6 Vectormeetkunde

Voorkennis Vergelijkingen van lijnen en de cosinusregel

Bladzijde 54

1 **a** $y = 0$ geeft $3x = 24$, dus $x = 8$.
Het snijpunt met de x-as is $(8, 0)$.
$x = 0$ geeft $4y = 24$, dus $y = 6$.
Het snijpunt met de y-as is $(0, 6)$.
b $3x + 4y = 24$
$4y = -3x + 24$
$y = -\frac{3}{4}x + 6$
Dus $rc_k = -\frac{3}{4}$.

2 **a** $4x - 3y = 10$
$-3y = -4x + 10$
$y = 1\frac{1}{3}x - 3\frac{1}{3}$
Dus $rc_k = 1\frac{1}{3}$.
$4x - 3y = 15$
$-3y = -4x + 15$
$y = 1\frac{1}{3}x - 5$
Dus $rc_l = 1\frac{1}{3}$.
k en l hebben dezelfde richtingscoëfficiënt, dus k en l zijn evenwijdig.
b m is evenwijdig met k en l als $a = 4$ en $b = -3$.
$\left.\begin{array}{l} m\text{: } 4x - 3y = c \\ \text{door } (6, 5) \end{array}\right\} c = 4 \cdot 6 - 3 \cdot 5 = 24 - 15 = 9$

3 **a** $\left.\begin{array}{l} m\text{: } 2x + 5y = c \\ \text{door } (3, 2) \end{array}\right\} c = 2 \cdot 3 + 5 \cdot 2 = 6 + 10 = 16$
Dus m: $2x + 5y = 16$.
b n is evenwijdig met l, dus n is van de vorm n: $3x + 5y = c$.
$\left.\begin{array}{l} n\text{: } 3x + 5y = c \\ \text{door } (0, 8) \end{array}\right\} c = 3 \cdot 0 + 5 \cdot 8 = 40$
$3x + 5y = 40$
$5y = -3x + 40$
$y = -\frac{3}{5}x + 8$
Dus n: $y = -\frac{3}{5}x + 8$.
c $y = 0$ geeft $3x = 12$ ofwel $x = 4$, dus l snijdt de x-as in het punt $(4, 0)$.
$rc_p = rc_k$, dus p: $2x + 5y = c$.
$\left.\begin{array}{l} p\text{: } 2x + 5y = c \\ \text{door } (4, 0) \end{array}\right\} c = 2 \cdot 4 + 5 \cdot 0 = 8 + 0 = 8$
Dus p: $2x + 5y = 8$.

Bladzijde 55

4 **a** $l\text{: } \frac{x}{6} + \frac{y}{-5} = 1$
$l\text{: } 5x - 6y = 30$
b $m\text{: } \frac{x}{-3} + \frac{y}{8} = 1$
$m\text{: } 8x - 3y = -24$
c $n\text{: } \frac{x}{p} + \frac{y}{4} = 1$
$n\text{: } 4x + py = 4p$
d $s\text{: } \frac{x}{5} + \frac{y}{q} = 1$
$s\text{: } qx + 5y = 5q$
e $t\text{: } \frac{x}{3r} + \frac{y}{2r} = 1$
$t\text{: } 2x + 3y = 6r$

Bladzijde 56

5 **a** $k: \frac{x}{p} + \frac{y}{6} = 1$

$k: 6x + py = 6p$

b $\left.\begin{array}{l} 6x + py = 6p \\ \text{door } (3, 2) \end{array}\right\} \begin{array}{l} 6 \cdot 3 + p \cdot 2 = 6p \\ 18 + 2p = 6p \\ -4p = -18 \\ p = 4\frac{1}{2} \end{array}$

c $k: 6x + py = 6p$ geeft $py = -6x + 6p$

$y = -\frac{6}{p}x + 6$

Dus $\text{rc}_k = -\frac{6}{p}$.

$k \,//\, l$ geeft $\text{rc}_k = \text{rc}_l$

$-\frac{6}{p} = 4$

$4p = -6$

$p = -1\frac{1}{2}$

6 **a** $k: \frac{x}{4} + \frac{y}{p} = 1$

$k: px + 4y = 4p$

$l: \frac{x}{2p} + \frac{y}{7} = 1$

$l: 7x + 2py = 14p$

b $\left.\begin{array}{l} px + 4y = 4p \\ \text{door } A(1, -2) \end{array}\right\} \begin{array}{l} p \cdot 1 + 4 \cdot -2 = 4p \\ p - 8 = 4p \\ -3p = 8 \\ p = -2\frac{2}{3} \end{array}$

$\left.\begin{array}{l} 7x + 2py = 14p \\ \text{door } A(1, -2) \end{array}\right\} \begin{array}{l} 7 \cdot 1 + 2p \cdot -2 = 14p \\ 7 - 4p = 14p \\ -18p = -7 \\ p = \frac{7}{18} \end{array}$

c $k: px + 4y = 4p$ geeft $4y = -px + 4p$

$y = -\frac{1}{4}px + p$

Dus $\text{rc}_k = -\frac{1}{4}p$.

$k \,//\, m$ geeft $\text{rc}_k = \text{rc}_m$

$-\frac{1}{4}p = -5$

$p = 20$

d $l: 7x + 2py = 14p$ geeft $2py = -7x + 14p$

$y = -\frac{7}{2p}x + 7$

Dus $\text{rc}_l = -\frac{7}{2p}$.

$n: 3x + 2y = 10$ geeft $2y = -3x + 10$

$y = -1\frac{1}{2}x + 5$

Dus $\text{rc}_n = -1\frac{1}{2}$.

$l \,//\, n$ geeft $\text{rc}_l = \text{rc}_n$

$-\frac{7}{2p} = -1\frac{1}{2}$

$\frac{7}{2p} = \frac{3}{2}$

$6p = 14$

$p = 2\frac{1}{3}$

7 $AB^2 = AC^2 + BC^2 - 2 \cdot AC \cdot BC \cdot \cos(\angle C)$

$64 = 36 + 49 - 2 \cdot 6 \cdot 7 \cdot \cos(\angle C)$

$84 \cos(\angle C) = 21$

$\cos(\angle C) = \frac{21}{84}$

$\angle C \approx 75{,}5°$

8 a $AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos(\angle ABC)$
$36 = 16 + 9 - 2 \cdot 4 \cdot 3 \cdot \cos(\angle ABC)$
$24\cos(\angle ABC) = -11$
$\cos(\angle ABC) = \frac{-11}{24}$
$\angle ABC \approx 117{,}3°$

b $AB^2 = AC^2 + BC^2 - 2 \cdot AC \cdot BC \cdot \cos(\angle ACB)$
$16 = 36 + 9 - 2 \cdot 6 \cdot 3 \cdot \cos(\angle ACB)$
$36\cos(\angle ACB) = 29$
$\cos(\angle ACB) = \frac{29}{36}$
$\angle ACB \approx 36{,}3°$

c $CS = \frac{1}{2}AC = 3$
Driehoek BCS is gelijkbenig met $BC = CS = 3$ en $\angle C \approx 36{,}3°$.
$\angle BSC \approx \frac{180° - 36{,}3°}{2} \approx 71{,}8°$

6.1 Vectoren en lijnen

Bladzijde 57

1 $OA^2 = 5^2 + 2^2 = 25 + 4 = 29$
$OA = \sqrt{29}$
$AB^2 = 3^2 + 4^2 = 9 + 16 = 25$
$AB = 5$
Dus de lengte van de wandeling is $5 + \sqrt{29}$.

Bladzijde 59

2 a $|\vec{a}| = \left|\begin{pmatrix} 4 \\ -1 \end{pmatrix}\right| = \sqrt{4^2 + (-1)^2} = \sqrt{16 + 1} = \sqrt{17}$

b $|\vec{b}| = \left|\begin{pmatrix} \sqrt{3} \\ \sqrt{6} \end{pmatrix}\right| = \sqrt{(\sqrt{3})^2 + (\sqrt{6})^2} = \sqrt{3 + 6} = \sqrt{9} = 3$

c $|\vec{c}| = \left|\begin{pmatrix} 0 \\ 5 \end{pmatrix}\right| = \sqrt{0^2 + 5^2} = \sqrt{0 + 25} = \sqrt{25} = 5$

d $|\vec{d}| = \left|\begin{pmatrix} -0{,}3 \\ 0{,}4 \end{pmatrix}\right| = \sqrt{(-0{,}3)^2 + (0{,}4)^2} = \sqrt{0{,}09 + 0{,}16} = \sqrt{0{,}25} = 0{,}5$

e $|\vec{e}| = 2 \cdot \left|\begin{pmatrix} 1 \\ 3 \end{pmatrix}\right| = 2 \cdot \sqrt{1^2 + 3^2} = 2 \cdot \sqrt{1 + 9} = 2\sqrt{10}$

f $|\vec{f}| = 0{,}6 \cdot \left|\begin{pmatrix} 6 \\ 8 \end{pmatrix}\right| = 0{,}6 \cdot \sqrt{6^2 + 8^2} = 0{,}6 \cdot \sqrt{36 + 64} = 0{,}6 \cdot \sqrt{100} = 0{,}6 \cdot 10 = 6$

Bladzijde 60

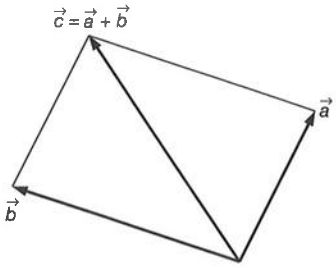
3 a $\vec{c} = 2\vec{a} + 3\vec{b} = 2 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 3 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ 18 \end{pmatrix}$

b $\vec{d} = \vec{a} - 2\vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 2 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

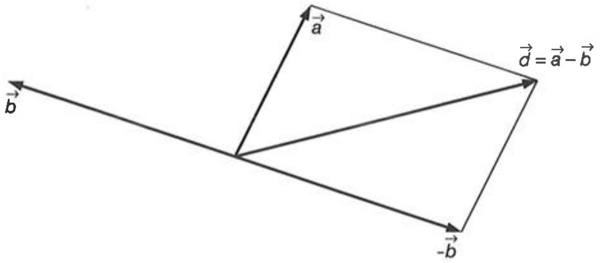
c $\vec{e} = 4\vec{a} - 3\vec{b} = 4 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 3 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix} - \begin{pmatrix} -3 \\ 12 \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix}$

d $\vec{f} = 2 \cdot (\vec{a} + \vec{b}) = 2 \cdot \left(\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}\right) = 2 \cdot \begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 14 \end{pmatrix}$

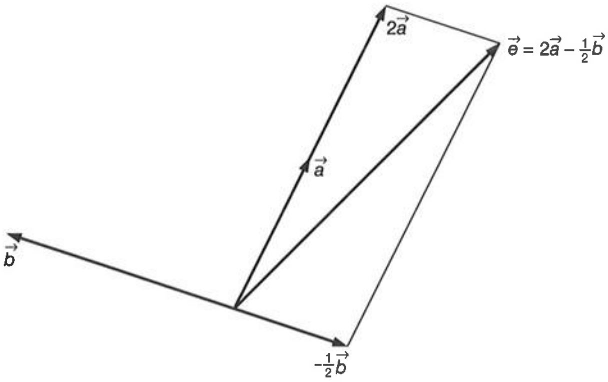
4 **a**



b



c



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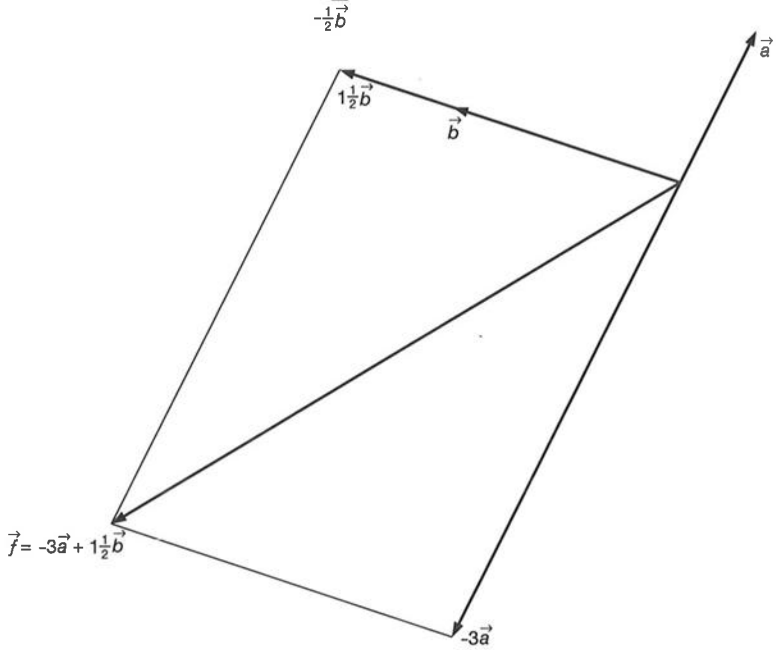
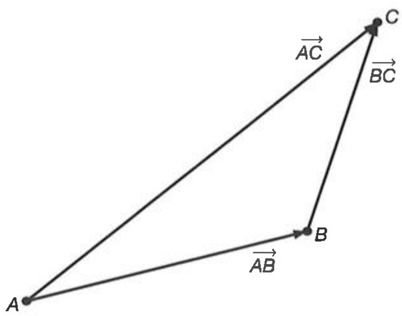
d



5 **a**

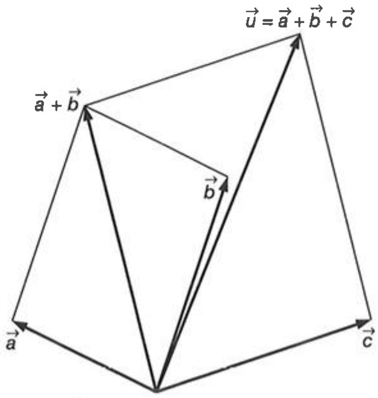


b $\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$

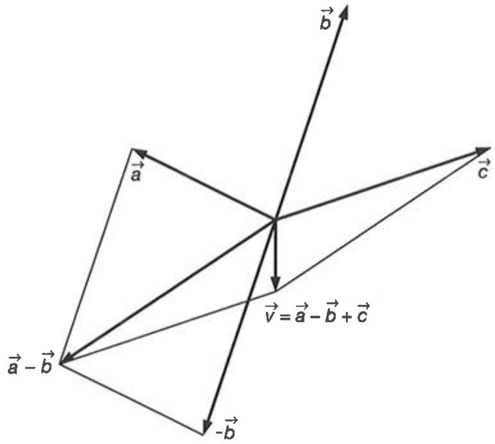
c $\overrightarrow{DE} + \overrightarrow{EF} = \overrightarrow{DF}$

d $\overrightarrow{AK} + \overrightarrow{KL} = \overrightarrow{AL}$

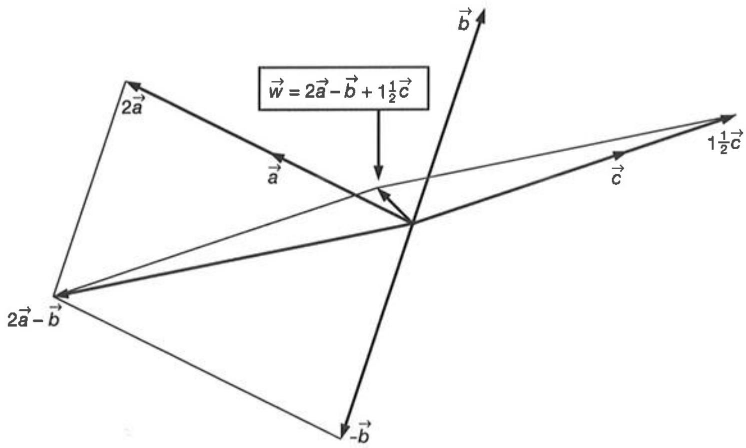
6 a


$\vec{u} = \vec{a} + \vec{b} + \vec{c}$
$\vec{a} + \vec{b}$
$\vec{b}$
$\vec{a}$
$\vec{c}$

