = 8 \cdot e^{\frac{1}{2}x} + 8x \cdot e^{\frac{1}{2}x} \cdot \frac{1}{2} = (4x + 8)e^{\frac{1}{2}x}$
$f'(x) = 0$ geeft $(4x + 8)e^{\frac{1}{2}x} = 0$
$4x + 8 = 0 \vee e^{\frac{1}{2}x} = 0$
$4x = -8$ geen opl.
$x = -2$



min. is $f(-2) = -16e^{-1} = -\dfrac{16}{e}$

b $f'(x) = (4x + 8)e^{\frac{1}{2}x}$ geeft $f''(x) = 4 \cdot e^{\frac{1}{2}x} + (4x + 8) \cdot e^{\frac{1}{2}x} \cdot \frac{1}{2} = 4e^{\frac{1}{2}x} + (2x + 4)e^{\frac{1}{2}x} = (2x + 8)e^{\frac{1}{2}x}$

$f''(x) = 0$ geeft $(2x + 8)e^{\frac{1}{2}x} = 0$, dus $2x + 8 = 0$ ofwel $x = -4$.

Stel k: $y = ax + b$ met $a = f'(-4) = (-16 + 8)e^{-2} = -\frac{8}{e^2}$.

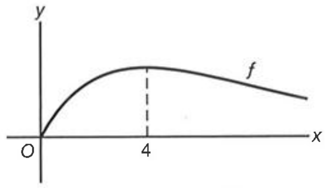
$$\left.\begin{array}{l} y = -\frac{8}{e^2}x + b \\ f(-4) = -32e^{-2} = -\frac{32}{e^2} \end{array}\right\} \begin{array}{l} -\frac{8}{e^2} \cdot -4 + b = -\frac{32}{e^2} \\ \frac{32}{e^2} + b = -\frac{32}{e^2} \\ b = -\frac{64}{e^2} \end{array}$$

Dus k: $y = -\frac{8}{e^2}x - \frac{64}{e^2}$.

18 $f(x) = x\,e^{-\sqrt{x}}$ geeft $f'(x) = 1 \cdot e^{-\sqrt{x}} + x \cdot e^{-\sqrt{x}} \cdot \frac{1}{-2\sqrt{x}} = e^{-\sqrt{x}} - \frac{x}{2\sqrt{x}}e^{-\sqrt{x}} = \left(1 - \frac{x}{2\sqrt{x}}\right)e^{-\sqrt{x}}$

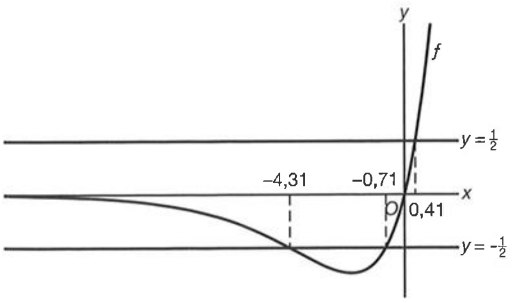
$f'(x) = 0$ geeft $\left(1 - \frac{x}{2\sqrt{x}}\right)e^{-\sqrt{x}} = 0$

$$\begin{array}{ll} 1 - \frac{x}{2\sqrt{x}} = 0 \vee e^{-\sqrt{x}} = 0 & \\ \frac{x}{2\sqrt{x}} = 1 & \text{geen opl.} \\ x = 2\sqrt{x} & \\ x^2 = \left(2\sqrt{x}\right)^2 & \\ x^2 = 4x & \\ x^2 - 4x = 0 & \\ x(x - 4) = 0 & \\ x = 0 \vee x = 4 & \\ \text{v.n.} \quad \text{vold.} & \end{array}$$



max. is $f(4) = 4e^{-\sqrt{4}} = 4e^{-2} = \frac{4}{e^2}$

19 **a** Voer in $y_1 = x\,e^{\frac{1}{2}x}$, $y_2 = -\frac{1}{2}$ en $y_3 = \frac{1}{2}$.
Intersect met y_1 en y_2 geeft $x \approx -4{,}31$ en $x \approx -0{,}71$.
Intersect met y_1 en y_3 geeft $x \approx 0{,}41$.



$-\frac{1}{2} < f(x) < \frac{1}{2}$ geeft $x < -4{,}31 \vee -0{,}71 < x < 0{,}41$

b $f(x) = g(x)$ geeft $x\,e^{\frac{1}{2}x} = x^2 e^{\frac{1}{2}x}$

$$\begin{array}{l} x\,e^{\frac{1}{2}x} - x^2 e^{\frac{1}{2}x} = 0 \\ (x - x^2)e^{\frac{1}{2}x} = 0 \\ x - x^2 = 0 \vee e^{\frac{1}{2}x} = 0 \\ x(1 - x) = 0 \quad \text{geen opl.} \\ x = 0 \vee x = 1 \end{array}$$

$f(0) = 0$ en $f(1) = 1 \cdot e^{\frac{1}{2}} = \sqrt{e}$

Dus de snijpunten zijn $(0, 0)$ en $\left(1, \sqrt{e}\right)$.

c $f(x) = x\,\mathrm{e}^{\frac{1}{2}x}$ geeft $f'(x) = 1 \cdot \mathrm{e}^{\frac{1}{2}x} + x \cdot \mathrm{e}^{\frac{1}{2}x} \cdot \frac{1}{2} = \left(\frac{1}{2}x + 1\right)\mathrm{e}^{\frac{1}{2}x}$

$f'(x) = \left(\frac{1}{2}x + 1\right)\mathrm{e}^{\frac{1}{2}x}$ geeft $f''(x) = \frac{1}{2} \cdot \mathrm{e}^{\frac{1}{2}x} + \left(\frac{1}{2}x + 1\right) \cdot \mathrm{e}^{\frac{1}{2}x} \cdot \frac{1}{2} = \frac{1}{2}\mathrm{e}^{\frac{1}{2}x} + \left(\frac{1}{4}x + \frac{1}{2}\right)\mathrm{e}^{\frac{1}{2}x} = \left(\frac{1}{4}x + 1\right)\mathrm{e}^{\frac{1}{2}x}$

$f''(x) = 0$ geeft $\left(\frac{1}{4}x + 1\right)\mathrm{e}^{\frac{1}{2}x} = 0$

$\frac{1}{4}x + 1 = 0 \vee \mathrm{e}^{\frac{1}{2}x} = 0$

$\frac{1}{4}x = -1$ geen opl.

$x = -4$

Stel k: $y = ax + b$ met $a = f'(-4) = (-2 + 1)\mathrm{e}^{-2} = -\frac{1}{\mathrm{e}^2}$.

$$\left.\begin{array}{l} y = -\frac{1}{\mathrm{e}^2}x + b \\ f(-4) = -4\,\mathrm{e}^{-2} = -\frac{4}{\mathrm{e}^2} \end{array}\right\} \begin{array}{l} -\frac{1}{\mathrm{e}^2} \cdot -4 + b = -\frac{4}{\mathrm{e}^2} \\ \frac{4}{\mathrm{e}^2} + b = -\frac{4}{\mathrm{e}^2} \\ b = -\frac{8}{\mathrm{e}^2} \end{array}$$

Dus k: $y = -\frac{1}{\mathrm{e}^2}x - \frac{8}{\mathrm{e}^2}$.

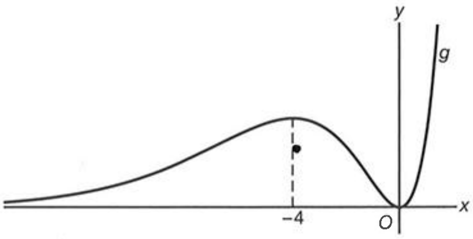
d $g(x) = x^2\mathrm{e}^{\frac{1}{2}x}$ geeft $g'(x) = 2x \cdot \mathrm{e}^{\frac{1}{2}x} + x^2 \cdot \mathrm{e}^{\frac{1}{2}x} \cdot \frac{1}{2} = \left(\frac{1}{2}x^2 + 2x\right)\mathrm{e}^{\frac{1}{2}x}$

$g'(x) = 0$ geeft $\left(\frac{1}{2}x^2 + 2x\right)\mathrm{e}^{\frac{1}{2}x} = 0$

$\frac{1}{2}x^2 + 2x = 0 \vee \mathrm{e}^{\frac{1}{2}x} = 0$

$\frac{1}{2}x(x + 4) = 0$ geen opl.

$x = 0 \vee x = -4$



max. is $g(-4) = 16\mathrm{e}^{-2} = \frac{16}{\mathrm{e}^2}$

min. is $g(0) = 0$

20 **a** $f_a(x) = x\mathrm{e}^{ax^3}$ geeft $f_a'(x) = 1 \cdot \mathrm{e}^{ax^3} + x \cdot \mathrm{e}^{ax^3} \cdot 3ax^2 = (3ax^3 + 1)\mathrm{e}^{ax^3}$

Een maximum voor $x = 1$, dus er geldt $f_a'(1) = 0$.

Dit geeft $(3a + 1)\mathrm{e}^a = 0$

$3a + 1 = 0 \vee \mathrm{e}^a = 0$

$3a = -1$ geen opl.

$a = -\frac{1}{3}$

Dus voor $a = -\frac{1}{3}$ heeft f_a een maximum.

b Een extreme waarde voor $x = 2$, dus er geldt $f_a'(2) = 0$.