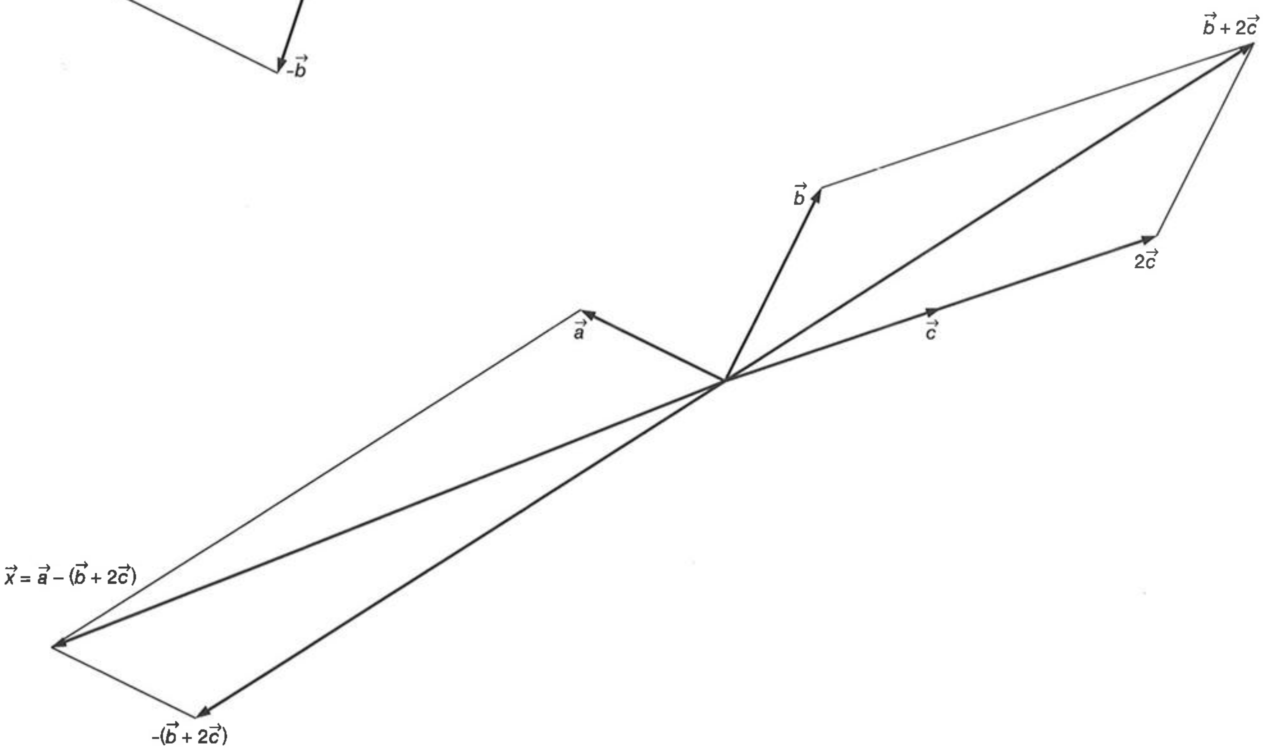
b


$\vec{b}$
$\vec{a}$
$\vec{c}$
$\vec{v} = \vec{a} - \vec{b} + \vec{c}$
$\vec{a} - \vec{b}$
$-\vec{b}$

c

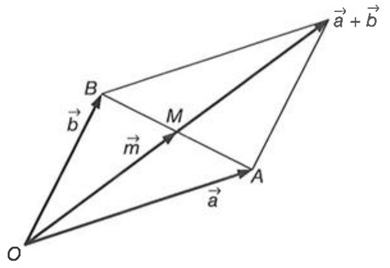

$\vec{b}$
$\vec{w} = 2\vec{a} - \vec{b} + 1\frac{1}{2}\vec{c}$
$2\vec{a}$
$\vec{a}$
$\vec{c}$
$1\frac{1}{2}\vec{c}$
$2\vec{a} - \vec{b}$
$-\vec{b}$

d


$\vec{b} + 2\vec{c}$
$\vec{b}$
$2\vec{c}$
$\vec{a}$
$\vec{c}$
$\vec{x} = \vec{a} - (\vec{b} + 2\vec{c})$
$-(\vec{b} + 2\vec{c})$

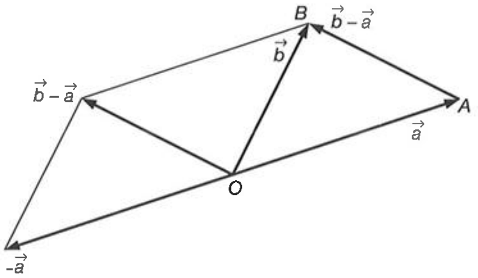
6

7 **a**



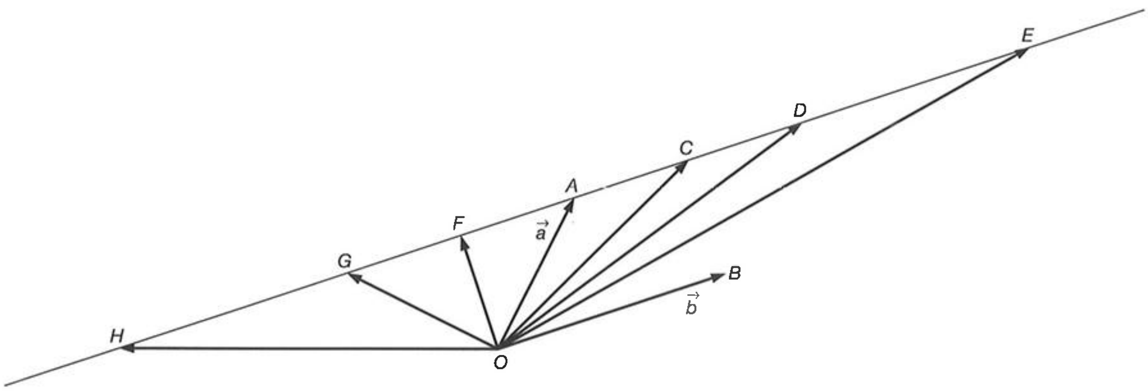
Met de parallellogramconstructie is de vector $\vec{a} + \vec{b}$ getekend.
In het parallellogram zijn de vector $\vec{a} + \vec{b}$ en het lijnstuk AB de diagonalen.
In een parallellogram delen de diagonalen elkaar middendoor.
Dus de vector $\overrightarrow{OM} = \vec{m}$ is de helft van de vector $\vec{a} + \vec{b}$.
Dus voor $\vec{m} = \overrightarrow{OM}$ geldt $\vec{m} = \frac{1}{2}(\vec{a} + \vec{b})$.

b



In de figuur is eerst de vector $\vec{a}$ getekend en daarna met de parallellogramconstructie de vector $\vec{b} - \vec{a}$.
De vector $\vec{b} - \vec{a}$ is vervolgens kop-staart gelegd met de vector $\vec{a}$, en dat is precies de vector $\overrightarrow{AB}$.
Dus $\vec{b} - \vec{a} = \overrightarrow{AB}$.

8 **a**



b De eindpunten van de in vraag a getekende vectoren liggen op de lijn door het eindpunt van de vector $\vec{a}$, die evenwijdig is met de vector $\vec{b}$.

Bladzijde 62

9 **a** Je kunt ook $\vec{q}$ als steunvector nemen.

$$\left.\begin{array}{l} l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{q} + \lambda(\vec{q} - \vec{p}) \\ \vec{q} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} \text{ en } \vec{q} - \vec{p} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \end{array}\right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

b De vectoren $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ en $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$ hebben dezelfde richting, dus $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$ is ook een richtingsvector van l.

Het punt (4, 5) is een punt van l, dus ook $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ is een steunvector van l.

Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ -1 \end{pmatrix}$.

10 **a**

$$\left.\begin{array}{l} k: \vec{v} = \vec{a} + \lambda(\vec{b} - \vec{a}) \\ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ en } \vec{b} - \vec{a} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \end{pmatrix} \end{array}\right\} k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} -6 \\ -1 \end{pmatrix}$$

b $\left.\begin{array}{l} l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{c} + \lambda(\vec{d} - \vec{c}) \\ \vec{c} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ en } \vec{d} - \vec{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \end{array}\right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

c $\left.\begin{array}{l} m: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{e} + \lambda(\vec{e} - \vec{f}) \\ \vec{e} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \text{ en } \vec{e} - \vec{f} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \end{array}\right\} m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

Bladzijde 63

11 lijn k

λ	0	1
punt	(1, 2)	(4, 3)

lijn l

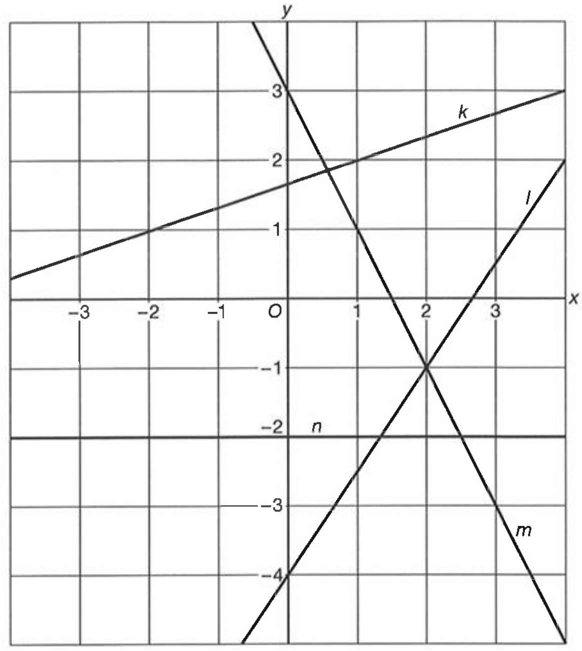
μ	0	1
punt	(2, −1)	(4, 2)

lijn m

ν	0	1
punt	(0, 3)	(1, 1)

lijn n

ρ	0	1
punt	(1, −2)	(2, −2)

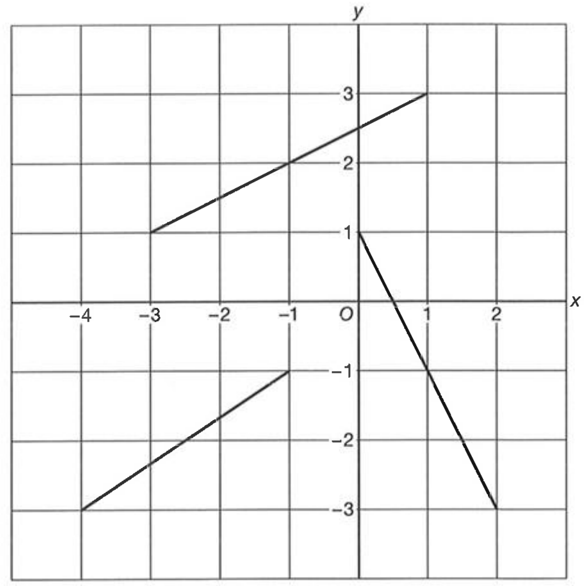


12

λ	−1	1
punt	(−3, 1)	(1, 3)

μ	1	3
punt	(0, 1)	(2, −3)

ν	−1	0
punt	(−4, −3)	(−1, −1)



13 **a** $\left.\begin{array}{l} k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + \lambda(\vec{b} - \vec{c}) \\ \vec{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \text{ en } \vec{b} - \vec{c} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} \end{array}\right\} k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 8 \\ 6 \end{pmatrix}$

b $\vec{m} = \frac{1}{2}(\vec{a} + \vec{c}) = \frac{1}{2} \cdot \left(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -5 \\ -2 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} -6 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$

$\left.\begin{array}{l} l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{b} + \lambda(\vec{b} - \vec{m}) \\ \vec{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ en } \vec{b} - \vec{m} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \end{array}\right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

c $\vec{n} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2}\cdot\left(\begin{pmatrix}-1\\2\end{pmatrix} + \begin{pmatrix}3\\4\end{pmatrix}\right) = \frac{1}{2}\cdot\begin{pmatrix}2\\6\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix}$

$$\left.\begin{array}{l} m: \begin{pmatrix}x\\y\end{pmatrix} = \vec{n} + \lambda(\vec{a} - \vec{c}) \\ \vec{n} = \begin{pmatrix}1\\3\end{pmatrix} \text{ en } \vec{a} - \vec{c} = \begin{pmatrix}-1\\2\end{pmatrix} - \begin{pmatrix}-5\\-2\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix} \end{array}\right\} m: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix} + \lambda\begin{pmatrix}4\\4\end{pmatrix}$$

d Stel eerst een vectorvoorstelling op van de lijn BC.

$$\left.\begin{array}{l} BC: \begin{pmatrix}x\\y\end{pmatrix} = \vec{b} + \lambda(\vec{b} - \vec{c}) \\ \vec{b} = \begin{pmatrix}3\\4\end{pmatrix} \text{ en } \vec{b} - \vec{c} = \begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}-5\\-2\end{pmatrix} = \begin{pmatrix}8\\6\end{pmatrix} \end{array}\right\} BC: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix} + \lambda\begin{pmatrix}8\\6\end{pmatrix}$$

Dus een vectorvoorstelling van het lijnstuk BC is $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix} + \lambda\begin{pmatrix}8\\6\end{pmatrix} \wedge -1 \le \lambda \le 0$.

14 **a** De lijn door de middens van AD en BC is evenwijdig met AB en CD en heeft dus dezelfde richting als AB en CD.

Dus een richtingsvector van deze lijn is $\overrightarrow{DC} = \vec{c} - \vec{d}$ (maar bijvoorbeeld ook $\overrightarrow{AB} = \vec{b} - \vec{a}$).

De vector vanuit O naar het midden van BC is $\frac{1}{2}(\vec{b} + \vec{c})$. Dus $\frac{1}{2}(\vec{b} + \vec{c})$ is een steunvector van de lijn door de middens van AD en BC. Dus $\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{2}(\vec{b} + \vec{c}) + \lambda(\vec{c} - \vec{d})$ is een vectorvoorstelling van de lijn door de middens van AD en BC.

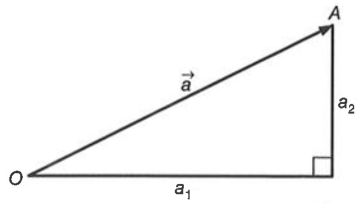
b
- $k: \begin{pmatrix}x\\y\end{pmatrix} = \vec{a} + \lambda(\vec{b} - \vec{d})$
- $l: \begin{pmatrix}x\\y\end{pmatrix} = \vec{b} + \mu\left(\frac{1}{2}(\vec{c} + \vec{d}) - \vec{b}\right)$ ofwel $l: \begin{pmatrix}x\\y\end{pmatrix} = \vec{b} + \mu\left(\frac{1}{2}\vec{c} + \frac{1}{2}\vec{d} - \vec{b}\right)$
- $m: \begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{2}(\vec{a} + \vec{c}) + \nu(\vec{b} - \vec{c})$

c I gaat door B en is evenwijdig met AC.
II gaat door het midden van AB en is evenwijdig met BD.
III gaat door A en het midden van BC.

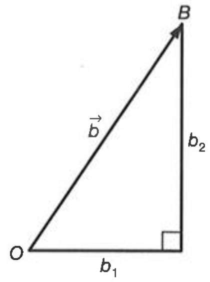
6.2 Vectoren en hoeken

Bladzijde 65

15 **a** De lengte van de vector $\vec{a}$ bereken je met de stelling van Pythagoras.
Dit geeft $|\vec{a}|^2 = a_1^2 + a_2^2$.

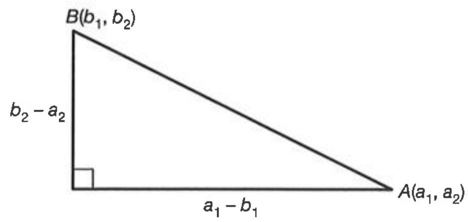


De lengte van de vector $\vec{b}$ bereken je met de stelling van Pythagoras.
Dit geeft $|\vec{b}|^2 = b_1^2 + b_2^2$.



b Zie de schets hiernaast.
De stelling van Pythagoras geeft

$$\begin{aligned} AB^2 &= (a_1 - b_1)^2 + (a_2 - b_2)^2 \\ &= (b_1 - a_1)^2 + (b_2 - a_2)^2 \\ &= b_1^2 - 2a_1b_1 + a_1^2 + b_2^2 - 2a_2b_2 + a_2^2 \\ &= a_1^2 + a_2^2 + b_1^2 + b_2^2 - 2a_1b_1 - 2a_2b_2. \end{aligned}$$



Bladzijde 67

16 a $\vec{a} \cdot \vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -6 \end{pmatrix} = 2 \cdot -1 + 3 \cdot -6 = -2 - 18 = -20$

b $\vec{c} \cdot \vec{d} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix} = -1 \cdot 2 + 0 \cdot 4 = -2 + 0 = -2$

c $\vec{e} \cdot \vec{f} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 2 \cdot 3 + -3 \cdot 2 = 6 - 6 = 0$

d $\vec{g} \cdot \vec{h} = \begin{pmatrix} \sqrt{2} \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 2\sqrt{2} \\ -\sqrt{3} \end{pmatrix} = \sqrt{2} \cdot 2\sqrt{2} + \sqrt{3} \cdot -\sqrt{3} = 4 - 3 = 1$

17 a $\cos(\angle(\vec{a}, \vec{b})) = \dfrac{\begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right|} = \dfrac{3 \cdot 2 + 4 \cdot 1}{\sqrt{25} \cdot \sqrt{5}} = \dfrac{10}{5\sqrt{5}} = \dfrac{2}{\sqrt{5}}$

Dus $\angle(\vec{a}, \vec{b}) \approx 26{,}6°$.

b $\cos(\angle(\vec{c}, \vec{d})) = \dfrac{\begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} 4 \\ -1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -1 \\ 3 \end{pmatrix}\right|} = \dfrac{4 \cdot -1 + -1 \cdot 3}{\sqrt{17} \cdot \sqrt{10}} = \dfrac{-7}{\sqrt{170}}$

Dus $\angle(\vec{c}, \vec{d}) \approx 125{,}5°$.

c $\cos(\angle(\vec{e}, \vec{f})) = \dfrac{\begin{pmatrix} 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \end{pmatrix}}{\left|\begin{pmatrix} 0 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ 5 \end{pmatrix}\right|} = \dfrac{0 \cdot 2 + 3 \cdot 5}{3 \cdot \sqrt{29}} = \dfrac{15}{3\sqrt{29}} = \dfrac{5}{\sqrt{29}}$

Dus $\angle(\vec{e}, \vec{f}) \approx 21{,}8°$.

d $\cos(\angle(\vec{g}, \vec{h})) = \dfrac{\begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ -5 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 5 \\ 2 \end{pmatrix}\right|} = \dfrac{2 \cdot 5 + -5 \cdot 2}{\sqrt{29} \cdot \sqrt{29}} = \dfrac{0}{29} = 0$

Dus $\angle(\vec{g}, \vec{h}) = 90°$.