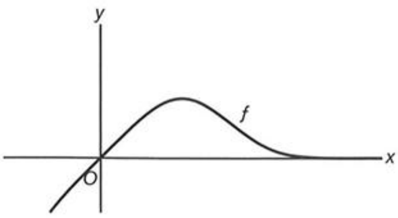
Dit geeft $(3a \cdot 8 + 1)\mathrm{e}^{8a} = 0$

$24a + 1 = 0 \vee \mathrm{e}^{8a} = 0$

$24a = -1$ geen opl.

$a = -\frac{1}{24}$

$a = -\frac{1}{24}$ geeft $f_{-\frac{1}{24}}(x) = x\,\mathrm{e}^{-\frac{1}{24}x^3}$



De extreme waarde voor $x = 2$ is een maximum.

12.2 De natuurlijke logaritme

Bladzijde 133

21
a $3^x = 20$ geeft $x = {}^3\log(20)$
b $e^x = 20$ geeft $x = {}^e\log(20)$

Bladzijde 134

22
a $\ln(e) = \ln(e^1) = 1$
b $\ln\left(e\sqrt{e}\right) = \ln\left(e^1 \cdot e^{\frac{1}{2}}\right) = \ln\left(e^{1\frac{1}{2}}\right) = 1\frac{1}{2}$
c $\ln\left(\dfrac{1}{e}\right) = \ln(e^{-1}) = -1$
d $\ln(1) = \ln(e^0) = 0$
e $3\ln\left(e \cdot \sqrt[3]{e}\right) = 3\ln\left(e^1 \cdot e^{\frac{1}{3}}\right) = 3\ln\left(e^{1\frac{1}{3}}\right) = 3 \cdot 1\frac{1}{3} = 4$
f $e^{\ln(7)} + e^{\ln(8)} = 7 + 8 = 15$
g $e^{2\ln(5)} = \left(e^{\ln(5)}\right)^2 = 5^2 = 25$
h $e^{\frac{1}{2}\ln(6)} = \left(e^{\ln(6)}\right)^{\frac{1}{2}} = 6^{\frac{1}{2}} = \sqrt{6}$

23
a $2\ln(3) + \ln(4) = \ln(3^2) + \ln(4) = \ln(9) + \ln(4) = \ln(9 \cdot 4) = \ln(36)$
b $\ln(20) - 3\ln(2) = \ln(20) - \ln(2^3) = \ln(20) - \ln(8) = \ln\left(\dfrac{20}{8}\right) = \ln\left(2\frac{1}{2}\right)$
c $-2\ln(4) + \ln(12) = \ln(4^{-2}) + \ln(12) = \ln\left(\frac{1}{4^2}\right) + \ln(12) = \ln\left(\frac{1}{16}\right) + \ln(12) = \ln\left(\frac{1}{16} \cdot 12\right) = \ln\left(\frac{12}{16}\right) = \ln\left(\frac{3}{4}\right)$
d $4 + \ln(3) = \ln(e^4) + \ln(3) = \ln(3e^4)$
e $5 - \ln(5) = \ln(e^5) - \ln(5) = \ln\left(\dfrac{e^5}{5}\right) = \ln\left(\frac{1}{5}e^5\right)$
f $1 + \ln(10) = \ln(e^1) + \ln(10) = \ln(10e)$
g $\frac{1}{2} + 2\ln(6) = \ln\left(e^{\frac{1}{2}}\right) + \ln(6^2) = \ln\left(\sqrt{e}\right) + \ln(36) = \ln\left(36\sqrt{e}\right)$
h $-3 + 4\ln\left(\frac{1}{2}\right) = \ln(e^{-3}) + \ln\left(\left(\frac{1}{2}\right)^4\right) = \ln\left(\dfrac{1}{e^3}\right) + \ln\left(\frac{1}{16}\right) = \ln\left(\dfrac{1}{e^3} \cdot \frac{1}{16}\right) = \ln\left(\dfrac{1}{16e^3}\right)$
i $e + \ln(2) = \ln(e^e) + \ln(2) = \ln(2e^e)$

24
a $\ln(x) = 2$
$x = e^2$
b $\ln(x) = -1$
$x = e^{-1} = \dfrac{1}{e}$
c $2\ln(x) = 5$
$\ln(x) = 2\frac{1}{2}$
$x = e^{2\frac{1}{2}} = e^2 \cdot e^{\frac{1}{2}} = e^2\sqrt{e}$
d $\ln(3x) = 3$
$3x = e^3$
$x = \frac{1}{3}e^3$
e $\ln(x + 1) = 2$
$x + 1 = e^2$
$x = e^2 - 1$
f $\ln^2(x) = 16$
$\ln(x) = 4 \vee \ln(x) = -4$
$x = e^4 \vee x = e^{-4}$
g $(2x - 5)\ln(x) = 0$
$2x - 5 = 0 \vee \ln(x) = 0$
$2x = 5 \vee x = e^0$
$x = 2\frac{1}{2} \vee x = 1$
h $x\ln(x + 2) = 0$
$x = 0 \vee \ln(x + 2) = 0$
$x = 0 \vee x + 2 = e^0$
$x = 0 \vee x + 2 = 1$
$x = 0 \vee x = -1$
i $x\ln(x) = 0$
$x = 0$ (vold. niet) $\vee \ln(x) = 0$
$x = e^0$
$x = 1$

25
a $e^x = 3$
$x = \ln(3)$
b $e^{3x} = 12$
$3x = \ln(12)$
$x = \frac{1}{3}\ln(12)$
c $5e^{2x} = 60$
$e^{2x} = 12$
$2x = \ln(12)$
$x = \frac{1}{2}\ln(12)$
d $4e^{1-x} = 20$
$e^{1-x} = 5$
$1 - x = \ln(5)$
$-x = -1 + \ln(5)$
$x = 1 - \ln(5)$
e $6 + e^{\frac{1}{2}x} = 10$
$e^{\frac{1}{2}x} = 4$
$\frac{1}{2}x = \ln(4)$
$x = 2\ln(4)$
f $\dfrac{3}{e^{2x}} = 10$
$10e^{2x} = 3$
$e^{2x} = \frac{3}{10}$
$2x = \ln\left(\frac{3}{10}\right)$
$x = \frac{1}{2}\ln\left(\frac{3}{10}\right)$

12

Bladzijde 135

26 a $\ln(x)(\ln(x) - 1) = 0$
$\ln(x) = 0 \lor \ln(x) - 1 = 0$
$\ln(x) = 0 \lor \ln(x) = 1$
$x = e^0 = 1 \lor x = e^1 = e$

b $2\ln(x) = \ln(\frac{1}{2}) - \ln(2)$
$2\ln(x) = \ln\left(\frac{\frac{1}{2}}{2}\right)$
$2\ln(x) = \ln(\frac{1}{4})$
$\ln(x) = \frac{1}{2}\ln(\frac{1}{4})$
$\ln(x) = \ln\left((\frac{1}{4})^{\frac{1}{2}}\right)$
$\ln(x) = \ln(\sqrt{\frac{1}{4}})$
$\ln(x) = \ln(\frac{1}{2})$
$x = \frac{1}{2}$

c $e^{x^2} = 100$
$x^2 = \ln(100)$
$x = \sqrt{\ln(100)} \lor x = -\sqrt{\ln(100)}$

d $5\,ex + 2 = \ln\left(\frac{1}{e^3}\right)$
$5\,ex + 2 = \ln(1) - \ln(e^3)$
$5\,ex + 2 = 0 - 3$
$5\,ex = -5$
$x = -\frac{1}{e}$

e $2\ln(x) + 3\ln(x) = 5 - \ln(32)$
$5\ln(x) = 5 - \ln(2^5)$
$5\ln(x) = 5 - 5\ln(2)$
$\ln(x) = 1 - \ln(2)$
$\ln(x) = \ln(e) - \ln(2)$
$\ln(x) = \ln\left(\frac{e}{2}\right)$
$x = \frac{e}{2} = \frac{1}{2}e$

f $\ln^2(x) - 2\ln(x) = 0$
$\ln(x)(\ln(x) - 2) = 0$
$\ln(x) = 0 \lor \ln(x) - 2 = 0$
$\ln(x) = 0 \lor \ln(x) = 2$
$x = e^0 = 1 \lor x = e^2$

27 a Voer in $y_1 = 140x^{0,75}$ en $y_2 = 11\,640\ln(x) - 20{,}4x - 44\,350$.
Intersect geeft de snijpunten (67, 3295) en (640, 17 806).
Bij een lichaamsgewicht van 67 kg en van 640 kg gebruikt een eland geen energie vóór het zoeken naar voedsel.
Dat betekent dat een eland die lichter is dan 67 kg niet zelf voor zijn voedsel kan zorgen. Hetzelfde geldt voor een eland die zwaarder is dan 640 kg.

b Voer in $y_1 = 11\,640\ln(x) - 20{,}4x - 44\,350 - 140x^{0,75}$.
De optie maximum geeft $x \approx 248$.
Het optimale gewicht is 248 kg.

c Op 1 juli weegt het kalf $11{,}2 + 30 \cdot 0{,}25 = 18{,}7$ kg.
Op 1 augustus weegt het kalf $18{,}7 + 31 \cdot 0{,}55 = 35{,}75$ kg.
Op 1 september weegt het kalf $35{,}75 + 31 \cdot 0{,}90 = 63{,}65$ kg.
Op 2 september weegt het kalf $63{,}65 + 1{,}2 = 64{,}85$ kg.
Op 3 september weegt het kalf $64{,}85 + 1{,}2 = 66{,}05$ kg.
Op 4 september weegt het kalf $66{,}05 + 1{,}2 = 67{,}25$ kg.
Dus op 4 september kan het kalf zelfstandig voor zijn voedsel zorgen.

28 a $2^x = (e^{\ln(2)})^x = e^{\ln(2)\cdot x} = e^{x\cdot\ln(2)}$

b $f(x) = 2^x = e^{x\cdot\ln(2)}$ geeft $f'(x) = e^{x\cdot\ln(2)} \cdot [x \cdot \ln(2)]' = e^{x\cdot\ln(2)} \cdot \ln(2) = 2^x \cdot \ln(2)$

Bladzijde 136

29 a $f(x) = 4^x$ geeft $f'(x) = 4^x \cdot \ln(4)$
b $f(x) = 5^{x+2}$ geeft $f'(x) = 5^{x+2} \cdot \ln(5)$
c $f(x) = 5 \cdot 6^x$ geeft $f'(x) = 5 \cdot 6^x \cdot \ln(6) = 5\ln(6) \cdot 6^x$
d $f(x) = 3 \cdot 10^{2x}$ geeft $f'(x) = 3 \cdot 10^{2x} \cdot \ln(10) \cdot 2 = 6\ln(10) \cdot 10^{2x}$
e $f(x) = (3x - 1) \cdot 2^x$ geeft $f'(x) = 3 \cdot 2^x + (3x - 1) \cdot 2^x \cdot \ln(2)$
f $f(x) = x^3 \cdot 3^x$ geeft $f'(x) = 3x^2 \cdot 3^x + x^3 \cdot 3^x \cdot \ln(3)$

30 Bij opgave 2b was $f'(0) \approx 0{,}693$ en $\ln(2) \approx 0{,}693$.
Bij opgave 2c was $f'(0) \approx 1{,}099$ en $\ln(3) \approx 1{,}099$.

31 a $f(x) = 2^{2x} - 2^x$ geeft $f'(x) = 2^{2x} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2) = 2^x \cdot \ln(2)(2 \cdot 2^x - 1)$
$f'(x) = 0$ geeft $2^x \cdot \ln(2)(2 \cdot 2^x - 1) = 0$
$2^x \cdot \ln(2) = 0 \lor 2 \cdot 2^x - 1 = 0$
geen opl. $\quad 2 \cdot 2^x = 1$
$2^x = \frac{1}{2}$
$2^x = 2^{-1}$
$x = -1$
min. is $f(-1) = 2^{-2} - 2^{-1} = \frac{1}{2^2} - \frac{1}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

b Stel k: $y = ax + b$ met $a = f'(1) = 2 \cdot \ln(2)(2 \cdot 2 - 1) = 2 \cdot \ln(2) \cdot 3 = 6\ln(2)$.

$\left.\begin{array}{l} y = 6\ln(2)x + b \\ f(1) = 2, \text{ dus } A(1, 2) \end{array}\right\} \begin{array}{l} 6\ln(2) \cdot 1 + b = 2 \\ b = 2 - 6\ln(2) \end{array}$

Dus k: $y = 6\ln(2)x + 2 - 6\ln(2)$.

Bladzijde 137

32 **a**

x	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	5	10
y_2	5	2,5	3	2	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{10}$

b Vermoeden: $f(x) = \ln(x)$ geeft $f'(x) = \frac{1}{x}$.

c $e^{\ln(x)} = x$ geeft $[e^{\ln(x)}]' = [x]'$

$$e^{\ln(x)} \cdot [\ln(x)]' = 1$$
$$x \cdot [\ln(x)]' = 1$$
$$[\ln(x)]' = \frac{1}{x}$$

33 **a** $g(x) = {}^2\log(x) = \dfrac{\ln(x)}{\ln(2)} = \dfrac{1}{\ln(2)} \cdot \ln(x)$ geeft $g'(x) = \dfrac{1}{\ln(2)} \cdot \dfrac{1}{x} = \dfrac{1}{x\ln(2)}$

b $h(x) = {}^3\log(x) = \dfrac{\ln(x)}{\ln(3)} = \dfrac{1}{\ln(3)} \cdot \ln(x)$ geeft $h'(x) = \dfrac{1}{\ln(3)} \cdot \dfrac{1}{x} = \dfrac{1}{x\ln(3)}$

Bladzijde 138

34 Manier I: $f(x) = \ln(2x)$ geeft $f'(x) = \dfrac{1}{2x} \cdot 2 = \dfrac{1}{x}$.

Manier II: $f(x) = \ln(2x) = \ln(2) + \ln(x)$ geeft $f'(x) = \dfrac{1}{x}$.

Kies zelf je eigen voorkeur.

35 **a** $f(x) = \ln(3x)$ geeft $f'(x) = \dfrac{1}{3x} \cdot 3 = \dfrac{1}{x}$

b $f(x) = \ln\left(\dfrac{1}{x}\right) = \ln(x^{-1}) = -\ln(x)$ geeft $f'(x) = -\dfrac{1}{x}$

c $f(x) = \ln(\sqrt{x}) = \ln(x^{\frac{1}{2}}) = \frac{1}{2}\ln(x)$ geeft $f'(x) = \frac{1}{2} \cdot \dfrac{1}{x} = \dfrac{1}{2x}$

d $f(x) = \ln(x^3) = 3\ln(x)$ geeft $f'(x) = 3 \cdot \dfrac{1}{x} = \dfrac{3}{x}$

e $f(x) = \ln\left(\dfrac{1}{x^3}\right) = \ln(x^{-3}) = -3\ln(x)$ geeft $f'(x) = -3 \cdot \dfrac{1}{x} = -\dfrac{3}{x}$

f $f(x) = \ln(2x - 5)$ geeft $f'(x) = \dfrac{1}{2x - 5} \cdot 2 = \dfrac{2}{2x - 5}$

36 **a** $f(x) = {}^2\log(3x)$ geeft $f'(x) = \dfrac{1}{3x\ln(2)} \cdot 3 = \dfrac{1}{x\ln(2)}$

b $f(x) = {}^3\log(4x)$ geeft $f'(x) = \dfrac{1}{4x\ln(3)} \cdot 4 = \dfrac{1}{x\ln(3)}$

c $f(x) = {}^2\log\left(\dfrac{1}{x}\right) = {}^2\log(x^{-1}) = -{}^2\log(x)$ geeft $f'(x) = -\dfrac{1}{x\ln(2)}$

d $f(x) = {}^{\frac{1}{2}}\log(x^5) = 5 \cdot {}^{\frac{1}{2}}\log(x)$ geeft $f'(x) = 5 \cdot \dfrac{1}{x\ln(\frac{1}{2})} = \dfrac{1}{x\ln(\frac{1}{2})}$

e $f(x) = \log(5x - 6)$ geeft $f'(x) = \dfrac{1}{(5x - 6)\ln(10)} \cdot 5 = \dfrac{5}{(5x - 6)\ln(10)}$

f $f(x) = \log(2x) + \log(3x)$ geeft $f'(x) = \dfrac{1}{2x\ln(10)} \cdot 2 + \dfrac{1}{3x\ln(10)} \cdot 3 = \dfrac{1}{x\ln(10)} + \dfrac{1}{x\ln(10)} = \dfrac{2}{x\ln(10)}$

12

37 **a** $f(x) = x^3 \cdot \ln(x)$ geeft $f'(x) = 3x^2 \cdot \ln(x) + x^3 \cdot \frac{1}{x} = 3x^2 \ln(x) + x^2$

b $f(x) = \frac{\ln(x)}{x}$ geeft $f'(x) = \frac{x \cdot \frac{1}{x} - \ln(x) \cdot 1}{x^2} = \frac{1 - \ln(x)}{x^2}$

c $f(x) = e^x \cdot \ln(5x)$ geeft $f'(x) = e^x \cdot \ln(5x) + e^x \cdot \frac{1}{5x} \cdot 5 = e^x \cdot \ln(5x) + e^x \cdot \frac{1}{x} = \left(\ln(5x) + \frac{1}{x}\right) e^x$

d $f(x) = \frac{x^2}{\ln(x)}$ geeft $f'(x) = \frac{\ln(x) \cdot 2x - x^2 \cdot \frac{1}{x}}{(\ln(x))^2} = \frac{2x\ln(x) - x}{\ln^2(x)}$

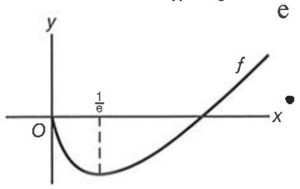
e $f(x) = x\ln\left(\sqrt{x}\right) = x\ln\left(x^{\frac{1}{2}}\right) = \frac{1}{2}x\ln(x)$ geeft $f'(x) = \frac{1}{2} \cdot \ln(x) + \frac{1}{2}x \cdot \frac{1}{x} = \frac{1}{2}\ln(x) + \frac{1}{2}$

f $f(x) = \frac{\ln(x) + 2}{\ln(x) + 1}$ geeft $f'(x) = \frac{(\ln(x) + 1) \cdot \frac{1}{x} - (\ln(x) + 2) \cdot \frac{1}{x}}{(\ln(x) + 1)^2} \cdot \frac{x}{x} = \frac{\ln(x) + 1 - (\ln(x) + 2)}{x(\ln(x) + 1)^2} = \frac{-1}{x(\ln(x) + 1)^2}$