Bladzijde 68

18 a $\cos(\angle(\vec{r_k}, \vec{r_l})) = \dfrac{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right|} = \dfrac{1 \cdot 2 + 2 \cdot 1}{\sqrt{5} \cdot \sqrt{5}} = \dfrac{4}{5}$

$\angle(\vec{r_k}, \vec{r_l}) \approx 36{,}9°$
Dus $\angle(k, l) \approx 36{,}9°$.

b $\cos(\angle(\vec{r_m}, \vec{r_n})) = \dfrac{\begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}}{\left|\begin{pmatrix} 3 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -1 \\ 2 \end{pmatrix}\right|} = \dfrac{3 \cdot -1 + 1 \cdot 2}{\sqrt{10} \cdot \sqrt{5}} = \dfrac{-1}{\sqrt{50}}$

$\angle(\vec{r_m}, \vec{r_n}) \approx 98{,}1°$
Dus $\angle(m, n) \approx 180° - 98{,}1° = 81{,}9°$.

c $\cos(\angle(\vec{r_p}, \vec{r_q})) = \dfrac{\begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} -1 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right|} = \dfrac{-1 \cdot 2 + 1 \cdot 1}{\sqrt{2} \cdot \sqrt{5}} = \dfrac{-1}{\sqrt{10}}$

$\angle(\vec{r_p}, \vec{r_q}) \approx 108{,}4°$
Dus $\angle(p, q) \approx 180° - 108{,}4° = 71{,}6°$.

19 **a** Een richtingsvector van de lijn AB is $\vec{b} - \vec{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

Een richtingsvector van de lijn BC is $\vec{b} - \vec{c} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

$$\cos(\angle(\vec{r}_{AB}, \vec{r}_{BC})) = \frac{\begin{pmatrix} 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix}}{\left|\begin{pmatrix} 3 \\ -1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 4 \end{pmatrix}\right|} = \frac{3 \cdot 1 + -1 \cdot 4}{\sqrt{10} \cdot \sqrt{17}} = \frac{-1}{\sqrt{170}}$$

$\angle(\vec{r}_{AB}, \vec{r}_{BC}) \approx 94{,}4°$
Dus $\angle(AB, BC) \approx 180° - 94{,}4° = 85{,}6°$.

b Een richtingsvector van de lijn AC is $\vec{c} - \vec{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

Een richtingsvector van de lijn BC is $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

$$\cos(\angle(\vec{r}_{AC}, \vec{r}_{BC})) = \frac{\begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ -5 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 4 \end{pmatrix}\right|} = \frac{2 \cdot 1 + -5 \cdot 4}{\sqrt{29} \cdot \sqrt{17}} = \frac{-18}{\sqrt{493}}$$

$\angle(\vec{r}_{AC}, \vec{r}_{BC}) \approx 144{,}2°$
Dus $\angle(AC, BC) \approx 180° - 144{,}2° = 35{,}8°$.

20 **a** $\overrightarrow{BA} = \vec{a} - \vec{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

$\overrightarrow{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$$\cos(\angle(\overrightarrow{BA}, \overrightarrow{BC})) = \frac{\begin{pmatrix} -2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} -2 \\ -2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -1 \\ 3 \end{pmatrix}\right|} = \frac{-2 \cdot -1 + -2 \cdot 3}{\sqrt{8} \cdot \sqrt{10}} = \frac{-4}{\sqrt{80}}$$

$\angle(\overrightarrow{BA}, \overrightarrow{BC}) \approx 116{,}6°$
Dus $\angle B \approx 116{,}6°$.

b $\overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$

$\overrightarrow{BE} = \vec{e} - \vec{b} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$

$$\cos(\angle(\overrightarrow{AD}, \overrightarrow{BE})) = \frac{\begin{pmatrix} -1 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} -1 \\ 6 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -1 \\ 5 \end{pmatrix}\right|} = \frac{-1 \cdot -5 + 6 \cdot 1}{\sqrt{37} \cdot \sqrt{26}} = \frac{11}{\sqrt{962}}$$

$\angle(\overrightarrow{AD}, \overrightarrow{BE}) \approx 69{,}2°$
Dus de hoek tussen de diagonalen AD en BE is $69{,}2°$.

21 **a** $\vec{a} \cdot \vec{b} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \end{pmatrix} = 4 \cdot 3 + 3 \cdot -4 = 12 - 12 = 0$
De hoek tussen de vectoren is 90°. Dus als het inproduct van twee vectoren nul is, dan staan die vectoren loodrecht op elkaar.

b $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$ staat loodrecht op $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$.

c $\begin{pmatrix} q \\ -p \end{pmatrix}$ staat loodrecht op $\begin{pmatrix} p \\ q \end{pmatrix}$.

22 **a** k: $2x + 3y = 6$ snijdt de assen in $A(0, 2)$ en $B(3, 0)$.
Dus $\vec{r}_k = \vec{b} - \vec{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

b $\vec{r}_k \cdot \vec{n} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 6 - 6 = 0$, dus $\vec{r}_k \perp \vec{n}$.
Dus $k \perp \vec{n}$.

Bladzijde 70

23 **a** $\vec{r}_k = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.
$\left.\begin{array}{l} k\text{: } 4x - 3y = c \\ (2, -1) \text{ op } k \end{array}\right\} c = 4 \cdot 2 - 3 \cdot -1 = 11$
k: $4x - 3y = 11$

b $\vec{r}_l = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, dus $\vec{n}_l = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
$\left.\begin{array}{l} l\text{: } 3x + 2y = c \\ (0, 4) \text{ op } l \end{array}\right\} c = 3 \cdot 0 + 2 \cdot 4 = 8$
l: $3x + 2y = 8$

c $\vec{r}_m = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$, dus $\vec{n}_m = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$.
$\left.\begin{array}{l} m\text{: } 7x + 4y = c \\ (0, 0) \text{ op } m \end{array}\right\} c = 0$
m: $7x + 4y = 0$

d $\vec{r}_n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, dus $\vec{n}_n = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.
$\left.\begin{array}{l} n\text{: } x = c \\ (0, 0) \text{ op } n \end{array}\right\} c = 0$
n: $x = 0$

24 **a** $\vec{n}_k = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, dus $\vec{r}_k = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$.

$(2, 0)$ op k, dus $\vec{s}_k = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

k: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

b $\vec{n}_l = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, dus $\vec{r}_l = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

$(-3, 0)$ op l, dus $\vec{s}_l = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$.

l: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

c $\vec{n}_m = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, dus $\vec{r}_m = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

$(0, 0)$ op m, dus $\vec{s}_m = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

m: $\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

d $\vec{n}_n = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, dus $\vec{r}_n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$(6, 0)$ op n, dus $\vec{s}_n = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$.

n: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Bladzijde 71

25 **a** Substitutie van $x = 3\lambda$ en $y = 2 + 2\lambda$ in $2x - 5y = 6$ geeft
$2 \cdot 3\lambda - 5 \cdot (2 + 2\lambda) = 6$
$6\lambda - 10 - 10\lambda = 6$
$-4\lambda = 16$
$\lambda = -4$
$\lambda = -4$ geeft $x = 3 \cdot -4 = -12$ en $y = 2 + 2 \cdot -4 = -6$
Dus het snijpunt is $(-12, -6)$.

b Substitutie van $x = 2 - \lambda$ en $y = -5 + 3\lambda$ in $3x + 4y = 10$ geeft
$3 \cdot (2 - \lambda) + 4 \cdot (-5 + 3\lambda) = 10$
$6 - 3\lambda - 20 + 12\lambda = 10$
$9\lambda = 24$
$\lambda = \frac{24}{9} = 2\frac{2}{3}$
$\lambda = 2\frac{2}{3}$ geeft $x = 2 - 2\frac{2}{3} = -\frac{2}{3}$ en $y = -5 + 3 \cdot 2\frac{2}{3} = 3$
Dus het snijpunt is $\left(-\frac{2}{3}, 3\right)$.

26 **a** $\vec{r}_l = \vec{r}_k = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, dus $\vec{n}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.
$\left.\begin{array}{l} l\text{: } 5x + 2y = c \\ \text{door } A(4, 1) \end{array}\right\} c = 5 \cdot 4 + 2 \cdot 1 = 22$
l: $5x + 2y = 22$

b $n \perp m$, dus $\vec{n}_n = \vec{r}_m = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
$\left.\begin{array}{l} n\text{: } 3x + 2y = c \\ \text{door } B(5, -1) \end{array}\right\} c = 3 \cdot 5 + 2 \cdot -1 = 13$
n: $3x + 2y = 13$

27 **a** $\vec{n}_k = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, dus $\vec{r}_k = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

$\vec{r}_l = \vec{r}_k = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$A(5, 2)$ op l, dus $\vec{s}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

l: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

b $n \perp m$, dus $\vec{r}_n = \vec{n}_m = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

$B(1, -3)$ op n, dus $\vec{s}_n = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

n: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

28 **a** $\vec{r}_k = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.
$\left.\begin{array}{l} k\text{: } 2x - y = c \\ \text{door } (3, 3) \end{array}\right\} c = 2 \cdot 3 - 3 = 3$
k: $2x - y = 3$
Substitutie van $x = -1 + 3\mu$ en $y = 5 - 4\mu$ in $2x - y = 3$ geeft
$2 \cdot (-1 + 3\mu) - (5 - 4\mu) = 3$
$-2 + 6\mu - 5 + 4\mu = 3$
$10\mu = 10$
$\mu = 1$
$\mu = 1$ geeft $x = -1 + 3 = 2$ en $y = 5 - 4 = 1$
Dus $S(2, 1)$.

b $\vec{r}_m = \vec{b} - \vec{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, dus $\vec{n}_m = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

$\left.\begin{array}{l} m: x + y = c \\ \text{door } A(-3, 5) \end{array}\right\} c = -3 + 5 = 2$

$m: x + y = 2$

$n \perp p$, dus $\vec{r}_n = \vec{n}_p = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.

$C(-4, -2)$ op n, dus $\vec{s}_n = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$.

$n: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

Substitutie van $x = -4 - \lambda$ en $y = -2 + 5\lambda$ in $x + y = 2$ geeft

$-4 - \lambda + -2 + 5\lambda = 2$

$4\lambda = 8$

$\lambda = 2$

$\lambda = 2$ geeft $x = -4 - 2 = -6$ en $y = -2 + 5 \cdot 2 = 8$

Dus $T(-6, 8)$.

29 **a** Snijden van k met de x-as geeft $y = 0$

$3 + \lambda = 0$

$\lambda = -3$

$\lambda = -3$ geeft $x_A = -2 + -3 \cdot 5 = -17$.

Dus $A(-17, 0)$.

$l \perp k$, dus $\vec{n}_l = \vec{r}_k = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$.

$\left.\begin{array}{l} l: 5x + y = c \\ \text{door } A(-17, 0) \end{array}\right\} c = 5 \cdot -17 + 0 = -85$

$l: 5x + y = -85$

b $\vec{r}_m = \vec{b} - \vec{c} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

$n \,//\, m$, dus $\vec{r}_n = \vec{r}_m = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

$(0, 0)$ op n, dus $\vec{s}_n = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

$n: \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

c $\vec{r}_{OE} = \vec{e} - \vec{o} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

$p \perp OE$, dus $\vec{n}_p = \vec{r}_{OE} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

$\left.\begin{array}{l} p: x - 4y = c \\ \text{door } D(-4, 7) \end{array}\right\} c = -4 - 4 \cdot 7 = -32$

$p: x - 4y = -32$

30 **a** $\vec{r}_k = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

$\vec{n}_l = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, dus $\vec{r}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

$$\cos(\angle(\vec{r}_k, \vec{r}_l)) = \frac{\begin{pmatrix} -4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix}}{\left|\begin{pmatrix} -4 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 5 \\ 2 \end{pmatrix}\right|} = \frac{-4 \cdot 5 + 1 \cdot 2}{\sqrt{17} \cdot \sqrt{29}} = \frac{-18}{\sqrt{493}}$$

$\angle(\vec{r}_k, \vec{r}_l) \approx 144{,}2°$

Dus $\angle(k, l) \approx 180° - 144{,}2° = 35{,}8°$.

6

b $\vec{n}_m = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$, dus $\vec{r}_m = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$.

$\vec{n}_n = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, dus $\vec{r}_n = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

$$\cos(\angle(\vec{r}_m, \vec{r}_n)) = \frac{\begin{pmatrix} 7 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix}}{\left|\begin{pmatrix} 7 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ -1 \end{pmatrix}\right|} = \frac{7 \cdot 2 + 3 \cdot -1}{\sqrt{58} \cdot \sqrt{5}} = \frac{11}{\sqrt{290}}$$

$\angle(\vec{r}_m, \vec{r}_n) \approx 49{,}8°$

Dus $\angle(m, n) \approx 49{,}8°$.

c $p: y = 2x + 5$ geeft $p: -2x + y = 5$

$\vec{n}_p = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, dus $\vec{r}_p = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

$q: y = -x + 4$ geeft $q: x + y = 4$

$\vec{n}_q = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, dus $\vec{r}_q = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

$$\cos(\angle(\vec{r}_p, \vec{r}_q)) = \frac{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}}{\left|\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right|} = \frac{1 \cdot 1 + 2 \cdot -1}{\sqrt{5} \cdot \sqrt{2}} = \frac{-1}{\sqrt{10}}$$

$\angle(\vec{r}_p, \vec{r}_q) \approx 108{,}4°$

Dus $\angle(p, q) \approx 180° - 108{,}4° = 71{,}6°$.

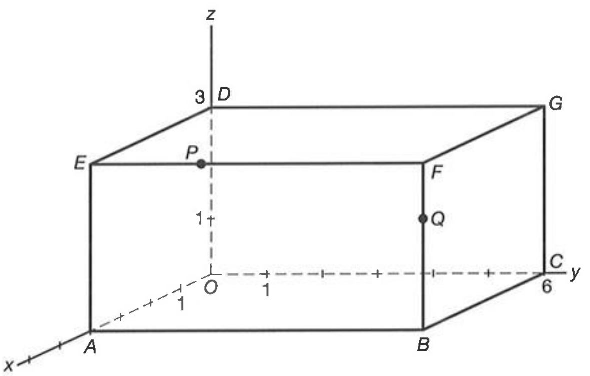
6.3 Coördinaten in de ruimte

Bladzijde 73

31 **a** M 4, $2\frac{1}{2}$ en 3
b N 2, 5 en 3
c B 4, 5 en 0
d G 0, 5 en 3
e E 4, 0 en 3
f C 0, 5 en 0

Bladzijde 74

32 **a,c**



b $O(0, 0, 0)$, $B(4, 6, 0)$, $E(4, 0, 3)$, $F(4, 6, 3)$ en $G(0, 6, 3)$

d $\vec{m} = \frac{1}{2}(\vec{p} + \vec{q}) = \frac{1}{2}\left(\begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \\ 2 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 8 \\ 8 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 2\frac{1}{2} \end{pmatrix}$

Dus $M\left(4, 4, 2\frac{1}{2}\right)$.

33 **a** $\vec{m} = \frac{1}{2}(\vec{c} + \vec{f}) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 6 \\ 12 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}$

Dus $M(3, 6, 3)$.

b $\vec{q} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}$

$\vec{r} = \frac{1}{2}(\vec{c} + \vec{q}) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 6 \\ 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 1\frac{1}{2} \end{pmatrix}$

Dus $R(3, 3, 1\frac{1}{2})$.

c $\vec{q} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}$ en $\vec{p} = \begin{pmatrix} 6 \\ 3 \\ 6 \end{pmatrix}$

$\vec{s} = \frac{1}{2}(\vec{p} + \vec{q}) = \frac{1}{2}\left(\begin{pmatrix} 6 \\ 3 \\ 6 \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 12 \\ 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 6 \\ 1\frac{1}{2} \\ 4\frac{1}{2} \end{pmatrix}$

Dus $S(6, 1\frac{1}{2}, 4\frac{1}{2})$.