38 **a** $f(x) = \ln(e^x) = x\ln(e) = x \cdot 1 = x$ geeft $f'(x) = 1$
b $f(x) = {}^g\log(g^x) = x$ geeft $f'(x) = 1$
c $f(x) = e^{\ln(x)} = x$ geeft $f'(x) = 1$ $(x > 0)$
d $f(x) = g^{{}^g\log(x)} = x$ geeft $f'(x) = 1$ $(x > 0)$

Bladzijde 139

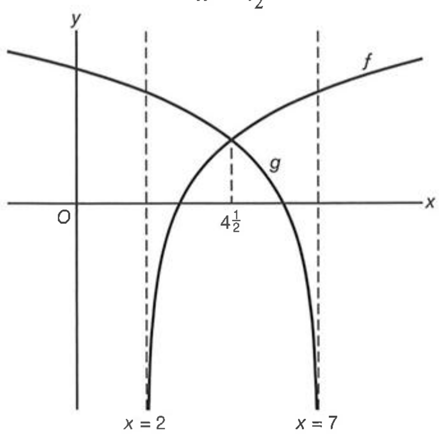
39 **a** $f(x) = x\ln(x)$ geeft $f'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1$
$f'(x) = 0$ geeft $\ln(x) + 1 = 0$
$\ln(x) = -1$
$x = e^{-1} = \frac{1}{e}$



min. is $f\left(\frac{1}{e}\right) = \frac{1}{e} \cdot \ln\left(\frac{1}{e}\right) = \frac{1}{e} \cdot \ln(e^{-1}) = \frac{1}{e} \cdot -1 = -\frac{1}{e}$

b Stel k: $y = ax + b$ met $a = f'(1) = \ln(1) + 1 = 0 + 1 = 1$.
$$\left.\begin{array}{l} y = x + b \\ f(1) = 0, \text{ dus } A(1, 0) \end{array}\right\} \begin{array}{l} 1 + b = 0 \\ b = -1 \end{array}$$
Dus k: $y = x - 1$.

40 **a** $x - 2 > 0$ geeft $x > 2$, dus $D_f = \langle 2, \rightarrow\rangle$.
$7 - x > 0$ geeft $x < 7$, dus $D_g = \langle \leftarrow, 7\rangle$.
b $f(x) = g(x)$ geeft $\ln(x - 2) = \ln(7 - x)$
$x - 2 = 7 - x$
$2x = 9$
$x = 4\frac{1}{2}$



$f(x) \leq g(x)$ geeft $2 < x \leq 4\frac{1}{2}$

c $s(x) = f(x) + g(x) = \ln(x-2) + \ln(7-x)$ geeft $s'(x) = \frac{1}{x-2} + \frac{1}{7-x} \cdot -1 = \frac{1}{x-2} - \frac{1}{7-x}$

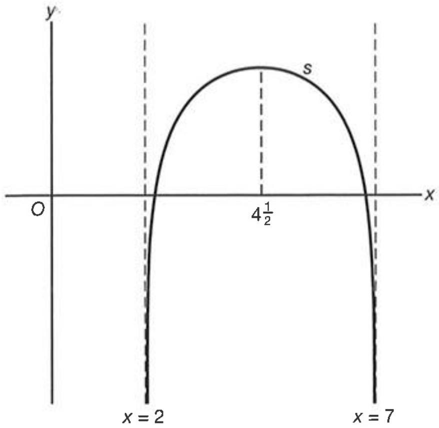
$s'(x) = 0$ geeft $\frac{1}{x-2} - \frac{1}{7-x} = 0$

$\frac{1}{x-2} = \frac{1}{7-x}$

$x - 2 = 7 - x$

$2x = 9$

$x = 4\frac{1}{2}$ vold.



max. is $s\left(4\frac{1}{2}\right) = \ln\left(2\frac{1}{2}\right) + \ln\left(2\frac{1}{2}\right) = 2\ln\left(2\frac{1}{2}\right)$

41 **a** $3 - x > 0 \ \wedge \ x + 2 > 0$

$-x > -3 \ \wedge \ x > -2$

$x < 3 \ \wedge \ x > -2$

Dus $D_f = \langle -2, 3 \rangle$.

b $f(x) = 2\ln(3-x) + \ln(x+2)$ geeft $f'(x) = 2 \cdot \frac{1}{3-x} \cdot -1 + \frac{1}{x+2} = \frac{1}{x+2} - \frac{2}{3-x}$

$f'(x) = 0$ geeft $\frac{1}{x+2} - \frac{2}{3-x} = 0$

$\frac{1}{x+2} = \frac{2}{3-x}$

$2(x+2) = 3 - x$

$2x + 4 = 3 - x$

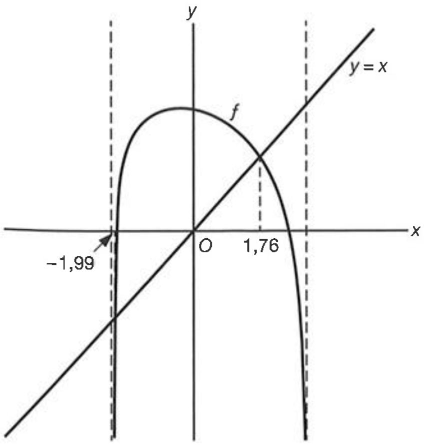
$3x = -1$

$x = -\frac{1}{3}$ vold.

$f\left(-\frac{1}{3}\right) = 2\ln\left(3\frac{1}{3}\right) + \ln\left(1\frac{2}{3}\right) = 2\ln\left(\frac{10}{3}\right) + \ln\left(\frac{5}{3}\right) = \ln\left(\left(\frac{10}{3}\right)^2\right) + \ln\left(\frac{5}{3}\right) = \ln\left(\frac{100}{9}\right) + \ln\left(\frac{5}{3}\right) = \ln\left(\frac{100}{9} \cdot \frac{5}{3}\right) = \ln\left(\frac{500}{27}\right) = \ln\left(18\frac{14}{27}\right)$

De top van de grafiek is $\left(-\frac{1}{3}, \ln\left(18\frac{14}{27}\right)\right)$.

c Voer in $y_1 = 2\ln(3-x) + \ln(x+2)$ en $y_2 = x$.
Intersect geeft $x \approx -1{,}99$ en $x \approx 1{,}76$.



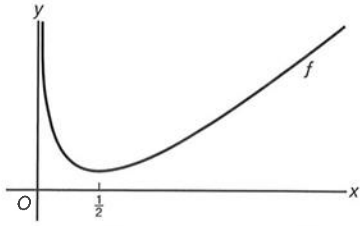
$f(x) > x$ geeft $-1{,}99 < x < 1{,}76$

42 **a** $f(x) = 2x - \ln(4x)$ geeft $f'(x) = 2 - \frac{1}{4x} \cdot 4 = 2 - \frac{1}{x}$

$f'(x) = 0$ geeft $2 - \frac{1}{x} = 0$

$\frac{1}{x} = 2$

$x = \frac{1}{2}$



min. is $f(\frac{1}{2}) = 2 \cdot \frac{1}{2} - \ln(4 \cdot \frac{1}{2}) = 1 - \ln(2) = \ln(\text{e}) - \ln(2) = \ln\left(\frac{\text{e}}{2}\right) = \ln(\frac{1}{2}\text{e})$

b $f'(x) = \frac{1}{2}$ geeft $2 - \frac{1}{x} = \frac{1}{2}$

$\frac{1}{x} = 1\frac{1}{2}$

$\frac{1}{x} = \frac{3}{2}$

$x = \frac{2}{3}$

$f(\frac{2}{3}) = 2 \cdot \frac{2}{3} - \ln(4 \cdot \frac{2}{3}) = 1\frac{1}{3} - \ln(2\frac{2}{3})$

Dus $A(\frac{2}{3}, 1\frac{1}{3} - \ln(2\frac{2}{3}))$.

c $f'(x) = 2$ geeft $2 - \frac{1}{x} = 2$

$\frac{1}{x} = 0$

geen opl.

De grafiek heeft geen raaklijn met richtingscoëfficiënt 2, want de vergelijking $f'(x) = 2$ heeft geen oplossing.

43 **a** $f(x) = \frac{x}{\ln(x)}$ geeft $f'(x) = \frac{\ln(x) \cdot 1 - x \cdot \frac{1}{x}}{(\ln(x))^2} = \frac{\ln(x) - 1}{\ln^2(x)}$

Stel k: $y = ax + b$ met $a = \frac{\ln\left(\frac{1}{\text{e}}\right) - 1}{\ln^2\left(\frac{1}{\text{e}}\right)} = \frac{\ln(\text{e}^{-1}) - 1}{\ln^2(\text{e}^{-1})} = \frac{-1 - 1}{(-1)^2} = -2.$

$$\left.\begin{array}{l} y = -2x + b \\ f\left(\frac{1}{\text{e}}\right) = -\frac{1}{\text{e}}, \text{ dus } A\left(\frac{1}{\text{e}}, -\frac{1}{\text{e}}\right) \end{array}\right\} -2 \cdot \frac{1}{\text{e}} + b = -\frac{1}{\text{e}}$$

$-\frac{2}{\text{e}} + b = -\frac{1}{\text{e}}$

$b = \frac{1}{\text{e}}$

Dus k: $y = -2x + \frac{1}{\text{e}}$.

b $f'(x) = -6$ geeft $\frac{\ln(x) - 1}{\ln^2(x)} = -6$

$\ln(x) - 1 = -6\ln^2(x)$

$6\ln^2(x) + \ln(x) - 1 = 0$

$D = 1^2 - 4 \cdot 6 \cdot -1 = 25$

$\ln(x) = \frac{-1 - 5}{12} = -\frac{1}{2} \vee \ln(x) = \frac{-1 + 5}{12} = \frac{1}{3}$

$x = \text{e}^{-\frac{1}{2}} = \frac{1}{\text{e}^{\frac{1}{2}}} = \frac{1}{\sqrt{\text{e}}} \vee x = \text{e}^{\frac{1}{3}} = \sqrt[3]{\text{e}}$

Dus de x-coördinaten van de raakpunten zijn $\frac{1}{\sqrt{\text{e}}}$ en $\sqrt[3]{\text{e}}$.