34 **a** $\vec{m} = \frac{1}{2}(\vec{b} + \vec{t}) = \frac{1}{2}\left(\begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \\ 8 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 9 \\ 9 \\ 8 \end{pmatrix} = \begin{pmatrix} 4\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix}$

Dus $M(4\frac{1}{2}, 4\frac{1}{2}, 4)$.

b $\vec{p} = \frac{1}{2}(\vec{c} + \vec{m}) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 4\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 4\frac{1}{2} \\ 10\frac{1}{2} \\ 4 \end{pmatrix} = \begin{pmatrix} 2\frac{1}{4} \\ 5\frac{1}{4} \\ 2 \end{pmatrix}$

Dus $P(2\frac{1}{4}, 5\frac{1}{4}, 2)$.

c $\vec{q} = \frac{1}{2}(\vec{m} + \vec{s}) = \frac{1}{2}\left(\begin{pmatrix} 4\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 7\frac{1}{2} \\ 7\frac{1}{2} \\ 4 \end{pmatrix} = \begin{pmatrix} 3\frac{3}{4} \\ 3\frac{3}{4} \\ 2 \end{pmatrix}$

Dus $Q(3\frac{3}{4}, 3\frac{3}{4}, 2)$.

Bladzijde 75

35 **a** $K(-6, 0, 6)$ en $L(-6, 6, 6)$

b $\frac{1}{2}(\vec{c} + \vec{k}) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} -6 \\ 6 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$

Dus het midden van het lijnstuk CK is $(-3, 3, 3)$.

c $\frac{1}{2}(\vec{a} + \vec{l}) = \frac{1}{2}\left(\begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -6 \\ 6 \\ 6 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 0 \\ 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$

Dus het midden van het lijnstuk AL is $(0, 3, 3)$.

d De punten A, D, E, K, O en P.

Bladzijde 76

36 **a** $A(3, 3, 0)$, $B(-3, 3, 0)$, $C(-3, -3, 0)$, $D(3, -3, 0)$, $E(3, 3, 6)$, $F(-3, 3, 6)$, $G(-3, -3, 6)$, $H(3, -3, 6)$ en $T(0, 0, 10)$.

b $\overrightarrow{TH} = \vec{h} - \vec{t} = \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}$, dus een richtingsvector van de lijn HT is $\begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}$.

$T(0, 0, 10)$ ligt op de lijn HT dus een steunvector is $\begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$.

Dus een vectorvoorstelling van de lijn HT is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}$.

c Het Oxy-vlak bestaat uit alle punten waarvan de z-coördinaat gelijk is aan 0.
P ligt in dit vlak dus $z_P = 0$.

d $\left.\begin{array}{l} z_P = 0 \\ z_P = 10 - 4\lambda \end{array}\right\}$ $10 - 4\lambda = 0$
$-4\lambda = -10$
$\lambda = 2\frac{1}{2}$

e $\lambda = 2\frac{1}{2}$ geeft $x_P = 2\frac{1}{2} \cdot 3 = 7\frac{1}{2}$ en $y_P = 2\frac{1}{2} \cdot -3 = -7\frac{1}{2}$
Dus $P\left(7\frac{1}{2}, -7\frac{1}{2}, 0\right)$.

Bladzijde 78

37 **a** Het Oxz-vlak bestaat uit alle punten waarvan de y-coördinaat gelijk is aan 0.
Q ligt in dit vlak dus $y_Q = 0$.
b Het Oyz-vlak bestaat uit alle punten waarvan de x-coördinaat gelijk is aan 0.
De x-coördinaat van deze punten is dus gelijk aan 0.

38 **a** $\vec{m} = \begin{pmatrix} 4 \\ 0 \\ 1\frac{1}{2} \end{pmatrix}$ en $\vec{p} - \vec{m} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 1\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \\ 1\frac{1}{2} \end{pmatrix}$, dus MP: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} -4 \\ 1 \\ 1\frac{1}{2} \end{pmatrix}$.

$z_T = 0$ geeft $3 + 1\frac{1}{2}\lambda = 0$, dus $\lambda = -2$.

$\lambda = -2$ geeft $\vec{t} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} - 2 \cdot \begin{pmatrix} -4 \\ 1 \\ 1\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \\ 0 \end{pmatrix}$

Dus $T(8, -1, 0)$.

b $\vec{p} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$ en $\vec{q} - \vec{p} = \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix}$, dus PQ: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix}$.

$z_Q = 0$ geeft $3 - \lambda = 0$, dus $\lambda = 3$.

$\lambda = 3$ geeft $\vec{q} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + 3 \cdot \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 12 \\ 13 \\ 0 \end{pmatrix}$

Dus $Q(12, 13, 0)$.

39 **a** $\vec{m} = \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix}$ en $\vec{p} - \vec{m} = \begin{pmatrix} 5\frac{1}{4} \\ 5\frac{1}{4} \\ 2 \end{pmatrix} - \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix} = \begin{pmatrix} 3\frac{3}{4} \\ \frac{3}{4} \\ -2 \end{pmatrix}$, dus MP: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix} + \lambda\begin{pmatrix} 3\frac{3}{4} \\ \frac{3}{4} \\ -2 \end{pmatrix}$.

$z_D = 0$ geeft $4 - 2\lambda = 0$, dus $\lambda = 2$.

$\lambda = 2$ geeft $\vec{d} = \begin{pmatrix} 1\frac{1}{2} \\ 4\frac{1}{2} \\ 4 \end{pmatrix} + 2\begin{pmatrix} 3\frac{3}{4} \\ \frac{3}{4} \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ 0 \end{pmatrix}$

Dus $D(9, 6, 0)$.

b $\vec{q} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix}$ en $\vec{p} - \vec{q} = \begin{pmatrix} 5\frac{1}{4} \\ 5\frac{1}{4} \\ 2 \end{pmatrix} - \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} = \begin{pmatrix} 1\frac{1}{2} \\ 3 \\ -4 \end{pmatrix}$, dus MQ: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} + \lambda\begin{pmatrix} 1\frac{1}{2} \\ 3 \\ -4 \end{pmatrix}$.

$x_E = 0$ geeft $3\frac{3}{4} + 1\frac{1}{2}\lambda = 0$, dus $\lambda = -2\frac{1}{2}$.

$\lambda = -2\frac{1}{2}$ geeft $\vec{e} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} - 2\frac{1}{2}\begin{pmatrix} 1\frac{1}{2} \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ -5\frac{1}{4} \\ 16 \end{pmatrix}$

Dus $E\left(0, -5\frac{1}{4}, 16\right)$.

c $\vec{q} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix}$ en $\vec{m} - \vec{q} = \begin{pmatrix} 5\frac{1}{4} \\ 5\frac{1}{4} \\ 2 \end{pmatrix} - \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} = \begin{pmatrix} 1\frac{1}{2} \\ 3 \\ -4 \end{pmatrix}$, dus MQ: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1\frac{1}{2} \\ 3 \\ -4 \end{pmatrix}$.

$y_E = 0$ geeft $3\frac{3}{4} + 1\frac{1}{2}\lambda = 0$, dus $\lambda = -2\frac{1}{2}$.

$\lambda = -2\frac{1}{2}$ geeft $\vec{e} = \begin{pmatrix} 3\frac{3}{4} \\ 2\frac{1}{4} \\ 6 \end{pmatrix} - 2\frac{1}{2}\begin{pmatrix} 1\frac{1}{2} \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ -5\frac{1}{4} \\ 16 \end{pmatrix}$

Dus $E\left(0, -5\frac{1}{4}, 16\right)$.

6.4 Vergelijkingen van vlakken

Bladzijde 79

40 **a** $A(3, 0, 0)$ invullen geeft $20 \cdot 3 + 15 \cdot 0 + 12 \cdot 0 = 60$ klopt.
$B(0, 4, 0)$ invullen geeft $20 \cdot 0 + 15 \cdot 4 + 12 \cdot 0 = 60$ klopt.
$C(0, 0, 5)$ invullen geeft $20 \cdot 0 + 15 \cdot 0 + 12 \cdot 5 = 60$ klopt.

b Het linker- en rechterlid delen door 60 geeft $\frac{20x}{60} + \frac{15y}{60} + \frac{12z}{60} = \frac{60}{60}$ ofwel $\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 1$.

c Je kunt de coördinaten van de snijpunten met de assen herkennen aan de getallen in de noemers.

Bladzijde 81

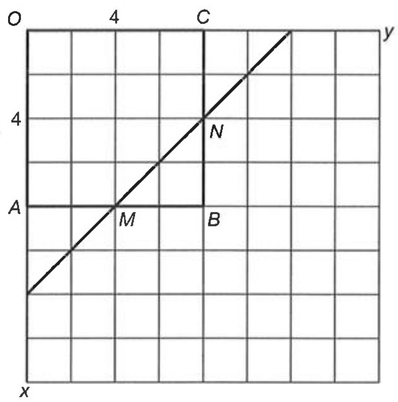
41 **a** ACD snijdt de x-as in $A(4, 0, 0)$, de y-as in $C(0, 4, 0)$ en de z-as in $D(0, 0, 4)$,
dus ACD: $\frac{x}{4} + \frac{y}{4} + \frac{z}{4} = 1$ ofwel $x + y + z = 4$.

Dus $\vec{n}_{ACD} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

b Van C naar M ga je 4 in de x-richting en −2 in de y-richting.
Ga je vanuit $M(4, 2, 0)$ opnieuw 4 in de x-richting en −2 in de y-richting,
dan kom je in het punt $(8, 0, 0)$.
De lijn CM en dus ook het vlak CMD snijdt de x-as in het punt $(8, 0, 0)$.
Dus CMD snijdt de x-as in $(8, 0, 0)$, de y-as in $C(0, 4, 0)$ en de z-as in $D(0, 0, 4)$,
dus CMD: $\frac{x}{8} + \frac{y}{4} + \frac{z}{4} = 1$ ofwel CMD: $x + 2y + 2z = 8$.

Dus $\vec{n}_{CMD} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$.

c Zie de figuur: MN snijdt de x-as in $(6, 0, 0)$ en de y-as in $(0, 6, 0)$.
MND snijdt de x-as in $(6, 0, 0)$, de y-as in $(0, 6, 0)$ en de z-as in
$D(0, 0, 4)$, dus MND: $\frac{x}{6} + \frac{y}{6} + \frac{z}{4} = 1$ ofwel MND: $2x + 2y + 3z = 12$.



Dus $\vec{n}_{MND} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$.

d MNF snijdt de x-as in $(6, 0, 0)$ en de y-as in $(0, 6, 0)$, dus
MNF: $\frac{x}{6} + \frac{y}{6} + \frac{z}{r} = 1$.
$F(4, 4, 4)$ invullen in $\frac{x}{6} + \frac{y}{6} + \frac{z}{r} = 1$ geeft $\frac{4}{6} + \frac{4}{6} + \frac{4}{r} = 1$

$$\frac{2}{3} + \frac{2}{3} + \frac{4}{r} = 1$$
$$\frac{4}{r} = -\frac{1}{3}$$
$$r = -12$$

MNF: $\frac{x}{6} + \frac{y}{6} - \frac{z}{12} = 1$ ofwel MNF: $2x + 2y - z = 12$

Dus $\vec{n}_{MNF} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$.

e $\vec{d} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$ en $\vec{b} - \vec{d} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -4 \end{pmatrix}$, dus BD: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

Substitutie van $x = \lambda, y = \lambda$ en $z = 4 - \lambda$ in $2x + 2y - z = 12$ geeft
$2\lambda + 2\lambda - (4 - \lambda) = 12$
$2\lambda + 2\lambda - 4 + \lambda = 12$
$5\lambda = 16$
$\lambda = 3\frac{1}{5}$
Dus het snijpunt is $\left(3\frac{1}{5}, 3\frac{1}{5}, \frac{4}{5}\right)$.

42 **a** ACD snijdt de x-as in $A(5, 0, 0)$, de y-as in $C(0, 6, 0)$ en de z-as in $D(0, 0, 4)$,
dus ACD: $\frac{x}{5} + \frac{y}{6} + \frac{z}{4} = 1$ ofwel $12x + 10y + 15z = 60$.

b Van C naar M ga je 5 in de x-richting en -3 in de y-richting.
Ga je vanuit $M(5, 3, 0)$ opnieuw 5 in de x-richting en -3 in de y-richting,
dan kom je in het punt $(10, 0, 0)$.
De lijn CM en dus ook het vlak CMD snijdt de x-as in het punt $(10, 0, 0)$.
Dus CMD snijdt de x-as in $(10, 0, 0)$, de y-as in $C(0, 6, 0)$ en de z-as in $D(0, 0, 4)$,
dus CMD: $\frac{x}{10} + \frac{y}{6} + \frac{z}{4} = 1$ ofwel CMD: $6x + 10y + 15z = 60$.

Dus $\vec{n}_{CMD} = \begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix}$.

c $\vec{d} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$ en $\vec{b} - \vec{d} = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ -4 \end{pmatrix}$, dus BD: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 6 \\ -4 \end{pmatrix}$.

ACF snijdt de x-as in $A(5, 0, 0)$, de y-as in $C(0, 6, 0)$ en de z-as in $(0, 0, -4)$,
dus ACF: $\frac{x}{5} + \frac{y}{6} - \frac{z}{4} = 1$ ofwel ACF: $12x + 10y - 15z = 60$.
Substitutie van $x = 5\lambda, y = 6\lambda$ en $z = 4 - 4\lambda$ in $12x + 10y - 15z = 60$ geeft
$12 \cdot 5\lambda + 10 \cdot 6\lambda - 15(4 - 4\lambda) = 60$
$60\lambda + 60\lambda - 60 + 60\lambda = 60$
$180\lambda = 120$
$\lambda = \frac{2}{3}$
Dus het snijpunt is $\left(3\frac{1}{3}, 4, 1\frac{1}{3}\right)$.

43 **a** ACP snijdt de x-as in $A(4, 0, 0)$ en de y-as in $C(0, 6, 0)$, dus ACP: $\frac{x}{4} + \frac{y}{6} + \frac{z}{r} = 1$.

$P\left(1, 1\frac{1}{2}, 2\frac{1}{2}\right)$ invullen in $\frac{x}{4} + \frac{y}{6} + \frac{z}{r} = 1$ geeft $\frac{1}{4} + \frac{1\frac{1}{2}}{6} + \frac{2\frac{1}{2}}{r} = 1$

$$\frac{1}{4} + \frac{1}{4} + \frac{2\frac{1}{2}}{r} = 1$$
$$\frac{2\frac{1}{2}}{r} = \frac{1}{2}$$
$$r = 5$$

ACP: $\frac{x}{4} + \frac{y}{6} + \frac{z}{5} = 1$ ofwel ACP: $15x + 10y + 12z = 60$.

b *MNP* snijdt de x-as in (6, 0, 0) en de y-as in (0, 9, 0), dus *MNP*: $\frac{x}{6}+\frac{y}{9}+\frac{z}{r}=1$.

$P(1, 1\frac{1}{2}, 2\frac{1}{2})$ invullen in $\frac{x}{6}+\frac{y}{9}+\frac{z}{r}=1$ geeft $\frac{1}{6}+\frac{1\frac{1}{2}}{9}+\frac{2\frac{1}{2}}{r}=1$

$$\frac{1}{6}+\frac{1}{6}+\frac{2\frac{1}{2}}{r}=1$$

$$\frac{2\frac{1}{2}}{r}=\frac{2}{3}$$

$$2r=7\tfrac{1}{2}$$

$$r=3\tfrac{3}{4}$$

MNP: $\frac{x}{6}+\frac{y}{9}+\frac{z}{3\frac{3}{4}}=1$ ofwel *MNP*: $15x+10y+24z=90$.

Dus $\vec{n}_{MNP}=\begin{pmatrix}15\\10\\24\end{pmatrix}$.

c $\vec{p}=\begin{pmatrix}1\\1\frac{1}{2}\\2\frac{1}{2}\end{pmatrix}$ en $\vec{b}-\vec{p}=\begin{pmatrix}4\\6\\0\end{pmatrix}-\begin{pmatrix}1\\1\frac{1}{2}\\2\frac{1}{2}\end{pmatrix}=\begin{pmatrix}3\\4\frac{1}{2}\\-2\frac{1}{2}\end{pmatrix}$, dus *BP*: $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}4\\6\\0\end{pmatrix}+\lambda\begin{pmatrix}6\\9\\-5\end{pmatrix}$.

ANT snijdt de x-as in $A(4, 0, 0)$ en de y-as in (0, 12, 0), dus *ANT*: $\frac{x}{4}+\frac{y}{12}+\frac{z}{r}=1$.

$T(2, 3, 5)$ invullen in $\frac{x}{4}+\frac{y}{12}+\frac{z}{r}=1$ geeft $\frac{2}{4}+\frac{3}{12}+\frac{5}{r}=1$

$$\frac{1}{2}+\frac{1}{4}+\frac{5}{r}=1$$

$$\frac{5}{r}=\frac{1}{4}$$

$$r=20$$

Dus *ANT*: $\frac{x}{4}+\frac{y}{12}+\frac{z}{20}=1$ ofwel *ANT*: $15x+5y+3z=60$.
Substitutie van $x=4+6\lambda, y=6+9\lambda$ en $z=-5\lambda$ in $15x+5y+3z=60$ geeft
$15(4+6\lambda)+5(6+9\lambda)+3\cdot -5\lambda=60$
$60+90\lambda+30+45\lambda-15\lambda=60$
$120\lambda=-30$
$\lambda=-\frac{1}{4}$
Dus het snijpunt is $(2\frac{1}{2}, 3\frac{3}{4}, 1\frac{1}{4})$.