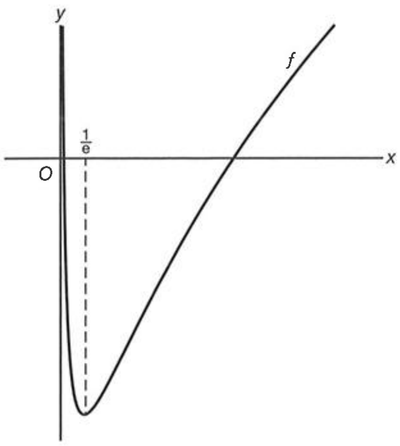
12

44 **a** $f(x) = 0$ geeft $\ln^2(x) + 2\ln(x) - 3 = 0$
$(\ln(x) - 1)(\ln(x) + 3) = 0$
$\ln(x) - 1 = 0 \vee \ln(x) + 3 = 0$
$\ln(x) = 1 \vee \ln(x) = -3$
$x = e^1 = e \vee x = e^{-3} = \frac{1}{e^3}$

De nulpunten zijn $\frac{1}{e^3}$ en e.

b $f(x) = \ln^2(x) + 2\ln(x) - 3 = (\ln(x))^2 + 2\ln(x) - 3$ geeft $f'(x) = 2\ln(x) \cdot \frac{1}{x} + 2 \cdot \frac{1}{x} = \frac{2\ln(x)}{x} + \frac{2}{x} = \frac{2\ln(x) + 2}{x}$

$f'(x) = 0$ geeft $2\ln(x) + 2 = 0$
$2\ln(x) = -2$
$\ln(x) = -1$
$x = e^{-1} = \frac{1}{e}$ vold.



min. is $f\left(\frac{1}{e}\right) = \ln^2\left(\frac{1}{e}\right) + 2\ln\left(\frac{1}{e}\right) - 3 = \ln^2(e^{-1}) + 2\ln(e^{-1}) - 3 = (-1)^2 + 2 \cdot -1 - 3 = 1 - 2 - 3 = -4$

c $f'(x) = \frac{2\ln(x) + 2}{x}$ geeft $f''(x) = \frac{x \cdot 2 \cdot \frac{1}{x} - (2\ln(x) + 2) \cdot 1}{x^2} = \frac{2 - 2\ln(x) - 2}{x^2} = \frac{-2\ln(x)}{x^2}$

$f''(x) = 0$ geeft $-2\ln(x) = 0$
$\ln(x) = 0$
$x = e^0 = 1$ vold.

Stel k: $y = ax + b$ met $a = f'(1) = \frac{2\ln(1) + 2}{1} = \frac{2 \cdot 0 + 2}{1} = 2$.

$\left.\begin{array}{l} y = 2x + b \\ f(1) = -3 \end{array}\right\} 2 \cdot 1 + b = -3$
$2 + b = -3$
$b = -5$

Dus k: $y = 2x - 5$.

12.3 Formules met e-machten en natuurlijke logaritmen

Bladzijde 141

45 **a** $y = 20x^3$
$\ln(y) = \ln(20x^3)$
$\ln(y) = \ln(20) + \ln(x^3)$
$\ln(y) = \ln(20) + 3\ln(x)$

b $N = 250 \cdot 1{,}18^t$
$\ln(N) = \ln(250 \cdot 1{,}18^t)$
$\ln(N) = \ln(250) + \ln(1{,}18^t)$
$\ln(N) = \ln(250) + t \cdot \ln(1{,}18)$

Bladzijde 142

46 **a** $y = 250x^{1,85}$
$\ln(y) = \ln(250x^{1,85})$
$\ln(y) = \ln(250) + \ln(x^{1,85})$
$\ln(y) = \ln(250) + 1{,}85\ln(x)$
$\ln(y) \approx 5{,}52 + 1{,}85\ln(x)$
Dus $\ln(y) = 5{,}52 + 1{,}85\ln(x)$.

b $y = \dfrac{2000}{x^3}$
$\ln(y) = \ln\left(\dfrac{2000}{x^3}\right)$
$\ln(y) = \ln(2000) - \ln(x^3)$
$\ln(y) = \ln(2000) - 3\ln(x)$
$\ln(y) \approx 7{,}60 - 3\ln(x)$
Dus $\ln(y) = 7{,}60 - 3\ln(x)$.

c $y = 80x^{-2,1}$
$\ln(y) = \ln(80x^{-2,1})$
$\ln(y) = \ln(80) + \ln(x^{-2,1})$
$\ln(y) = \ln(80) - 2{,}1\ln(x)$
$\ln(y) \approx 4{,}38 - 2{,}1\ln(x)$
Dus $\ln(y) = 4{,}38 - 2{,}1\ln(x)$.

d $y = \dfrac{300}{x^2 \cdot \sqrt{x}} = \dfrac{300}{x^2 \cdot x^{\frac{1}{2}}} = \dfrac{300}{x^{2\frac{1}{2}}}$
$\ln(y) = \ln\left(\dfrac{300}{x^{2\frac{1}{2}}}\right)$
$\ln(y) = \ln(300) - \ln\left(x^{2\frac{1}{2}}\right)$
$\ln(y) = \ln(300) - 2\tfrac{1}{2}\ln(x)$
$\ln(y) \approx 5{,}70 - 2\tfrac{1}{2}\ln(x)$
Dus $\ln(y) = 5{,}70 - 2\tfrac{1}{2}\ln(x)$.

47 **a** $N = 6000 \cdot 1{,}25^t$
$\ln(N) = \ln(6000 \cdot 1{,}25^t)$
$\ln(N) = \ln(6000) + \ln(1{,}25^t)$
$\ln(N) = \ln(6000) + t\ln(1{,}25)$
$\ln(N) \approx 0{,}223t + 8{,}70$
Dus $\ln(N) = 0{,}223t + 8{,}70$.

b $N = 18000 \cdot 0{,}85^t$
$\ln(N) = \ln(18\,000 \cdot 0{,}85^t)$
$\ln(N) = \ln(18\,000) + \ln(0{,}85^t)$
$\ln(N) = \ln(18\,000) + t\ln(0{,}85)$
$\ln(N) \approx -0{,}163t + 9{,}80$
Dus $\ln(N) = -0{,}163t + 9{,}80$.

c $N = 300\,e^{t-1}$
$\ln(N) = \ln(300\,e^{t-1})$
$\ln(N) = \ln(300) + \ln(e^{t-1})$
$\ln(N) = \ln(300) + t - 1$
$\ln(N) \approx t + 4{,}70$
Dus $\ln(N) = t + 4{,}70$.

d $N = 12\,e^{2t+1}$
$\ln(N) = \ln(12\,e^{2t+1})$
$\ln(N) = \ln(12) + \ln(e^{2t+1})$
$\ln(N) = \ln(12) + 2t + 1$
$\ln(N) \approx 2t + 3{,}48$
Dus $\ln(N) = 2t + 3{,}48$.

48 **a** $N = 0{,}15t^{1,25}$
$\ln(N) = \ln(0{,}15t^{1,25})$
$\ln(N) = \ln(0{,}15) + \ln(t^{1,25})$
$\ln(N) = \ln(0{,}15) + 1{,}25\ln(t)$
$\ln(N) \approx -1{,}90 + 1{,}25\ln(t)$
Dus $\ln(N) = -1{,}90 + 1{,}25\ln(t)$.

b $y = 50 \cdot 0{,}9^{x-1}$
$\ln(y) = \ln(50 \cdot 0{,}9^{x-1})$
$\ln(y) = \ln(50) + \ln(0{,}9^{x-1})$
$\ln(y) = \ln(50) + (x-1)\ln(0{,}9)$
$\ln(y) = \ln(50) + x\ln(0{,}9) - \ln(0{,}9)$
$\ln(y) \approx -0{,}11x + 4{,}02$
Dus $\ln(y) = -0{,}11x + 4{,}02$.

c $N = 750 \cdot 1{,}23^{t-1}$
$\ln(N) = \ln(750 \cdot 1{,}23^{t-1})$
$\ln(N) = \ln(750) + \ln(1{,}23^{t-1})$
$\ln(N) = \ln(750) + (t-1)\ln(1{,}23)$
$\ln(N) = \ln(750) + t\ln(1{,}23) - \ln(1{,}23)$
$t\ln(1{,}23) = \ln(1{,}23) - \ln(750) + \ln(N)$
$t = \dfrac{\ln(1{,}23) - \ln(750)}{\ln(1{,}23)} + \dfrac{1}{\ln(1{,}23)} \cdot \ln(N)$
$t \approx -30{,}98 + 4{,}83\ln(\text{N})$
Dus $t = -30{,}98 + 4{,}83\ln(N)$.

d $N = 400\,e^{2t-1}$
$\ln(N) = \ln(400\,e^{2t-1})$
$\ln(N) = \ln(400) + \ln(e^{2t-1})$
$\ln(N) = \ln(400) + 2t - 1$
$2t = 1 - \ln(400) + \ln(N)$
$t = \tfrac{1}{2} - \tfrac{1}{2}\ln(400) + \tfrac{1}{2}\ln(N)$
$t \approx -2{,}5 + \tfrac{1}{2}\ln(N)$
Dus $t = -2{,}5 + \tfrac{1}{2}\ln(N)$.