Bladzijde 82

44 **a** $\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\cdot\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}=a_1(a_2b_3-a_3b_2)+a_2(a_3b_1-a_1b_3)+a_3(a_1b_2-a_2b_1)=$

$a_1a_2b_3-a_1a_3b_2+a_2a_3b_1-a_1a_2b_3+a_1a_3b_2-a_2a_3b_1=0$

b $\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}\cdot\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}=b_1(a_2b_3-a_3b_2)+b_2(a_3b_1-a_1b_3)+b_3(a_1b_2-a_2b_1)=$

$a_2b_1b_3-a_3b_1b_2+a_3b_1b_2-a_1b_2b_3+a_1b_2b_3-a_2b_1b_3=0$

Bladzijde 84

45 **a**

$$\begin{array}{ccl} 3 & 1 & \\ 1 & 2 & 1\cdot 3 - 2\cdot 2 = -1 \\ 2 & 3 & 2\cdot 1 - 3\cdot 3 = -7 \\ 3 & 1 & 3\cdot 2 - 1\cdot 1 = 5 \\ 1 & 2 & \\ 2 & 3 & \end{array}$$

Dus $\vec{a} \times \vec{b} = \begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix}$.

b

$$\begin{array}{ccl} 3 & 12 & \\ 1 & 4 & 1\cdot 8 - 2\cdot 4 = 0 \\ 2 & 8 & 2\cdot 12 - 3\cdot 8 = 0 \\ 3 & 12 & 3\cdot 4 - 1\cdot 12 = 0 \\ 1 & 4 & \\ 2 & 8 & \end{array}$$

Dus $\vec{a} \times \vec{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

c

$$\begin{array}{ccl} 1 & -3 & \\ 2 & -1 & 2\cdot -2 - 3\cdot -1 = -1 \\ 3 & -2 & 3\cdot -3 - 1\cdot -2 = -7 \\ 1 & -3 & 1\cdot -1 - 2\cdot -3 = 5 \\ 2 & -1 & \\ 3 & -2 & \end{array}$$

Dus $\vec{b} \times \vec{d} = \begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix}$.

d

$$\begin{array}{ccl} 1 & 2 & \\ 2 & 4 & 2\cdot 6 - 3\cdot 4 = 0 \\ 3 & 6 & 3\cdot 2 - 1\cdot 6 = 0 \\ 1 & 2 & 1\cdot 4 - 2\cdot 2 = 0 \\ 2 & 4 & \\ 3 & 6 & \end{array}$$

Dus $\vec{b} \times \vec{e} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

e

$$\begin{array}{ccl} 1 & 3 & \\ 2 & 1 & 2\cdot 2 - 3\cdot 1 = 1 \\ 3 & 2 & 3\cdot 3 - 1\cdot 2 = 7 \\ 1 & 3 & 1\cdot 1 - 2\cdot 3 = -5 \\ 2 & 1 & \\ 3 & 2 & \end{array}$$

Dus $\vec{b} \times \vec{a} = \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix}$.

f

$$\begin{array}{ccl} -3 & 1 & \\ -1 & 2 & -1\cdot 3 - -2\cdot 2 = 1 \\ -2 & 3 & -2\cdot 1 - -3\cdot 3 = 7 \\ -3 & 1 & -3\cdot 2 - -1\cdot 1 = -5 \\ -1 & 2 & \\ -2 & 3 & \end{array}$$

Dus $\vec{d} \times \vec{b} = \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix}$.

46 **a** $\vec{a} \cdot \vec{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3$

$\vec{b} \cdot \vec{a} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = b_1a_1 + b_2a_2 + b_3a_3 = a_1b_1 + a_2b_2 + a_3b_3$

Dus $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ is waar voor elk tweetal vectoren.

b Bij 45c vond je $\vec{b} \times \vec{d} = \begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix}$.

Bij 45f vond je $\vec{d} \times \vec{b} = \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix}$.

Dus $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$ is niet waar voor elk tweetal vectoren.

47 **a** $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1\cdot 1 - 2\cdot -2 \\ 2\cdot 4 - 3\cdot 1 \\ 3\cdot -2 - -1\cdot 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$

b $\begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5\cdot 1 - 1\cdot 0 \\ 1\cdot 2 - 0\cdot 1 \\ 0\cdot 0 - 5\cdot 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ -10 \end{pmatrix}$

c $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1\cdot 0 - 1\cdot 1 \\ 1\cdot 0 - 1\cdot 0 \\ 1\cdot 1 - 1\cdot 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

d $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0\cdot 0 - 0\cdot 1 \\ 0\cdot 0 - 1\cdot 0 \\ 1\cdot 1 - 0\cdot 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

48 $\vec{r}_k \times \vec{r}_m = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \cdot 1 - 2 \cdot 1 \\ 2 \cdot 1 - 2 \cdot 1 \\ 2 \cdot 1 - -1 \cdot 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}$, dus $\vec{r}_l = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$.

$l\colon \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + v\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$.

49 **a** $C(0, 2, 0)$ en $G(0, 2, 2)$, dus $M(0, 2, 1)$.

$\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$

$\overrightarrow{BM} = \vec{m} - \vec{b} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$

$\overrightarrow{AB} \times \overrightarrow{BM} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 - 0 \cdot 0 \\ 0 \cdot -2 - 0 \cdot 1 \\ 0 \cdot 0 - 2 \cdot -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$

De vector $\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ staat loodrecht op $\overrightarrow{AB}$ en $\overrightarrow{BM}$, dus de vector $\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ staat loodrecht op het vlak waar $\overrightarrow{AB}$ en $\overrightarrow{BM}$ richtingsvectoren van zijn. Dat is het vlak ABM.

Dus de vector $\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ staat loodrecht op het vlak ABM en dus is $\vec{n}_{ABM} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$.

b $\vec{n}_{ABM} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$, dus $ABM\colon 2x + 4z = d$.

$A(2, 0, 0)$ invullen geeft $d = 4$.
Dus $ABM\colon 2x + 4z = 4$ ofwel $ABM\colon x + 2z = 2$.

Bladzijde 86

50 **a** $\overrightarrow{EG} = \vec{g} - \vec{e} = \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$\overrightarrow{EP} = \vec{p} - \vec{e} = \begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$

$\vec{n}_{EGP} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ -5 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}$

$\left.\begin{array}{l} EGP\colon 2x + 2y + 5z = d \\ \text{door } E(5, 0, 5) \end{array}\right\} d = 35$

Dus $EGP\colon 2x + 2y + 5z = 35$.

b $AD \perp OEF$, dus $\overrightarrow{AD} = \vec{n}_{OEF}$.

$\overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 5 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$\left.\begin{array}{l} OEF\colon x - z = d \\ \text{door } O(0, 0, 0) \end{array}\right\} d = 0$

Dus $OEF\colon x - z = 0$.

6

c $\overrightarrow{DE} = \vec{e} - \vec{d} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\overrightarrow{EP} = \vec{p} - \vec{e} = \begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$

$\vec{n}_{DEP} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix}$

$\left.\begin{array}{l} DEP\text{: } 2y + 5z = d \\ \text{door } D(0, 0, 5) \end{array}\right\} d = 25$

Dus DEP: $2y + 5z = 25$.

51 **a** $BD \perp ACT$, dus $\overrightarrow{BD} = \vec{n}_{ACT}$.

$\overrightarrow{BD} = \vec{d} - \vec{b} = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\left.\begin{array}{l} ACT\text{: } x + y = d \\ \text{door } A(2, -2, 0) \end{array}\right\} d = 0$

Dus ACT: $x + y = 0$.

b $C(-2, 2, 0)$ en $T(0, 0, 6)$, dus $M(-1, 1, 3)$.

$\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\overrightarrow{AM} = \vec{m} - \vec{a} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$\vec{n}_{ABM} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\left.\begin{array}{l} ABM\text{: } x + z = d \\ \text{door } B(2, 2, 0) \end{array}\right\} d = 2$

Dus ABM: $x + z = 2$.

c $\overrightarrow{BD} = \vec{d} - \vec{b} = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\overrightarrow{BM} = \vec{m} - \vec{b} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix}$

$\vec{n}_{BDM} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$

$\left.\begin{array}{l} BDM\text{: } 3x - 3y + 2z = d \\ \text{door } B(2, 2, 0) \end{array}\right\} d = 0$

Dus BDM: $3x - 3y + 2z = 0$.

52 **a** $\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 6 \\ 0 \end{pmatrix}$

$\overrightarrow{CG} = \vec{g} - \vec{c} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\vec{n}_{ACG} = \begin{pmatrix} -5 \\ 6 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \\ 0 \end{pmatrix}$

$\left.\begin{array}{l} ACG\text{: } 6x + 5y = d \\ \text{door } A(5, 0, 0) \end{array}\right\} d = 30$

Dus ACG: $6x + 5y = 30$.

b $\overrightarrow{BE} = \vec{e} - \vec{b} = \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$

$\overrightarrow{BG} = \vec{g} - \vec{b} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 4 \end{pmatrix}$

$\vec{n}_{BEG} = \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} \times \begin{pmatrix} -5 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -12 \\ -10 \\ -15 \end{pmatrix} \triangleq \begin{pmatrix} 12 \\ 10 \\ 15 \end{pmatrix}$

6.5 Hoeken in de ruimte

Bladzijde 88

53 **a** $D(0, 0, 2)$ en $G(0, 2, 2)$, dus $M\left(\frac{0+0}{2}, \frac{0+2}{2}, \frac{2+2}{2}\right) = M(0, 1, 2)$.

$B(2, 2, 0)$ en $C(0, 2, 0)$, dus $N\left(\frac{2+0}{2}, \frac{2+2}{2}, \frac{0+0}{2}\right) = N(1, 2, 0)$.

$\overrightarrow{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$

$\overrightarrow{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

b $|\overrightarrow{AM}| = \sqrt{(-2)^2 + 1^2 + 2^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$

$|\overrightarrow{MN}| = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$

Bladzijde 90

54 De vectoren $\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$ en $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ hebben dezelfde richting én de vectoren $\begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$ en $\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$ hebben dezelfde richting, dus $\angle\left(\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}\right) = \angle\left(\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}\right)$.

De vectoren $\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$ en $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ hebben dezelfde richting maar de vectoren $\begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$ en $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ hebben niet dezelfde richting, dus $\angle\left(\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}\right) \neq \angle\left(\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}\right)$.

55 $d(A, B) = |\overrightarrow{AB}| = |\vec{b} - \vec{a}| = \left|\begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{pmatrix}\right| = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2} =$

$\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2 + b_1{}^2 + b_2{}^2 + b_3{}^2 - 2a_1b_1 - 2a_2b_2 - 2a_3b_3}$

$d(O, A) = |\vec{a}| = \left|\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}\right| = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}$

$d(O, B) = |\vec{b}| = \left|\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}\right| = \sqrt{b_1{}^2 + b_2{}^2 + b_3{}^2}$

De cosinusregel in driehoek OAB geeft

$d(A, B)^2 = d(O, A)^2 + d(O, B)^2 - 2 \cdot d(O, A) \cdot d(O, B) \cdot \cos(\varphi)$

$a_1{}^2 + a_2{}^2 + a_3^2 + b_1{}^2 + b_2{}^2 + b_3{}^2 - 2a_1b_1 - 2a_2b_2 - 2a_3b_3 = a_1{}^2 + a_2{}^2 + a_3{}^2 + b_1{}^2 + b_2{}^2 + b_3{}^2 - 2 \cdot |\vec{a}| \cdot |\vec{b}| \cdot \cos(\varphi)$

$2 \cdot |\vec{a}| \cdot |\vec{b}| \cdot \cos(\varphi) = 2a_1b_1 + 2a_2b_2 + 2a_3b_3$

$\cos(\varphi) = \dfrac{2a_1b_1 + 2a_2b_2 + 2a_3b_3}{2 \cdot |\vec{a}| \cdot |\vec{b}|} = \dfrac{a_1b_1 + a_2b_2 + a_3b_3}{|\vec{a}| \cdot |\vec{b}|} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$

56 **a** $\vec{r}_{EM} = \vec{m} - \vec{e} = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

$\vec{r}_{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

$\cos(\angle(\vec{r}_{EM}, \vec{r}_{MN})) = \dfrac{\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}}{\left|\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}\right|} = \dfrac{-2 + 1 + 0}{\sqrt{5} \cdot \sqrt{6}} = \dfrac{-1}{\sqrt{30}}$

$\angle(\vec{r}_{EM}, \vec{r}_{MN}) \approx 100{,}5°$

Dus $\angle(EM, MN) \approx 180° - 100{,}5° = 79{,}5°$.

b $\vec{r}_{AG} = \vec{g} - \vec{a} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$\vec{r}_{CE} = \vec{e} - \vec{c} = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$\cos(\angle(\vec{r}_{AG}, \vec{r}_{CE})) = \dfrac{\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}\right|} = \dfrac{-1 - 1 + 1}{\sqrt{3} \cdot \sqrt{3}} = -\tfrac{1}{3}$

$\angle(\vec{r}_{AG}, \vec{r}_{CE}) \approx 109{,}5°$

Dus $\angle(AG, CE) \approx 180° - 109{,}5° = 70{,}5°$.

c $\angle ENG = \angle(\overrightarrow{NE}, \overrightarrow{NG})$

$$\overrightarrow{NE} = \vec{e} - \vec{n} = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}$$

$$\overrightarrow{NG} = \vec{g} - \vec{n} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{NE}, \overrightarrow{NG})) = \frac{\begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}\right|} = \frac{-4 + 0 + 16}{6 \cdot \sqrt{20}} = \frac{12}{6\sqrt{20}} = \frac{2}{\sqrt{20}}$$

$\angle ENG = \angle(\overrightarrow{NE}, \overrightarrow{NG}) \approx 63{,}4°$

57 **a** $$\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 3 \end{pmatrix}$$

$$\vec{r}_{CE} = \vec{c} - \vec{e} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_{BD}, \vec{r}_{CE})) = \frac{\begin{pmatrix} -6 \\ -4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}}{\left|\begin{pmatrix} -6 \\ -4 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}\right|} = \frac{36 - 16 - 9}{\sqrt{61} \cdot \sqrt{61}} = \frac{11}{61}$$

Dus $\angle(BD, CE) = \angle(\vec{r}_{BD}, \vec{r}_{CE}) \approx 79{,}6°$.

b $$\vec{r}_{OG} = \vec{g} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$\vec{r}_{EG} = \vec{g} - \vec{e} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_{OG}, \vec{r}_{EG})) = \frac{\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}}{\left|\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}\right|} = \frac{0 + 8 - 1}{\sqrt{5} \cdot \sqrt{53}} = \frac{7}{\sqrt{265}}$$

Dus $\angle(OG, EG) = \angle(\vec{r}_{OG}, \vec{r}_{EG}) \approx 64{,}5°$.

c $\angle BGD = \angle(\overrightarrow{GB}, \overrightarrow{GD})$

$$\overrightarrow{GB} = \vec{b} - \vec{g} = \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ -2 \end{pmatrix}$$

$$\overrightarrow{GD} = \vec{d} - \vec{g} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{GB}, \overrightarrow{GD})) = \frac{\begin{pmatrix} 6 \\ 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} 6 \\ 0 \\ -2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}\right|} = \frac{0 + 0 - 2}{\sqrt{40} \cdot \sqrt{17}} = \frac{-2}{\sqrt{680}}$$

$\angle BGD = \angle(\overrightarrow{GB}, \overrightarrow{GD}) \approx 94{,}4°$

58 **a** $C(0, 4, 0)$ en $T(2, 2, 6)$, dus $M(1, 3, 3)$.

$$\vec{r}_{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{r}_{BT} = \vec{t} - \vec{b} = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_{AM}, \vec{r}_{BT})) = \frac{\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}\right|} = \frac{1 - 1 + 3}{\sqrt{3} \cdot \sqrt{11}} = \frac{3}{\sqrt{33}}$$

Dus $\angle(AM, BT) = \angle(\vec{r}_{AM}, \vec{r}_{BT}) \approx 58{,}5°$.

b $\vec{r}_{AM} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$$\vec{r}_{CT} = \vec{t} - \vec{c} = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_{AM}, \vec{r}_{CT})) = \frac{\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}\right|} = \frac{-1 - 1 + 3}{\sqrt{3} \cdot \sqrt{11}} = \frac{1}{\sqrt{33}}$$

Dus $\angle(AM, CT) = \angle(\vec{r}_{AM}, \vec{r}_{CT}) \approx 80{,}0°$.

c $\angle OMB = \angle(\overrightarrow{MO}, \overrightarrow{MB})$

$$\overrightarrow{MO} = \vec{o} - \vec{m} = \begin{pmatrix} -1 \\ -3 \\ -3 \end{pmatrix}$$

$$\overrightarrow{MB} = \vec{b} - \vec{m} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -3 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{MO}, \overrightarrow{MB})) = \frac{\begin{pmatrix} -1 \\ -3 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ -3 \end{pmatrix}}{\left|\begin{pmatrix} -1 \\ -3 \\ -3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 3 \\ 1 \\ -3 \end{pmatrix}\right|} = \frac{-3 - 3 + 9}{\sqrt{19} \cdot \sqrt{19}} = \frac{3}{19}$$

$\angle OMB = \angle(\overrightarrow{MO}, \overrightarrow{MB}) \approx 80{,}9°$

Bladzijde 91

59 **a** $\vec{r}_{OT} = \vec{t} = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$, dus OT: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$.

$\lambda = 0$ geeft $O(0, 0, 0)$ en $\lambda = 2$ geeft $T(2, 2, 6)$.
Dus $P(\lambda, \lambda, 3\lambda)$ met $0 \leq \lambda \leq 2$ is voor elke λ een punt op het lijnstuk OT.

b $\vec{r}_{BP} = \vec{p} - \vec{b} = \begin{pmatrix} \lambda \\ \lambda \\ 3\lambda \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda - 4 \\ \lambda - 4 \\ 3\lambda \end{pmatrix}$ en $\vec{r}_{OT} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$.

$BP \perp OT$, dus $\vec{r}_{BP} \cdot \vec{r}_{OT} = 0$

$$\begin{pmatrix} \lambda - 4 \\ \lambda - 4 \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = 0$$

$\lambda - 4 + \lambda - 4 + 9\lambda = 0$

$11\lambda = 8$

$\lambda = \frac{8}{11}$

Dus $P(\frac{8}{11}, \frac{8}{11}, 2\frac{2}{11})$.

c De hoek tussen BP en OT is maximaal 90° voor $P(\frac{8}{11}, \frac{8}{11}, 2\frac{2}{11})$.
De hoek tussen BP en OT is minimaal als $P = O$ of als $P = T$.
Bereken dus $\angle BOT$ en $\angle BTO$.

$\tan(\angle BOT) = \frac{TS}{OS} = \frac{6}{2\sqrt{2}}$, dus $\angle BOT \approx 64{,}8°$.

$\angle BTO = 180° - 2\angle BOT \approx 50{,}5°$ (hoekensom driehoek)
Dus $\angle(BP, OT)$ kan alle waarden aannemen van 50,5° tot en met 90°.

d $\angle(BP, OT) = 55°$

$$\cos(55°) = \frac{\begin{pmatrix} \lambda-4 \\ \lambda-4 \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} \lambda-4 \\ \lambda-4 \\ 3\lambda \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}\right|} \vee \cos(180° - 55°) = \frac{\begin{pmatrix} \lambda-4 \\ \lambda-4 \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} \lambda-4 \\ \lambda-4 \\ 3\lambda \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}\right|}$$

$$\cos(55°) = \frac{\lambda - 4 + \lambda - 4 + 9\lambda}{\sqrt{(\lambda-4)^2 + (\lambda-4)^2 + 9\lambda^2} \cdot \sqrt{11}} \vee \cos(125°) = \frac{\lambda - 4 + \lambda - 4 + 9\lambda}{\sqrt{(\lambda-4)^2 + (\lambda-4)^2 + 9\lambda^2} \cdot \sqrt{11}}$$

$$\cos(55°) = \frac{11\lambda - 8}{\sqrt{11(\lambda-4)^2 + 11(\lambda-4)^2 + 99\lambda^2}} \vee \cos(125°) = \frac{11\lambda - 8}{\sqrt{11(\lambda-4)^2 + 11(\lambda-4)^2 + 99\lambda^2}}$$

$$\cos(55°) = \frac{11\lambda - 8}{\sqrt{22(\lambda-4)^2 + 99\lambda^2}} \vee \cos(125°) = \frac{11\lambda - 8}{\sqrt{22(\lambda-4)^2 + 99\lambda^2}}$$

Voer in $y_1 = \dfrac{11x - 8}{\sqrt{22(x-4)^2 + 99x^2}}$, $y_2 = \cos(55°)$ en $y_3 = \cos(125°)$.

Intersect met y_1 en y_2 geeft $x = 1{,}8075...$

y_1 en y_3 snijden elkaar niet.

Dus $P(1{,}81;\ 1{,}81;\ 5{,}42)$.

60 De tangens van de gevraagde hoek is $\dfrac{50}{40} = 1{,}25$ dus de gevraagde hoek is ongeveer $51{,}3°$.

6

Bladzijde 92

61 **a** Lijn n is een normaal van ACD.

$\vec{r}_{OF} = \vec{f} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$ en $\vec{r}_n = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$.

$$\cos(\angle(\vec{r}_{OF}, \vec{r}_n)) = \frac{\begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}}{\left|\begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}\right|} = \frac{12 + 12 + 12}{\sqrt{29} \cdot \sqrt{61}} = \frac{36}{\sqrt{1769}}$$

$\angle(\vec{r}_{OF}, \vec{r}_n) \approx 31{,}1°$

Dus $\angle(OF, ACD) \approx 90° - 31{,}1° = 58{,}9°$.

b $\vec{r}_{CE} = \vec{e} - \vec{c} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix}$

Lijn n is een normaal van OBG.

$\vec{r}_{OB} = \vec{b} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}$ en $\vec{r}_{OG} = \vec{g} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$

$\vec{r}_n = \vec{n}_{OBG} = \vec{r}_{OB} \times \vec{r}_{OG} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \\ 12 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}$

$$\cos(\angle(\vec{r}_{CE}, \vec{r}_n)) = \frac{\begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}}{\left|\begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}\right|} = \frac{12 + 12 + 12}{\sqrt{29} \cdot \sqrt{61}} = \frac{36}{\sqrt{1769}}$$

$\angle(\vec{r}_{OF}, \vec{r}_n) \approx 31{,}1°$

Dus $\angle(CE, OBG) \approx 90° - 31{,}1° = 58{,}9°$.

Bladzijde 93

62 **a** $\vec{r}_{AG} = \vec{g} - \vec{a} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

Lijn n is een normaal van EBG.

$OF \perp EBG$, dus $\vec{r}_n = \overrightarrow{OF} = \vec{f} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

$$\cos(\angle(\vec{r}_{AG}, \vec{r}_n)) = \frac{\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right|} = \frac{-1+1+1}{\sqrt{3} \cdot \sqrt{3}} = \frac{1}{3}$$

$\angle(\vec{r}_{AG}, \vec{r}_n) \approx 70{,}5°$
Dus $\angle(AG, EBG) \approx 90° - 70{,}5° = 19{,}5°$.

b $\vec{r}_{DM} = \vec{m} - \vec{d} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

Lijn n is een normaal van ACD.

$OF \perp ACD$, dus $\vec{r}_n = \overrightarrow{OF} = \vec{f} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

$$\cos(\angle(\vec{r}_{DM}, \vec{r}_n)) = \frac{\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right|} = \frac{2+1-2}{3 \cdot \sqrt{3}} = \frac{1}{3\sqrt{3}}$$

$\angle(\vec{r}_{DM}, \vec{r}_n) \approx 78{,}9°$
Dus $\angle(DM, ACD) \approx 90° - 78{,}9° = 11{,}1°$.

c $\vec{r}_{GM} = \vec{m} - \vec{g} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$

Lijn n is een normaal van CDM.

$\vec{r}_{CD} = \vec{d} - \vec{c} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

$\vec{r}_{CM} = \vec{m} - \vec{c} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$

$\vec{r}_n = \vec{n}_{CDM} = \vec{r}_{CD} \times \vec{r}_{CM} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

$$\cos(\angle(\vec{r}_{GM}, \vec{r}_n)) = \frac{\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}\right|} = \frac{2-2-4}{3 \cdot 3} = \frac{-4}{9}$$

$\angle(\vec{r}_{GM}, \vec{r}_n) \approx 116{,}4°$
Dus $\angle(GM, n) \approx 180° - 116{,}4° = 63{,}6°$ en $\angle(GM, CDM) \approx 90° - 63{,}6° = 26{,}4°$.

63 Voor de hellingshoek φ geldt $\cos(\varphi) = \frac{3,0}{3,8}$, dus $\varphi \approx 37,9°$.

Bladzijde 94

64 Het vlak MND snijdt de assen in (6, 0, 0), (0, 6, 0) en (0, 0, 4).

MND: $\frac{x}{6} + \frac{y}{6} + \frac{z}{4} = 1$ ofwel MND: $2x + 2y + 3z = 12$

Dus $\vec{r}_m = \vec{n}_{MND} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$.

65 **a** Lijn m is een normaal van MNF en lijn n is een normaal van OND.

$\vec{r}_m = \vec{n}_{MNF} = \overrightarrow{MN} \times \overrightarrow{MF} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ en $\vec{r}_n = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$

$$\cos(\angle(\vec{r}_m, \vec{r}_n)) = \frac{\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}\right|} = \frac{4-2+0}{3 \cdot \sqrt{5}} = \frac{2}{3\sqrt{5}}$$

Dus $\angle(MNF, OND) = \angle(m, n) \approx 72,7°$.

b Lijn m is een normaal van MND en lijn n is een normaal van OMD.

$\vec{r}_m = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$ en $\vec{r}_n = \vec{n}_{OMD} = \overrightarrow{OM} \times \overrightarrow{OD} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ -16 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$

$$\cos(\angle(\vec{r}_m, \vec{r}_n)) = \frac{\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}\right|} = \frac{2-4+0}{\sqrt{17} \cdot \sqrt{5}} = \frac{-2}{\sqrt{85}}$$

$\angle(\vec{r}_m, \vec{r}_n) \approx 102,5°$

Dus $\angle(MND, OMD) = \angle(m, n) \approx 180° - 102,5° = 77,5°$.

Bladzijde 95

66 **a** Lijn m is een normaal van AMN en lijn n is een normaal van OAM.

MN snijdt de y-as in (0, 9, 0) en de z-as in (0, 0, 6).

AMN: $\frac{x}{5} + \frac{y}{9} + \frac{z}{6} = 1$

$18x + 10y + 15z = 90$

Dus $\vec{r}_n = \vec{n}_{AMN} = \begin{pmatrix} 18 \\ 10 \\ 15 \end{pmatrix}$.

$\vec{r}_n = \vec{n}_{OAM} = \overrightarrow{OA} \times \overrightarrow{OM} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -20 \\ 15 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}$

$$\cos(\angle(\vec{r}_m, \vec{r}_n)) = \frac{\begin{pmatrix} 18 \\ 10 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} 18 \\ 10 \\ 15 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}\right|} = \frac{0-40+45}{\sqrt{649} \cdot 5} = \frac{1}{\sqrt{649}}$$

Dus $\angle(AMN, OAM) = \angle(m, n) \approx 87,8°$.

b Lijn m is een normaal van EMN en lijn n is een normaal van BCD.

EMN: $\frac{x}{r} + \frac{y}{9} + \frac{z}{6} = 1$

$E(5, 0, 4)$ invullen in $\frac{x}{r} + \frac{y}{9} + \frac{z}{6} = 1$ geeft $\frac{5}{r} + \frac{0}{9} + \frac{4}{6} = 1$

$$\frac{5}{r} + \frac{2}{3} = 1$$

$$\frac{5}{r} = \frac{1}{3}$$

$$r = 15$$

EMN: $\frac{x}{15} + \frac{y}{9} + \frac{z}{6} = 1$

$6x + 10y + 15z = 90$

Dus $\vec{r}_m = \vec{n}_{EMN} = \begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix}$.

$$\vec{r}_n = \vec{n}_{BCD} = \overrightarrow{BC} \times \overrightarrow{CD} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} -5 \\ -6 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -20 \\ -30 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_m, \vec{r}_n)) = \frac{\begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}\right|} = \frac{0 + 20 + 45}{19 \cdot \sqrt{13}} = \frac{65}{19\sqrt{13}}$$

Dus $\angle(EMN, BCD) = \angle(m, n) \approx 18{,}4°$.

67 **a** $\vec{r}_{DT} = \vec{t} - \vec{d} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$

Lijn n is een normaal van ABN.

$C(-2, 2, 0)$ en $T(0, 0, 3)$, dus $N(-1, 1, 1\frac{1}{2})$.

$$\vec{r}_{AN} = \vec{n} - \vec{a} = \begin{pmatrix} -1 \\ 1 \\ 1\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 1\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$$

$$\vec{r}_{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{ABN} = \vec{r}_{AN} \times \vec{r}_{AB} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_{DT}, \vec{r}_n)) = \frac{\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}\right|} = \frac{2 + 0 + 6}{\sqrt{17} \cdot \sqrt{5}} = \frac{8}{\sqrt{85}}$$

$\angle(DT, n) \approx 29{,}8°$

Dus $\angle(DT, ABN) \approx 90° - 29{,}8° = 60{,}2°$.

b $\vec{r}_{MN} = \vec{n} - \vec{m} = \begin{pmatrix} -1 \\ 1 \\ 1\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 1\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix}$

$\vec{r}_{CT} = \vec{t} - \vec{c} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$

$$\cos(\angle(\vec{r}_{MN}, \vec{r}_{CT})) = \frac{\begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}\right|} = \frac{-4 - 12 + 9}{7 \cdot \sqrt{17}} = \frac{-1}{\sqrt{17}}$$

$\angle(\vec{r}_{MN}, \vec{r}_{CT}) \approx 104{,}0°$
Dus $\angle(MN, CT) \approx 180° - 104{,}0° = 76{,}0°$.

c Lijn m is een normaal van ABN en lijn n is een normaal van ACT.

$\vec{r}_m = \vec{r}_{AN} \times \vec{r}_{AB} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ (zie a)

$\vec{r}_{AC} = \vec{c} - \vec{a} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$\vec{r}_{AT} = \vec{t} - \vec{a} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}$

$\vec{r}_n = \vec{n}_{ACT} = \vec{r}_{AC} \times \vec{r}_{AT} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$$\cos(\angle(\vec{r}_m, \vec{r}_n)) = \frac{\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}{\left|\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right|} = \frac{1 + 0 + 0}{\sqrt{5} \cdot \sqrt{2}} = \frac{1}{\sqrt{10}}$$

$\angle(\vec{r}_m, \vec{r}_n) \approx 71{,}6°$
Dus $\angle(ABN, ACT) = \angle(m, n) \approx 71{,}6°$.

d $\vec{r}_{MN} = \begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix}$ (zie b)

Lijn n is een normaal van ACT.

$\vec{r}_n = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ (zie c)

$$\cos(\angle(\vec{r}_{MN}, \vec{r}_n)) = \frac{\begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}{\left|\begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right|} = \frac{-2 + 6 + 0}{7 \cdot \sqrt{2}} = \frac{4}{7\sqrt{2}}$$

$\angle(\vec{r}_{MN}, \vec{r}_n) \approx 66{,}2°$
Dus $\angle(MN, ACT) \approx 90° - 66{,}2° = 23{,}8°$.

6.6 Afstanden en vectoren

Bladzijde 97

68 a $\overrightarrow{QR} = \vec{r} - \vec{q} = \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix}$

$|\overrightarrow{QR}| = \sqrt{(-4)^2 + (-1)^2 + 2^2} = \sqrt{16 + 1 + 4} = \sqrt{21}$

b $\overrightarrow{AR} = \vec{r} - \vec{a} = \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \\ 4 \end{pmatrix}$

$|\overrightarrow{AR}| = \sqrt{(-4)^2 + 3^2 + 4^2} = \sqrt{16 + 9 + 16} = \sqrt{41}$

Dus de lengte van het lijnstuk AR is $\sqrt{41}$.

c $\overrightarrow{PQ} = \vec{q} - \vec{p} = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$

$|\overrightarrow{PQ}| = \sqrt{2^2 + 4^2 + (-2)^2} = \sqrt{4 + 16 + 4} = \sqrt{24} = 2\sqrt{6}$

Dus de afstand tussen de punten P en Q is $2\sqrt{6}$.

Bladzijde 98

69 a $C(0, 4, 0)$ en $G(0, 4, 2)$, dus $M(0, 4, 1)$.

$\overrightarrow{MP} = \vec{p} - \vec{m} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$

$d(M, P) = |\overrightarrow{MP}| = \sqrt{3^2 + (-4)^2 + 1^2} = \sqrt{9 + 16 + 1} = \sqrt{26}$

b $\vec{r}_{DG} = \vec{g} - \vec{d} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

V is het vlak door B, loodrecht op DG.

$\left.\begin{array}{l} V\colon y = d \\ \text{door } B(4, 4, 0) \end{array}\right\} d = 4$

Dus $V\colon y = 4$.

$DG\colon \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ snijden met V geeft $\lambda = 4$.

Het snijpunt is $G(0, 4, 2)$.

$d(B, DG) = d(B, G) = |\overrightarrow{BG}| = \left|\begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix}\right| = \sqrt{20} = 2\sqrt{5}$

c $\vec{r}_{MP} = \vec{p} - \vec{m} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$

V is het vlak door F, loodrecht op MP.

$\left.\begin{array}{l} V\colon 3x - 4y + z = d \\ \text{door } F(4, 4, 2) \end{array}\right\} d = -2$

Dus $V\colon 3x - 4y + z = -2$.

$MP\colon \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$ snijden met V geeft $3 \cdot 3\lambda - 4(4 - 4\lambda) + 1 + \lambda = -2$

$$\begin{aligned} 9\lambda - 16 + 16\lambda + 1 + \lambda &= -2 \\ 26\lambda &= 13 \\ \lambda &= \tfrac{1}{2} \end{aligned}$$

Het snijpunt is $S\left(1\tfrac{1}{2}, 2, 1\tfrac{1}{2}\right)$.