12

49 **a** $t = 0$ en $T = 80$ geeft $20 + p\mathrm{e}^{-\frac{1}{6}\cdot 0} = 80$

$p\mathrm{e}^0 = 60$

$p \cdot 1 = 60$

$p = 60$

b $T = 45$ geeft $20 + 60\mathrm{e}^{-\frac{1}{6}t} = 45$

$60\mathrm{e}^{-\frac{1}{6}t} = 25$

$\mathrm{e}^{-\frac{1}{6}t} = \frac{25}{60}$

$-\frac{1}{6}t = \ln\left(\frac{25}{60}\right)$

$t = -6\ln\left(\frac{25}{60}\right) = 5{,}25...$

Dus na 5 minuten en 0,25... × 60 ≈ 15 seconden is de temperatuur van de koffie 45 °C.

c $T = 20 + 60\mathrm{e}^{-\frac{1}{6}t}$ geeft $\frac{\mathrm{d}T}{\mathrm{d}t} = 60\mathrm{e}^{-\frac{1}{6}t} \cdot -\frac{1}{6} = -10\mathrm{e}^{-\frac{1}{6}t}$

$\left[\frac{\mathrm{d}T}{\mathrm{d}t}\right]_{t=2} = -10\mathrm{e}^{-\frac{1}{6}\cdot 2} \approx -7{,}2$

Twee minuten nadat de koffie is ingeschonken neemt de temperatuur van de koffie af met een snelheid van 7,2 °C per minuut.

d $T = 20 + 60\mathrm{e}^{-\frac{1}{6}t}$

$T - 20 = 60\mathrm{e}^{-\frac{1}{6}t}$

$60\mathrm{e}^{-\frac{1}{6}t} = T - 20$

$\mathrm{e}^{-\frac{1}{6}t} = \frac{1}{60}T - \frac{1}{3}$

$\ln\left(\mathrm{e}^{-\frac{1}{6}t}\right) = \ln\left(\frac{1}{60}T - \frac{1}{3}\right)$

$-\frac{1}{6}t = \ln\left(\frac{1}{60}T - \frac{1}{3}\right)$

$t = -6\ln\left(\frac{1}{60}T - \frac{1}{3}\right)$

Dus $a = -6$, $b = \frac{1}{60}$ en $c = -\frac{1}{3}$.

50 **a** In opgave 45a is de formule $y = 20 \cdot x^3$ herleid tot $\ln(y) = \ln(20) + 3\ln(x)$, dus deze formules komen op hetzelfde neer.

b In opgave 45b is de formule $N = 250 \cdot 1{,}18^t$ herleid tot $\ln(N) = \ln(250) + t \cdot \ln(1{,}18)$, dus deze formules komen op hetzelfde neer.

Bladzijde 143

51 **a** $\ln(y) = 1{,}7 + 1{,}45\ln(x)$

$\ln(y) = \ln(\mathrm{e}^{1,7}) + \ln(x^{1,45})$

$\ln(y) = \ln(\mathrm{e}^{1,7} \cdot x^{1,45})$

$y = \mathrm{e}^{1,7} \cdot x^{1,45}$

$y \approx 5{,}5 \cdot x^{1,45}$

Dus $y = 5{,}5 \cdot x^{1,45}$.

b $\ln(y) = 3{,}5 - 1{,}12\ln(x)$

$\ln(y) = \ln(\mathrm{e}^{3,5}) + \ln(x^{-1,12})$

$\ln(y) = \ln(\mathrm{e}^{3,5} \cdot x^{-1,12})$

$y = \mathrm{e}^{3,5} \cdot x^{-1,12}$

$y \approx 33{,}1 \cdot x^{-1,12}$

Dus $y = 33{,}1 \cdot x^{-1,12}$.

c $\ln(x) = 0{,}5\ln(y) - 2{,}75$

$0{,}5\ln(y) = 2{,}75 + \ln(x)$

$\ln(y) = 5{,}5 + 2\ln(x)$

$\ln(y) = \ln(\mathrm{e}^{5,5}) + \ln(x^2)$

$\ln(y) = \ln(\mathrm{e}^{5,5} \cdot x^2)$

$y = \mathrm{e}^{5,5} \cdot x^2$

$y \approx 244{,}7 \cdot x^2$

Dus $y = 244{,}7 \cdot x^2$.

d $3\ln(x) - 4\ln(y) + 5 = 0$

$4\ln(y) = 5 + 3\ln(x)$

$\ln(y) = 1{,}25 + 0{,}75\ln(x)$

$\ln(y) = \ln(\mathrm{e}^{1,25}) + \ln(x^{0,75})$

$\ln(y) = \ln(\mathrm{e}^{1,25} \cdot x^{0,75})$

$y = \mathrm{e}^{1,25} \cdot x^{0,75}$

$y \approx 3{,}5 \cdot x^{0,75}$

Dus $y = 3{,}5 \cdot x^{0,75}$.

52 **a** $\ln(N) = 0{,}95t + 2{,}45$
$N = e^{0{,}95t + 2{,}45}$
$N = e^{0{,}95t} \cdot e^{2{,}45}$
$N = e^{2{,}45} \cdot e^{0{,}95t}$
$N \approx 11{,}6 \cdot e^{0{,}95t}$
Dus $N = 11{,}6 \cdot e^{0{,}95t}$.

b $\ln(N) = 4{,}12 - 1{,}12t$
$N = e^{4{,}12 - 1{,}12t}$
$N = e^{4{,}12} \cdot e^{-1{,}12t}$
$N \approx 61{,}6 \cdot e^{-1{,}12t}$
Dus $N = 61{,}6 \cdot e^{-1{,}12t}$.

c $t = 4\ln(N) - 3$
$4\ln(N) = 3 + t$
$\ln(N) = 0{,}75 + 0{,}25t$
$N = e^{0{,}75 + 0{,}25t}$
$N = e^{0{,}75} \cdot e^{0{,}25t}$
$N \approx 2{,}1 \cdot e^{0{,}25t}$
Dus $N = 2{,}1 \cdot e^{0{,}25t}$.

d $t = 50 - 10\ln(N)$
$10\ln(N) = 50 - t$
$\ln(N) = 5 - 0{,}1t$
$N = e^{5 - 0{,}1t}$
$N = e^{5} \cdot e^{-0{,}1t}$
$N \approx 148{,}4 \cdot e^{-0{,}1t}$
Dus $N = 148{,}4 \cdot e^{-0{,}1t}$.

Bladzijde 144

53 **a** $\ln(N) = 8 + 2{,}1\ln(t)$
$\ln(N) = \ln(e^{8}) + \ln(t^{2{,}1})$
$\ln(N) = \ln(e^{8} \cdot t^{2{,}1})$
$N = e^{8} \cdot t^{2{,}1}$
$N \approx 2981 \cdot t^{2{,}1}$
Dus $N = 2981 \cdot t^{2{,}1}$.

b $\ln(y) = 1{,}65x + 3{,}16$
$y = e^{1{,}65x + 3{,}16}$
$y = e^{1{,}65x} \cdot e^{3{,}16}$
$y = e^{3{,}16} \cdot e^{1{,}65x}$
$y = e^{3{,}16} \cdot (e^{1{,}65})^{x}$
$y \approx 23{,}57 \cdot 5{,}21^{x}$
Dus $y = 23{,}57 \cdot 5{,}21^{x}$.

c $K = 8\ln(P) - 16$
$8\ln(P) = 16 + K$
$\ln(P) = 2 + 0{,}125K$
$P = e^{2 + 0{,}125K}$
$P = e^{2} \cdot e^{0{,}125K}$
$P \approx 7{,}389 \cdot e^{0{,}125K}$
Dus $P = 7{,}389 \cdot e^{0{,}125K}$.

54 **a** $D = 50$ geeft $\ln(N) = 12{,}1 - 1{,}7\ln(50) = 5{,}44...$, dus $N = e^{5{,}44...} \approx 233$.
Dus 233 bomen per ha.

b $N = \dfrac{2000}{8} = 250$
Los op $\ln(250) = 12{,}1 - 1{,}7\ln(D)$.
Voer in $y_1 = \ln(250)$ en $y_2 = 12{,}1 - 1{,}7\ln(x)$.
Intersect geeft $x \approx 47{,}9$.
Dus $D \approx 48$.

c $\ln(N) = 12{,}1 - 1{,}7\ln(D)$
$1{,}7\ln(D) = 12{,}1 - \ln(N)$
$\ln(D) = \dfrac{12{,}1}{1{,}7} - \dfrac{1}{1{,}7}\ln(N)$
$\ln(D) = \ln\left(e^{\frac{12{,}1}{1{,}7}}\right) + \ln\left(N^{-\frac{1}{1{,}7}}\right)$
$\ln(D) = \ln\left(e^{\frac{12{,}1}{1{,}7}} \cdot N^{-\frac{1}{1{,}7}}\right)$
$D = e^{\frac{12{,}1}{1{,}7}} \cdot N^{-\frac{1}{1{,}7}}$
$D \approx 1234 \cdot N^{-0{,}59}$
Dus $a = 1230$ en $b = -0{,}59$.

55 a $T = -2{,}57 \ln\left(\dfrac{87 - \frac{1}{0{,}314}L^*}{63} \cdot \dfrac{0{,}314}{0{,}314}\right) = -2{,}57 \ln\left(\dfrac{27{,}318 - L^*}{19{,}782}\right)$

b $L^* = 25$ geeft $T = -2{,}57 \ln\left(\dfrac{27{,}318 - 25}{19{,}782}\right) = 5{,}51...$
De leeftijd is 5 jaar en 6 maanden.

c Los op $-2{,}57 \ln\left(\dfrac{27{,}318 - L^*}{19{,}782}\right) = 10$.

Voer in $y_1 = -2{,}57 \ln\left(\dfrac{27{,}318 - x}{19{,}782}\right)$ en $y_2 = 10$.
Intersect geeft $x \approx 26{,}9$.
De lengte is 26,9 meter.

d $T = -2{,}57 \ln\left(\dfrac{87 - L}{63}\right)$

$\ln\left(\dfrac{87 - L}{63}\right) = \dfrac{1}{-2{,}57}T$

$\ln\left(\dfrac{87 - L}{63}\right) = \ln\left(e^{-\frac{1}{2{,}57}T}\right)$

$\dfrac{87 - L}{63} = e^{-\frac{1}{2{,}57}T}$

$87 - L = 63\,e^{-\frac{1}{2{,}57}T}$

$-L = -87 + 63\,e^{-\frac{1}{2{,}57}T}$

$L \approx 87 - 63\,e^{-0{,}39T}$

Dus $a = 87$, $b = -63$ en $c = -0{,}39$.

Diagnostische toets

Bladzijde 146

1 a $3e^3 - 8e^3 = -5e^3$

b $5e^{5a} - 3e^{5a} = 2e^{5a}$

c $\dfrac{6\,e^{8x} + 3\,e^{4x}}{e^{2x}} = \dfrac{6\,e^{8x}}{e^{2x}} + \dfrac{3\,e^{4x}}{e^{2x}} = 6\,e^{6x} + 3\,e^{2x}$

d $(2\,e^a - 1)^2 = 4\,e^{2a} - 4\,e^a + 1$

2 a $(x^2 - 4x - 5)\,e^x = 0$
$x^2 - 4x - 5 = 0 \lor e^x = 0$
$(x + 1)(x - 5) = 0$ geen opl.
$x = -1 \lor x = 5$

b $e^{2x+1} - 1 = 0$
$e^{2x+1} = 1$
$e^{2x+1} = e^0$
$2x + 1 = 0$
$2x = -1$
$x = -\frac{1}{2}$

c $e^{x+1} - e^2 \cdot \sqrt{e} = 0$
$e^{x+1} = e^2 \cdot \sqrt{e}$
$e^{x+1} = e^2 \cdot e^{\frac{1}{2}}$
$e^{x+1} = e^{2\frac{1}{2}}$
$x + 1 = 2\frac{1}{2}$
$x = 1\frac{1}{2}$

d $x^2 e^{2x} = x\,e^{2x}$
$x^2 e^{2x} - x\,e^{2x} = 0$
$(x^2 - x)\,e^{2x} = 0$
$x^2 - x = 0 \lor e^{2x} = 0$
$x(x - 1) = 0$ geen opl.
$x = 0 \lor x = 1$

3 a $f(x) = e^x + 2x^2 - 5x$ geeft $f'(x) = e^x + 4x - 5$

b $f(x) = e^{2x^2 - 5x}$ geeft $f'(x) = e^{2x^2 - 5x} \cdot (4x - 5) = (4x - 5)\,e^{2x^2 - 5x}$

c $f(x) = \dfrac{3x + 1}{e^{3x}}$ geeft $f'(x) = \dfrac{e^{3x} \cdot 3 - (3x + 1) \cdot e^{3x} \cdot 3}{(e^{3x})^2} = \dfrac{3\,e^{3x} - 9x\,e^{3x} - 3\,e^{3x}}{(e^{3x})^2} = \dfrac{-9x\,e^{3x}}{(e^{3x})^2} = -\dfrac{9x}{e^{3x}}$

d $f(x) = (2x^2 - 5x)\,e^x$ geeft $f'(x) = (4x - 5) \cdot e^x + (2x^2 - 5x) \cdot e^x = (2x^2 - x - 5)\,e^x$

e Gebruik $y = \dfrac{1}{x} = x^{-1}$ geeft $\dfrac{dy}{dx} = -x^{-2} = -\dfrac{1}{x^2}$.

$f(x) = x^2 \cdot e^{\frac{1}{x}}$ geeft $f'(x) = 2x \cdot e^{\frac{1}{x}} + x^2 \cdot e^{\frac{1}{x}} \cdot -\dfrac{1}{x^2} = 2x \cdot e^{\frac{1}{x}} - e^{\frac{1}{x}} = (2x - 1)\,e^{\frac{1}{x}}$

f $f(x) = \dfrac{e^{2x}}{e^{2x} - 1}$ geeft $f'(x) = \dfrac{(e^{2x} - 1) \cdot e^{2x} \cdot 2 - e^{2x} \cdot e^{2x} \cdot 2}{(e^{2x} - 1)^2} = \dfrac{2\,e^{4x} - 2\,e^{2x} - 2\,e^{4x}}{(e^{2x} - 1)^2} = \dfrac{-2\,e^{2x}}{(e^{2x} - 1)^2}$

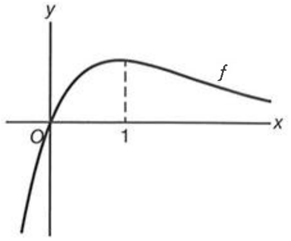
4 **a** $f(x) = 2x\mathrm{e}^{-x}$ geeft $f'(x) = 2 \cdot \mathrm{e}^{-x} + 2x \cdot \mathrm{e}^{-x} \cdot -1 = 2\mathrm{e}^{-x} - 2x\mathrm{e}^{-x} = (2 - 2x)\mathrm{e}^{-x}$

$f'(x) = 0$ geeft $(2 - 2x)\mathrm{e}^{-x} = 0$

$2 - 2x = 0 \vee \mathrm{e}^{-x} = 0$

$2x = 2$ geen opl.

$x = 1$



max. is $f(1) = 2\mathrm{e}^{-1} = \dfrac{2}{\mathrm{e}}$

b $f'(x) = (2 - 2x)\mathrm{e}^{-x}$ geeft $f''(x) = -2 \cdot \mathrm{e}^{-x} + (2 - 2x) \cdot \mathrm{e}^{-x} \cdot -1 = -2\mathrm{e}^{-x} + (2x - 2)\mathrm{e}^{-x} = (2x - 4)\mathrm{e}^{-x}$

$f''(x) = 0$ geeft $(2x - 4)\mathrm{e}^{-x} = 0$ ofwel $2x - 4 = 0$, dus $x = 2$.

Stel k: $y = ax + b$ met $a = f'(2) = -2\mathrm{e}^{-2} = -\dfrac{2}{\mathrm{e}^2}$.

$$\left.\begin{array}{l} y = -\dfrac{2}{\mathrm{e}^2}x + b \\ f(2) = 4\mathrm{e}^{-2} = \dfrac{4}{\mathrm{e}^2} \end{array}\right\} \begin{array}{l} -\dfrac{2}{\mathrm{e}^2} \cdot 2 + b = \dfrac{4}{\mathrm{e}^2} \\ -\dfrac{4}{\mathrm{e}^2} + b = \dfrac{4}{\mathrm{e}^2} \\ b = \dfrac{8}{\mathrm{e}^2} \end{array}$$

Dus k: $y = -\dfrac{2}{\mathrm{e}^2}x + \dfrac{4}{\mathrm{e}^2}$.