$d(F, MP) = d(F, S) = |\overrightarrow{FS}| = \left|\begin{pmatrix} -2\frac{1}{2} \\ -2 \\ -\frac{1}{2} \end{pmatrix}\right| = \sqrt{10\tfrac{1}{2}} = \tfrac{1}{2}\sqrt{42}$

Bladzijde 99

70 **a** V is het vlak door P, loodrecht op k.

$\left.\begin{array}{l} V: y + 2z = d \\ \text{door } P(8, 10, 0) \end{array}\right\} d = 10$

Dus $V: y + 2z = 10$.

$k: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ snijden met V geeft $\lambda + 2 \cdot 2\lambda = 10$

$5\lambda = 10$

$\lambda = 2$

Het snijpunt is $S(4, 2, 4)$.

$d(P, k) = d(P, S) = |\overrightarrow{PS}| = \left| \begin{pmatrix} -4 \\ -8 \\ 4 \end{pmatrix} \right| = \sqrt{96} = 4\sqrt{6}$

b V is het vlak door Q, loodrecht op l.

$\left.\begin{array}{l} V: -x + 3z = d \\ \text{door } Q(1, 5, 6) \end{array}\right\} d = 17$

Dus $V: -x + 3z = 17$.

$l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$ snijden met V geeft $-(-\mu) + 3(-1 + 3\mu) = 17$

$10\mu = 20$

$\mu = 2$

Het snijpunt is $S(-2, 1, 5)$.

$d(Q, l) = d(Q, S) = |\overrightarrow{QS}| = \left| \begin{pmatrix} -3 \\ -4 \\ -1 \end{pmatrix} \right| = \sqrt{26}$

c V is het vlak door R, loodrecht op m.

$\left.\begin{array}{l} V: x + 6y + z = d \\ \text{door } R(5, 4, 9) \end{array}\right\} d = 38$

Dus $V: x + 6y + z = 38$.

$m: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = v \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix}$ snijden met V geeft $v + 6 \cdot 6v + v = 38$

$38v = 38$

$v = 1$

Het snijpunt is $S(1, 6, 1)$.

$d(R, m) = d(R, S) = |\overrightarrow{RS}| = \left| \begin{pmatrix} -4 \\ 2 \\ -8 \end{pmatrix} \right| = \sqrt{84} = 2\sqrt{21}$

71 **a** $\vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix}$

V is het vlak door A, loodrecht op BC.

$\left.\begin{array}{l} V: -3x + 2z = d \\ \text{door } A(-3, 0, 3) \end{array}\right\} d = -3 \cdot -3 + 2 \cdot 3 = 15$

Dus $V: -3x + 2z = 15$.

$BC: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix}$ snijden met V geeft $-3 \cdot -3\lambda + 2(1 + 2\lambda) = 15$

$9\lambda + 2 + 4\lambda = 15$

$13\lambda = 13$

$\lambda = 1$

Het snijpunt is $S(-3, 1, 3)$.

$d(A, BC) = d(A, S) = \sqrt{0^2 + 1^2 + 0^2} = \sqrt{1} = 1$

b $O(\triangle ABC) = \frac{1}{2} \cdot d(B, C) \cdot d(A, BC) = \frac{1}{2} \cdot \sqrt{3^2 + 0^2 + 2^2} \cdot 1 = \frac{1}{2}\sqrt{13}$

72 $A(8, 0, 0)$ en $E(8, 0, 8)$, dus $M(8, 0, 4)$.
$E(8, 0, 8)$ en $F(8, 8, 8)$, dus $N(8, 4, 8)$.

$$\vec{r}_{MN} = \vec{n} - \vec{m} = \begin{pmatrix} 8 \\ 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

V is het vlak door $C(0, 8, 0)$, loodrecht op MN.

$$\left.\begin{array}{l} V\text{: } y + z = d \\ \text{door } C(0, 8, 0) \end{array}\right\} d = 8$$

Dus V: $y + z = 8$.

$$MN\text{: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \text{ snijden met } V \text{ geeft } \begin{array}{l} \lambda + 4 + \lambda = 8 \\ 2\lambda = 4 \\ \lambda = 2 \end{array}$$

Het snijpunt is $S(8, 2, 6)$.

$$d(C, MN) = d(C, S) = |\overrightarrow{CS}| = \left| \begin{pmatrix} 8 \\ -6 \\ 6 \end{pmatrix} \right| = \sqrt{136}$$

$$O(\triangle CMN) = \tfrac{1}{2} \cdot d(C, MN) \cdot d(M, N) = \tfrac{1}{2} \cdot \sqrt{136} \cdot \sqrt{4^2 + 4^2} = \tfrac{1}{2}\sqrt{136} \cdot \sqrt{32} = \tfrac{1}{2}\sqrt{4362} = 8\sqrt{17}$$

Bladzijde 100

73
$$\vec{r}_{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$C(0, 4, 0)$ en $T(2, 2, 6)$, dus $M(1, 3, 3)$.

$$\vec{r}_{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{n}_{ABM} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\left.\begin{array}{l} ABM\text{: } x + z = d \\ \text{door } A(4, 0, 0) \end{array}\right\} d = 4$$

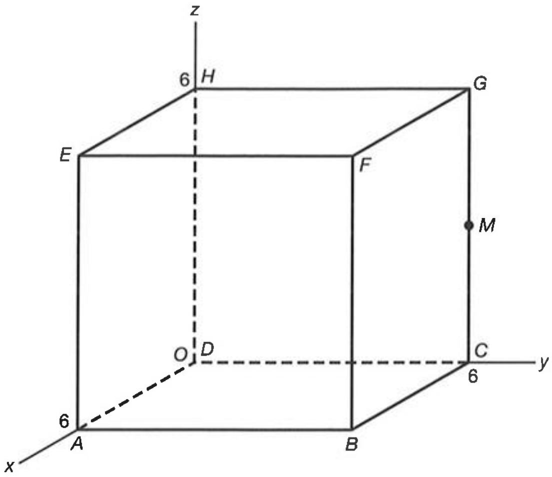
Dus ABM: $x + z = 4$.
$B(4, 4, 0)$ en $T(2, 2, 6)$, dus $N(3, 3, 3)$.
k is de lijn door N, loodrecht op ABM.

$$\vec{s}_k = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \text{ en } \vec{r}_k = \vec{n}_{ABM} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \text{ dus } k\text{: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

k snijden met ABM geeft
$3 + \lambda + 3 + \lambda = 4$
$2\lambda = -2$
$\lambda = -1$
Het snijpunt is $Q(2, 3, 2)$.

$$d(N, ABM) = d(N, Q) = |\overrightarrow{NQ}| = \left| \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} \right| = \sqrt{2}$$

74 Breng een assenstelsel aan zoals in de figuur hieronder.



HM snijdt de *y*-as in (0, 12, 0).

$$\left.\begin{array}{l} BMH\text{: } \frac{x}{r} + \frac{y}{12} + \frac{z}{6} = 1 \\ B(6, 6, 0) \end{array}\right\} \begin{array}{l} \frac{6}{r} + \frac{6}{12} + \frac{0}{6} = 1 \\ \frac{6}{r} = \frac{1}{2} \\ r = 12 \end{array}$$

$BMH\text{: } \frac{x}{12} + \frac{y}{12} + \frac{z}{6} = 1$ ofwel $BMH\text{: } x + y + 2z = 12$

k is de lijn door *F*(6, 6, 6), loodrecht op *BMH*.

$\vec{s}_k = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$ en $\vec{r}_k = \vec{n}_{BMH} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, dus $k\text{: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

k snijden met *BMH* geeft

$6 + \lambda + 6 + \lambda + 2(6 + 2\lambda) = 12$

$6\lambda = -12$

$\lambda = -2$

Het snijpunt is *Q*(4, 4, 2).

$$d(F, BMH) = d(F, Q) = |\vec{FQ}| = \left|\begin{pmatrix} -2 \\ -2 \\ -4 \end{pmatrix}\right| = \sqrt{24} = 2\sqrt{6}$$

Diagnostische toets

Bladzijde 102

1 **a** $\vec{c} = 3\vec{a} + 2\vec{b} = 3 \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix} + 2 \cdot \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 12 \\ 6 \end{pmatrix} + \begin{pmatrix} -6 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$|\vec{c}| = \left|\begin{pmatrix} 6 \\ 8 \end{pmatrix}\right| = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

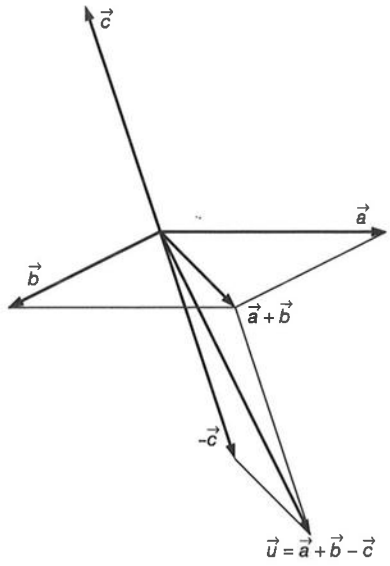
b $\vec{d} = -\vec{a} + 4\vec{b} = -\begin{pmatrix} 4 \\ 2 \end{pmatrix} + 4 \cdot \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} + \begin{pmatrix} -12 \\ 4 \end{pmatrix} = \begin{pmatrix} -16 \\ 2 \end{pmatrix}$

$|\vec{d}| = \left|\begin{pmatrix} -16 \\ 2 \end{pmatrix}\right| = \sqrt{(-16)^2 + 2^2} = \sqrt{260} = 2\sqrt{65}$

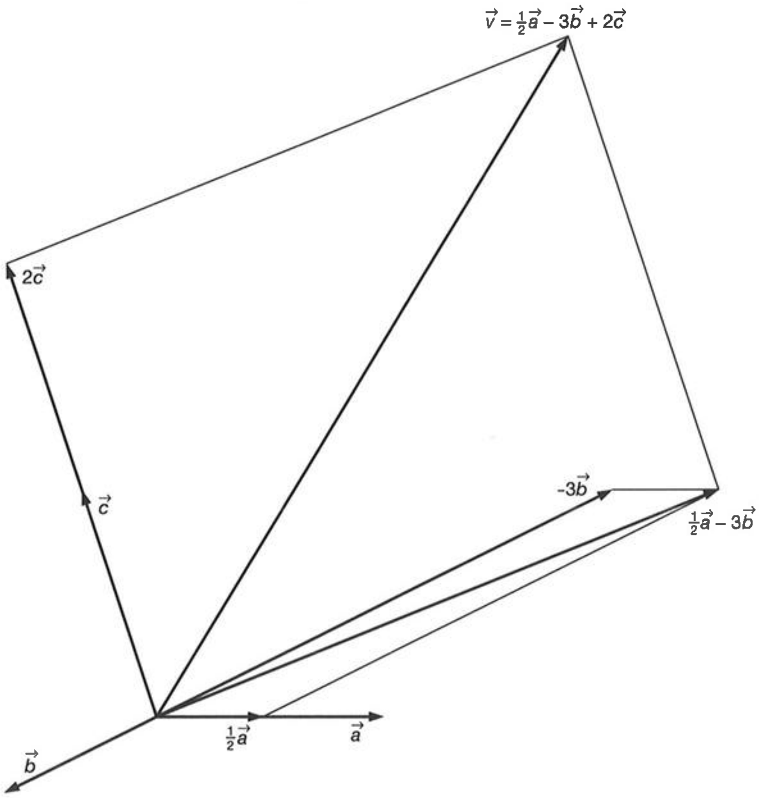
c $\vec{e} = 3(\vec{a} + \vec{b}) = 3\left(\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \end{pmatrix}\right) = 3 \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$

$|\vec{e}| = \left|\begin{pmatrix} 3 \\ 9 \end{pmatrix}\right| = \sqrt{3^2 + 9^2} = \sqrt{90} = 3\sqrt{10}$

2 **a**

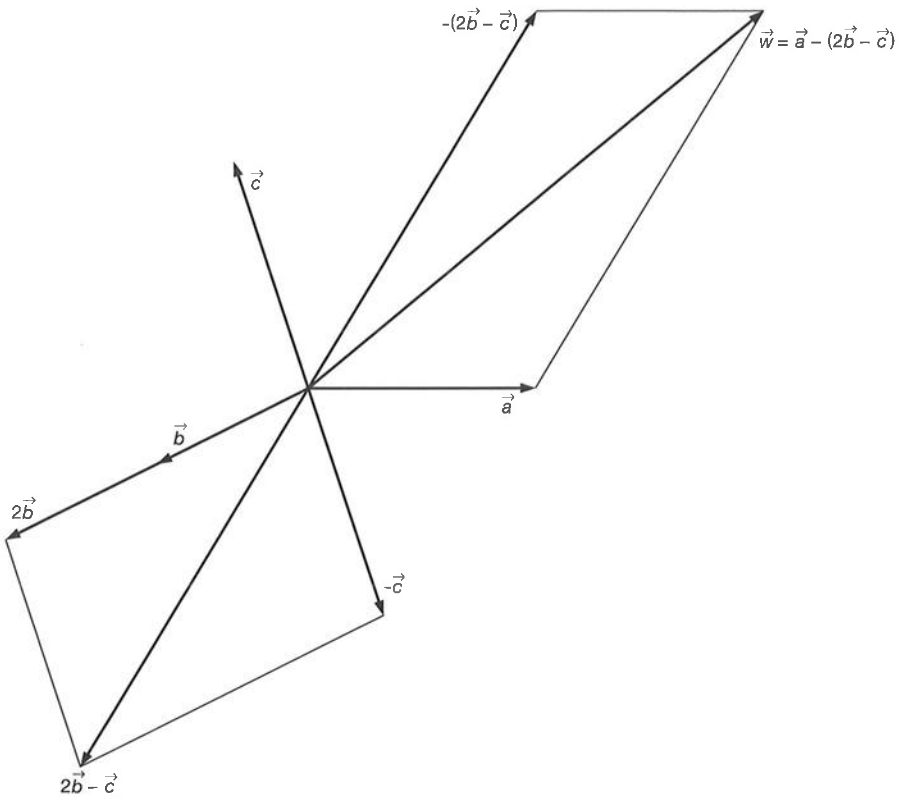

$\vec{c}$
$\vec{a}$
$\vec{b}$
$\vec{a}+\vec{b}$
$-\vec{c}$
$\vec{u}=\vec{a}+\vec{b}-\vec{c}$

b


$\vec{v}=\frac{1}{2}\vec{a}-3\vec{b}+2\vec{c}$
$2\vec{c}$
$\vec{c}$
$-3\vec{b}$
$\frac{1}{2}\vec{a}-3\vec{b}$
$\vec{b}$
$\frac{1}{2}\vec{a}$
$\vec{a}$

6

c



3 **a**

$$\left.\begin{array}{l} k: \begin{pmatrix}x\\y\end{pmatrix} = \vec{b} + \lambda(\vec{a} - \vec{c}) \\ \vec{b} = \begin{pmatrix}4\\2\end{pmatrix} \text{ en } \vec{a} - \vec{c} = \begin{pmatrix}1\\-3\end{pmatrix} - \begin{pmatrix}-2\\1\end{pmatrix} = \begin{pmatrix}3\\-4\end{pmatrix} \end{array}\right\} k: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}4\\2\end{pmatrix} + \lambda\begin{pmatrix}3\\-4\end{pmatrix}$$

b M is het midden van BC, dus $\vec{m} = \frac{1}{2}(\vec{b} + \vec{c}) = \frac{1}{2} \cdot \left(\begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}-2\\1\end{pmatrix}\right) = \frac{1}{2} \cdot \begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}1\\1\frac{1}{2}\end{pmatrix}$.

$$\left.\begin{array}{l} l: \begin{pmatrix}x\\y\end{pmatrix} = \vec{a} + \lambda(\vec{m} - \vec{a}) \\ \vec{a} = \begin{pmatrix}1\\-3\end{pmatrix} \text{ en } \vec{m} - \vec{a} = \begin{pmatrix}1\\1\frac{1}{2}\end{pmatrix} - \begin{pmatrix}1\\-3\end{pmatrix} = \begin{pmatrix}0\\4\frac{1}{2}\end{pmatrix} \end{array}\right\} l: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix} + \lambda\begin{pmatrix}0\\4\frac{1}{2}\end{pmatrix} \text{ ofwel } l: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix} + \lambda\begin{pmatrix}0\\1\end{pmatrix}$$

c N is het midden van AB, dus $\vec{n} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2} \cdot \left(\begin{pmatrix}1\\-3\end{pmatrix} + \begin{pmatrix}4\\2\end{pmatrix}\right) = \frac{1}{2} \cdot \begin{pmatrix}5\\-1\end{pmatrix} = \begin{pmatrix}2\frac{1}{2}\\-\frac{1}{2}\end{pmatrix}$.

$$\left.\begin{array}{l} m: \begin{pmatrix}x\\y\end{pmatrix} = \vec{n} + \lambda(\vec{b} - \vec{c}) \\ \vec{n} = \begin{pmatrix}2\frac{1}{2}\\-\frac{1}{2}\end{pmatrix} \text{ en } \vec{b} - \vec{c} = \begin{pmatrix}4\\2\end{pmatrix} - \begin{pmatrix}-2\\1\end{pmatrix} = \begin{pmatrix}6\\1\end{pmatrix} \end{array}\right\} m: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2\frac{1}{2}\\-\frac{1}{2}\end{pmatrix} + \lambda\begin{pmatrix}6\\1\end{pmatrix}$$

d Stel eerst een vectorvoorstelling op van de lijn AC.

$$\left.\begin{array}{l} AC: \begin{pmatrix}x\\y\end{pmatrix} = \vec{a} + \lambda(\vec{a} - \vec{c}) \\ \vec{a} = \begin{pmatrix}1\\-3\end{pmatrix} \text{ en } \vec{a} - \vec{c} = \begin{pmatrix}1\\-3\end{pmatrix} - \begin{pmatrix}-2\\1\end{pmatrix} = \begin{pmatrix}3\\-4\end{pmatrix} \end{array}\right\} AC: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix} + \lambda\begin{pmatrix}3\\-4\end{pmatrix}$$

$\lambda = 0$ geeft $\begin{pmatrix}1\\-3\end{pmatrix}$ en $\lambda = -1$ geeft $\begin{pmatrix}-2\\1\end{pmatrix}$.