5 **a** $\mathrm{e}^x = 4$
$x = \ln(4)$

b $\mathrm{e}^{x-2} = 3$
$x - 2 = \ln(3)$
$x = 2 + \ln(3)$

c $\mathrm{e}^{\frac{1}{2}x} = 5$
$\frac{1}{2}x = \ln(5)$
$x = 2\ln(5)$

d $\ln(x) = -2$
$x = \mathrm{e}^{-2}$
$x = \dfrac{1}{\mathrm{e}^2}$

e $\ln(2 - x) = 4$
$2 - x = \mathrm{e}^4$
$-x = -2 + \mathrm{e}^4$
$x = 2 - \mathrm{e}^4$

f $\ln^2(x) = 25$
$\ln(x) = 5 \vee \ln(x) = -5$
$x = \mathrm{e}^5 \vee x = \mathrm{e}^{-5}$
$x = \mathrm{e}^5 \vee x = \dfrac{1}{\mathrm{e}^5}$

g $\ln(x^2) = 6$
$x^2 = \mathrm{e}^6$
$x = \mathrm{e}^3 \vee x = -\mathrm{e}^3$

h $\mathrm{e}^{2x} \cdot \ln\left(\frac{1}{2}x\right) = 0$
$\mathrm{e}^{2x} = 0 \vee \ln\left(\frac{1}{2}x\right) = 0$
geen opl. $\frac{1}{2}x = 1$
$x = 2$

i $(x - \mathrm{e})\ln(x) = 0$
$x - \mathrm{e} = 0 \vee \ln(x) = 0$
$x = \mathrm{e} \vee x = 1$

j $\ln(x)(\ln(x) + 3) = 0$
$\ln(x) = 0 \vee \ln(x) + 3 = 0$
$x = 1 \vee \ln(x) = -3$
$x = 1 \vee x = \mathrm{e}^{-3}$
$x = 1 \vee x = \dfrac{1}{\mathrm{e}^3}$

6 **a** $f(x) = 3^{4x-1}$ geeft $f'(x) = 3^{4x-1} \cdot \ln(3) \cdot 4 = 4\ln(3) \cdot 3^{4x-1}$

b $g(x) = 5 \cdot 2^{3x}$ geeft $g'(x) = 5 \cdot 2^{3x} \cdot \ln(2) \cdot 3 = 15\ln(2) \cdot 2^{3x}$

c $h(x) = (x + 1) \cdot 2^x$ geeft $h'(x) = 1 \cdot 2^x + (x + 1) \cdot 2^x \cdot \ln(2) = (1 + x\ln(2) + \ln(2)) \cdot 2^x$

d $k(x) = x^4 \cdot 4^x$ geeft $k'(x) = 4x^3 \cdot 4^x + x^4 \cdot 4^x \cdot \ln(4)$

7 **a** $f(x) = x^3 + \ln(x^3)$ geeft $f'(x) = 3x^2 + \dfrac{1}{x^3} \cdot 3x^2 = 3x^2 + \dfrac{3}{x}$

b $f(x) = x^2 \cdot {}^2\log(x)$ geeft $f'(x) = 2x \cdot {}^2\log(x) + x^2 \cdot \dfrac{1}{x\ln(2)} = 2x \cdot {}^2\log(x) + \dfrac{x}{\ln(2)}$

c Gebruik $y = \frac{1}{x\sqrt{x}} = \frac{1}{x^{1\frac{1}{2}}} = x^{-1\frac{1}{2}}$ geeft $\frac{\mathrm{d}y}{\mathrm{d}x} = -1\frac{1}{2}x^{-2\frac{1}{2}} = -\frac{3}{2x^{2\frac{1}{2}}} = -\frac{3}{2x^2\sqrt{x}}$.

$f(x) = \sqrt{x} \cdot \ln\left(x\sqrt{x}\right)$ geeft $f'(x) = \frac{1}{2\sqrt{x}} \cdot \ln\left(x\sqrt{x}\right) + \sqrt{x} \cdot \frac{1}{x\sqrt{x}} \cdot -\frac{3}{2x^2\sqrt{x}} = \frac{1}{2\sqrt{x}} \cdot \ln\left(x\sqrt{x}\right) - \frac{3}{2x^3\sqrt{x}}$

d $f(x) = \frac{\ln(x)}{x^2}$ geeft $f'(x) = \frac{x^2 \cdot \frac{1}{x} - \ln(x) \cdot 2x}{(x^2)^2} = \frac{x - 2x\ln(x)}{x^4} = \frac{1 - 2\ln(x)}{x^3}$

e $f(x) = {}^4\log(4x)$ geeft $f'(x) = \frac{1}{4x\ln(4)} \cdot 4 = \frac{1}{x\ln(4)}$

f $f(x) = \frac{\ln(x)}{\ln(x) - 1}$ geeft $f'(x) = \frac{(\ln(x) - 1) \cdot \frac{1}{x} - \ln(x) \cdot \frac{1}{x}}{(\ln(x) - 1)^2} \cdot \frac{x}{x} = \frac{\ln(x) - 1 - \ln(x)}{x(\ln(x) - 1)^2} = \frac{-1}{x(\ln(x) - 1)^2}$

Bladzijde 147

8 **a** $D_f = \langle 0, \rightarrow \rangle$

b $f(x) = 0$ geeft $x^2 \cdot \ln(x) = 0$

$x^2 = 0 \vee \ln(x) = 0$

$x = 0$ (vold. niet) $\vee$ $x = 1$

$f(x) = x^2 \cdot \ln(x)$ geeft $f'(x) = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} = 2x\ln(x) + x$

Stel k: $y = ax + b$ met $a = f'(1) = 2 \cdot 1 \cdot 0 + 1 = 1$.

$$\left.\begin{array}{l} y = x + b \\ A(1, 0) \end{array}\right\} \begin{array}{l} 1 + b = 0 \\ b = -1 \end{array}$$

Dus k: $y = x - 1$.

c $f'(x) = 0$ geeft $2x\ln(x) + x = 0$

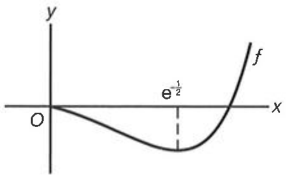
$x(2\ln(x) + 1) = 0$

$x = 0$ (vold. niet) $\vee$ $2\ln(x) + 1 = 0$

$2\ln(x) = -1$

$\ln(x) = -\frac{1}{2}$

$x = e^{-\frac{1}{2}}$



min. is $f\left(e^{-\frac{1}{2}}\right) = e^{-1} \cdot \ln\left(e^{-\frac{1}{2}}\right) = e^{-1} \cdot -\frac{1}{2} = -\frac{1}{2e}$

9 **a** $y = 250 \cdot x^{-2,5}$

$\ln(y) = \ln(250 \cdot x^{-2,5})$

$\ln(y) = \ln(250) + \ln(x^{-2,5})$

$\ln(y) = \ln(250) - 2,5\ln(x)$

$\ln(y) \approx 5,52 - 2,5\ln(x)$

Dus $\ln(y) = 5,52 - 2,5\ln(x)$.

b $N = 750 \cdot 0,65^{3t}$

$\ln(N) = \ln(750 \cdot 0,65^{3t})$

$\ln(N) = \ln(750) + \ln(0,65^{3t})$

$\ln(N) = \ln(750) + 3t\ln(0,65)$

$3t\ln(0,65) = -\ln(750) + \ln(N)$

$t = -\frac{\ln(750)}{3\ln(0,65)} + \frac{1}{3\ln(0,65)}\ln(N)$

$t \approx 5,12 - 0,77\ln(N)$

Dus $t = 5,12 - 0,77\ln(N)$.

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c $\ln(y) = 2{,}3 + 0{,}8\ln(x)$
$\ln(y) = \ln(e^{2,3}) + \ln(x^{0,8})$
$\ln(y) = \ln(e^{2,3} \cdot x^{0,8})$
$y = e^{2,3} \cdot x^{0,8}$
$y \approx 9{,}97 \cdot x^{0,8}$
Dus $y = 9{,}97 \cdot x^{0,8}$.

d $\ln(N) = 0{,}8t + 3{,}2$
$\ln(N) = \ln(e^{0,8t}) + \ln(e^{3,2})$
$\ln(N) = \ln(e^{0,8t} \cdot e^{3,2})$
$N = e^{0,8t} \cdot e^{3,2}$
$N = e^{3,2} \cdot e^{0,8t}$
$N \approx 24{,}5\, e^{0,8t}$
Dus $N = 24{,}5\, e^{0,8t}$.

e $2{,}5t + 2\ln(N) = 6$
$2\ln(N) = 6 - 2{,}5t$
$\ln(N) = 3 - 1{,}25t$
$N = e^{3 - 1,25t}$
$N = e^{3} \cdot e^{-1,25t}$
$N = e^{3} \cdot (e^{-1,25})^{t}$
$N \approx 20 \cdot 0{,}287^{t}$
Dus $N = 20 \cdot 0{,}287^{t}$.

10 **a** $t = 0$ geeft $T = 200 - 180e^{0} = 200 - 180 \cdot 1 = 20$
Dus op het moment dat het verwarmingselement wordt aangezet is de temperatuur in de sauna 20 °C.

b $T = 100$ geeft $200 - 180e^{-0,29t} = 100$

$$-180e^{-0,29t} = -100$$
$$e^{-0,29t} = \frac{-100}{-180} = \frac{5}{9}$$
$$-0{,}29t = \ln\left(\tfrac{5}{9}\right)$$
$$t = \frac{\ln\left(\frac{5}{9}\right)}{-0{,}29} = 2{,}026...$$

$2{,}026... \times 60 \approx 122$
Dus om 17:02 uur wordt het opwarmen stopgezet.

c $T = 200 - 180e^{-0,29t}$
$180e^{-0,29t} = -T + 200$
$e^{-0,29t} = -\frac{1}{180}T + 1\frac{1}{9}$
$-0{,}29t = \ln\left(-\frac{1}{180}T + 1\frac{1}{9}\right)$
$t = -\dfrac{1}{0{,}29} \cdot \ln\left(-\frac{1}{180}T + 1\frac{1}{9}\right)$
$t \approx -3{,}45\ln\left(-\frac{1}{180}T + 1\frac{1}{9}\right)$
Dus $a \approx -3{,}45$, $b = -\frac{1}{180}$ en $c = 1\frac{1}{9}$.

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