Dus een vectorvoorstelling van het lijnstuk AC is $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix} + \lambda\begin{pmatrix}3\\-4\end{pmatrix} \wedge -1 \leq \lambda \leq 0$.

4 **a** $\vec{r}_{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

$\vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -7 \\ -5 \end{pmatrix}$

$$\cos(\angle(\vec{r}_{AB}, \vec{r}_{BC})) = \frac{\begin{pmatrix} 6 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -7 \\ -5 \end{pmatrix}}{\left|\begin{pmatrix} 6 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -7 \\ -5 \end{pmatrix}\right|} = \frac{6 \cdot -7 + 1 \cdot -5}{\sqrt{37} \cdot \sqrt{74}} = \frac{-47}{\sqrt{2738}}$$

$\angle(\vec{r}_{AB}, \vec{r}_{BC}) \approx 153{,}9°$
Dus $\angle(AB, BC) \approx 180° - 153{,}9° = 26{,}1°$.

b $\vec{r}_{AB} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ en $\vec{r}_{AC} = \vec{c} - \vec{a} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$

$$\cos(\angle(\vec{r}_{AB}, \vec{r}_{AC})) = \frac{\begin{pmatrix} 6 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -4 \end{pmatrix}}{\left|\begin{pmatrix} 6 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -1 \\ -4 \end{pmatrix}\right|} = \frac{6 \cdot -1 + 1 \cdot -4}{\sqrt{37} \cdot \sqrt{17}} = \frac{-10}{\sqrt{629}}$$

$\angle(\vec{r}_{AB}, \vec{r}_{AC}) \approx 113{,}5°$
Dus $\angle(AB, AC) \approx 180° - 113{,}5° = 66{,}5°$.

5 **a** $\vec{r}_l = \vec{n}_k = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(2, 2) op l, dus $\vec{s}_l = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$.

Dus l: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

b $\vec{r}_n = \vec{r}_m = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$, dus $\vec{n}_n = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

$\left.\begin{array}{l} n\text{: } 3x + 5y = c \\ \text{door } (3, 4) \end{array}\right\} c = 3 \cdot 3 + 5 \cdot 4 = 29$

l: $3x + 5y = 29$

c (1, 1) ligt op q, dus $\vec{s}_q = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

$q \perp p$, dus $\vec{r}_q = \vec{n}_p = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

q: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 5 \\ -1 \end{pmatrix}$

d $\left.\begin{array}{l} s \perp r\text{, dus } s\text{: } 4x - y = c \\ \text{door } (-2, 3) \end{array}\right\} c = 4 \cdot -2 - 3 = -11$

s: $4x - y = -11$

6 Substitutie van $x = -2 + 2\lambda$ en $y = 3 - \lambda$ in $2x - 3y = 22$ geeft
$2(-2 + 2\lambda) - 3(3 - \lambda) = 22$
$-4 + 4\lambda - 9 + 3\lambda = 22$
$7\lambda = 35$
$\lambda = 5$
$\lambda = 5$ geeft $x = 8$ en $y = -2$.
Dus $S(8, -2)$.

Bladzijde 103

7 **a** $A(4, 0, 0)$ en $T(0, 0, 6)$ geeft $M(2, 0, 3)$.

$$\vec{n} = \vec{b} + \tfrac{1}{4}\overrightarrow{BT} = \vec{b} + \tfrac{1}{4}(\vec{t} - \vec{b}) = \begin{pmatrix}4\\4\\0\end{pmatrix} + \tfrac{1}{4}\left(\begin{pmatrix}0\\0\\6\end{pmatrix} - \begin{pmatrix}4\\4\\0\end{pmatrix}\right) = \begin{pmatrix}4\\4\\0\end{pmatrix} + \tfrac{1}{4}\begin{pmatrix}-4\\-4\\6\end{pmatrix} = \begin{pmatrix}3\\3\\1\frac{1}{2}\end{pmatrix}$$

Dus $N\left(3, 3, 1\frac{1}{2}\right)$.

$$\vec{p} = \vec{c} + \tfrac{3}{4}\overrightarrow{CT} = \vec{c} + \tfrac{3}{4}(\vec{t} - \vec{c}) = \begin{pmatrix}0\\4\\0\end{pmatrix} + \tfrac{3}{4}\left(\begin{pmatrix}0\\0\\6\end{pmatrix} - \begin{pmatrix}0\\4\\0\end{pmatrix}\right) = \begin{pmatrix}0\\4\\0\end{pmatrix} + \tfrac{3}{4}\begin{pmatrix}0\\-4\\6\end{pmatrix} = \begin{pmatrix}0\\1\\4\frac{1}{2}\end{pmatrix}$$

Dus $P\left(0, 1, 4\frac{1}{2}\right)$.

b $\vec{m} = \begin{pmatrix}2\\0\\3\end{pmatrix}$ en $\vec{n} - \vec{m} = \begin{pmatrix}3\\3\\1\frac{1}{2}\end{pmatrix} - \begin{pmatrix}2\\0\\3\end{pmatrix} = \begin{pmatrix}1\\3\\-1\frac{1}{2}\end{pmatrix} \triangleq \begin{pmatrix}2\\6\\-3\end{pmatrix}$, dus $MN\colon \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2\\0\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\6\\-3\end{pmatrix}$.

$x_D = 0$ geeft $2 + 2\lambda = 0$, dus $\lambda = -1$.

$\lambda = -1$ geeft $\vec{d} = \begin{pmatrix}2\\0\\3\end{pmatrix} - \begin{pmatrix}2\\6\\-3\end{pmatrix} = \begin{pmatrix}0\\-6\\6\end{pmatrix}$

Dus $D(0, -6, 6)$.

c $\vec{m} = \begin{pmatrix}2\\0\\3\end{pmatrix}$ en $\vec{p} - \vec{m} = \begin{pmatrix}0\\1\\4\frac{1}{2}\end{pmatrix} - \begin{pmatrix}2\\0\\3\end{pmatrix} = \begin{pmatrix}-2\\1\\1\frac{1}{2}\end{pmatrix} \triangleq \begin{pmatrix}-4\\2\\3\end{pmatrix}$, dus $MP\colon \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2\\0\\3\end{pmatrix} + \lambda\begin{pmatrix}-4\\2\\3\end{pmatrix}$.

$z_E = 0$ geeft $3 + 3\lambda = 0$, dus $\lambda = -1$.

$\lambda = -1$ geeft $\vec{e} = \begin{pmatrix}2\\0\\3\end{pmatrix} - \begin{pmatrix}-4\\2\\3\end{pmatrix} = \begin{pmatrix}6\\-2\\0\end{pmatrix}$

Dus $E(6, -2, 0)$.

8 **a** ACD snijdt de x-as in $A(6, 0, 0)$, de y-as in $C(0, 8, 0)$ en de z-as in $D(0, 0, 5)$,
dus $ACD\colon \frac{x}{6} + \frac{y}{8} + \frac{z}{5} = 1$ ofwel $20x + 15y + 24z = 120$.

b CDF snijdt de y-as in $(0, 8, 0)$ en de z-as in $(0, 0, 5)$, dus $CDF\colon \frac{x}{r} + \frac{y}{8} + \frac{z}{5} = 1$.

$F(6, 8, 5)$ invullen in $\frac{x}{r} + \frac{y}{8} + \frac{z}{5} = 1$ geeft $\frac{6}{r} + \frac{8}{8} + \frac{5}{5} = 1$

$$\frac{6}{r} + 1 + 1 = 1$$
$$\frac{6}{r} = -1$$
$$r = -6$$

$CDF\colon \frac{x}{-6} + \frac{y}{8} + \frac{z}{5} = 1$ ofwel $CDF\colon -20x + 15y + 24z = 120$

Dus $\vec{n}_{CDF} = \begin{pmatrix}-20\\15\\24\end{pmatrix}$.

d Lijn m is een normaal van DEF en lijn n is een normaal van AFC.

$$\vec{r}_{DE} = \vec{e} - \vec{d} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{r}_{DF} = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} \text{ (zie b)}$$

$$\vec{r}_m = \vec{n}_{DEF} = \vec{r}_{DE} \times \vec{r}_{DF} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 6 \end{pmatrix}$$

$$\vec{r}_{AF} = \vec{f} - \vec{a} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$$

$$\vec{r}_{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{AFC} = \vec{r}_{AF} \times \vec{r}_{AC} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} \times \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_m, \vec{r}_n)) = \frac{\begin{pmatrix} 0 \\ -1 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} 0 \\ -1 \\ 6 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix}\right|} = \frac{0 + 2 + 18}{\sqrt{37} \cdot \sqrt{22}} = \frac{20}{\sqrt{814}}$$

$\angle(\vec{r}_m, \vec{r}_n) \approx 45{,}5°$

Dus $\angle(DEF,\ AFC) = \angle(m, n) \approx 45{,}5°$.

e Lijn m is een normaal van BGD en lijn n is een normaal van OBE.

$$\vec{r}_{BG} = \vec{g} - \vec{b} = \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix}$$

$$\vec{r}_m = \vec{n}_{BGD} = \vec{r}_{BG} \times \vec{r}_{BD} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \\ 6 \end{pmatrix}$$

$$\vec{r}_{OB} = \vec{b} = \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$$\vec{r}_{OE} = \vec{e} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{OBE} = \vec{r}_{OB} \times \vec{r}_{OE} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ -6 \\ -12 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_m, \vec{r}_n)) = \frac{\begin{pmatrix} 6 \\ -1 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}}{\left|\begin{pmatrix} 6 \\ -1 \\ 6 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}\right|} = \frac{18 + 2 - 24}{\sqrt{73} \cdot \sqrt{29}} = \frac{-4}{\sqrt{2117}}$$

$\angle(\vec{r}_m, \vec{r}_n) \approx 95{,}0°$
Dus $\angle(BGD, OBE) \approx 180° - 95{,}0° = 85{,}0°$.

f $$\vec{r}_{CE} = \vec{e} - \vec{c} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix}$$

Lijn m is een normaal van BDF.

$$\vec{r}_{BD} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} \text{ (zie e)}$$

$$\vec{r}_{BF} = \vec{f} - \vec{b} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{r}_n = \vec{n}_{BDF} = \vec{r}_{BD} \times \vec{r}_{BF} = \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_{CE}, \vec{r}_n)) = \frac{\begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}}{\left|\begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}\right|} = \frac{-12 - 12 + 0}{\sqrt{61} \cdot \sqrt{13}} = \frac{-24}{\sqrt{793}}$$

$\angle(\vec{r}_{CE}, \vec{r}_n) \approx 148{,}5°$
Dus $\angle(CE, n) \approx 180° - 148{,}5° = 31{,}5°$ en $\angle(CE, BDF) \approx 90° - 31{,}5° = 58{,}5°$.

11 **a** $\vec{m} = \frac{1}{2}(\vec{d} + \vec{g}) = \frac{1}{2}\left(\begin{pmatrix}0\\0\\4\end{pmatrix} + \begin{pmatrix}0\\8\\4\end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix}0\\8\\8\end{pmatrix} = \begin{pmatrix}0\\4\\4\end{pmatrix}$

$\overrightarrow{MB} = \vec{b} - \vec{m} = \begin{pmatrix}8\\8\\0\end{pmatrix} - \begin{pmatrix}0\\4\\4\end{pmatrix} = \begin{pmatrix}8\\4\\-4\end{pmatrix}$

$d(M, B) = |\overrightarrow{MB}| = \left|\begin{pmatrix}8\\4\\-4\end{pmatrix}\right| = \sqrt{8^2 + 4^2 + (-4)^2} = \sqrt{64 + 16 + 16} = \sqrt{96} = 4\sqrt{6}$

b $\vec{r}_{AC} = \vec{c} - \vec{a} = \begin{pmatrix}0\\8\\0\end{pmatrix} - \begin{pmatrix}8\\0\\0\end{pmatrix} = \begin{pmatrix}-8\\8\\0\end{pmatrix} \triangleq \begin{pmatrix}-1\\1\\0\end{pmatrix}$

V is het vlak door M, loodrecht op AC.

$\left.\begin{array}{l} V\text{: } -x + y = d \\ \text{door } M(0, 4, 4) \end{array}\right\} d = 4$

Dus V: $-x + y = 4$.

AC: $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}8\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}-1\\1\\0\end{pmatrix}$ snijden met $-x + y = 4$ geeft $-(8 - \lambda) + \lambda = 4$

$-8 + \lambda + \lambda = 4$

$2\lambda = 12$

$\lambda = 6$

Het snijpunt is $S(2, 6, 0)$.

$d(M, AC) = d(M, S) = |\overrightarrow{MS}| = \left|\begin{pmatrix}2\\2\\-4\end{pmatrix}\right| = \sqrt{24} = 2\sqrt{6}$

c $\vec{r}_{OB} = \vec{b} = \begin{pmatrix}8\\8\\0\end{pmatrix} \triangleq \begin{pmatrix}1\\1\\0\end{pmatrix}$ en $\vec{r}_{OE} = \vec{e} = \begin{pmatrix}8\\0\\4\end{pmatrix} \triangleq \begin{pmatrix}2\\0\\1\end{pmatrix}$

$\vec{n}_{OBE} = \begin{pmatrix}1\\1\\0\end{pmatrix} \times \begin{pmatrix}2\\0\\1\end{pmatrix} = \begin{pmatrix}1\\-1\\-2\end{pmatrix}$

$\left.\begin{array}{l} OBE\text{: } x - y - 2z = d \\ \text{door } O(0, 0, 0) \end{array}\right\} d = 0$

Dus OBE: $x - y - 2z = 0$.

k is de lijn door M, loodrecht op OBE.

$\vec{s}_k = \vec{m} = \begin{pmatrix}0\\4\\4\end{pmatrix}$ en $\vec{r}_k = \vec{n}_{OBE} = \begin{pmatrix}1\\-1\\-2\end{pmatrix}$, dus k: $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\4\\4\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\-2\end{pmatrix}$.

k snijden met OBE geeft

$\lambda - (4 - \lambda) - 2(4 - 2\lambda) = 0$

$\lambda - 4 + \lambda - 8 + 4\lambda = 0$

$6\lambda = 12$

$\lambda = 2$

Het snijpunt is $S(2, 2, 0)$.

$d(M, OBE) = d(M, S) = |\overrightarrow{MS}| = \left|\begin{pmatrix}2\\-2\\-4\end{pmatrix}\right| = \sqrt{24} = 2\sqrt{6}$

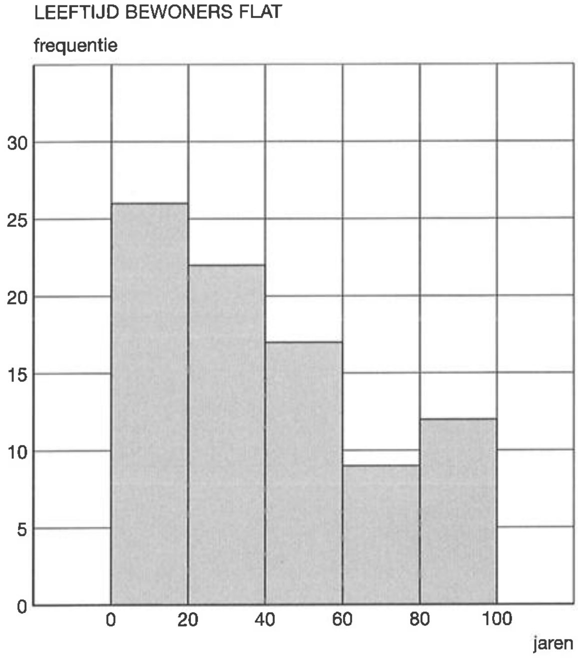
6

7 De normale verdeling

Voorkennis Frequentiepolygonen en centrummaten

Bladzijde 107

1 **a**



b