5 Machten, exponenten en logaritmen

Voorkennis Herleiden van machten

Bladzijde 10

1
- **a** $x^2 \cdot x^3 = x^5$
- **b** $2p^3 \cdot 3p^2 = 6p^5$
- **c** $4a^2b \cdot 5a^3b^2 = 20a^5b^3$
- **d** $-2p^4q^3 \cdot -3pq = 6p^5q^4$
- **e** $5x^2y \cdot 2x - 3x^3y = 10x^3y - 3x^3y = 7x^3y$
- **f** $12a^4b \cdot \frac{1}{4}ab - 8ab = 3a^5b^2 - 8ab$

2
- **a** $(p^2q)^3 = p^6q^3$
- **b** $(3x^2)^3 = 27x^6$
- **c** $(-5x^2y^3)^2 = 25x^4y^6$
- **d** $(-4ab^4)^2 = 16a^2b^8$
- **e** $(3a)^2 \cdot (2a^2)^3 = 9a^2 \cdot 8a^6 = 72a^8$
- **f** $(3a^3)^2 + (2a^2)^3 = 9a^6 + 8a^6 = 17a^6$

3
- **a** $\dfrac{12x^6}{4x^2} = 3x^4$
- **b** $\dfrac{5x^{10}}{15x^5} = \dfrac{x^5}{3} = \frac{1}{3}x^5$
- **c** $\dfrac{24a^4b^2}{6ab} = 4a^3b$
- **d** $\dfrac{-15p^6q}{5p^2q} = -3p^4$
- **e** $\dfrac{10x^3y^2}{5x^2y} = 2xy$
- **f** $\dfrac{(2ab)^3}{(3ab)^2} = \dfrac{8a^3b^3}{9a^2b^2} = \frac{8}{9}ab$

4
- **a** $\dfrac{-28a^6}{7a} = -4a^5$
- **b** $-(3a^4)^2 = -9a^8$
- **c** $(-2a^2)^5 = -32a^{10}$
- **d** $(-a^3)^3 = -a^9$
- **e** $(5a)^3 \cdot -3a = 125a^3 \cdot -3a = -375a^4$
- **f** $\left(\dfrac{9a^4}{a}\right)^2 = (9a^3)^2 = 81a^6$

5
- **a** $(ab)^4 \cdot a = a^4b^4 \cdot a = a^5b^4$
- **b** $(-2ab)^3 \cdot b = -8a^3b^3 \cdot b = -8a^3b^4$
- **c** $(3a)^2 + (2b)^2 = 9a^2 + 4b^2$
- **d** $(3a)^3 - 8a^3 = 27a^3 - 8a^3 = 19a^3$
- **e** $(\frac{1}{2}a)^2 + (-a)^2 = \frac{1}{4}a^2 + a^2 = 1\frac{1}{4}a^2$
- **f** $(5a^4)^2 + (-a^2)^4 = 25a^8 + a^8 = 26a^8$

6
- **a** $a^{2n} \cdot a^{n-1} = a^{2n+n-1} = a^{3n-1}$
- **b** $a^{n^2-1} \cdot a^{n-1} = a^{n^2-1+n-1} = a^{n^2+n-2}$
- **c** $\dfrac{a^{n^2-n}}{a^{n-1}} = a^{n^2-n-(n-1)} = a^{n^2-n-n+1} = a^{n^2-2n+1}$

5.1 Machten met negatieve en gebroken exponenten

Bladzijde 11

1
- **a** De exponenten worden telkens 1 minder en de getallen worden steeds door 2 gedeeld,

  dus $2^1 = \frac{4}{2} = 2$, $2^0 = \frac{2}{2} = 1$, $2^{-1} = \frac{1}{2}$ en $2^{-2} = \dfrac{\frac{1}{2}}{2} = \frac{1}{4}$.
- **b** $2^{-3} = \dfrac{1}{2^3}$

  $2^{-4} = \dfrac{1}{2^4}$
- **c** $x^0 = 1$

  $x^{-1} = \dfrac{1}{x}$

  $x^{-3} = \dfrac{1}{x^3}$

Bladzijde 12

2

a $\frac{1}{a^2} = a^{-2}$

b $a^4 \cdot \frac{1}{a^6} = a^4 \cdot a^{-6} = a^{-2}$

c $\frac{a^n}{\left(\frac{1}{a^4}\right)} = \frac{a^n}{a^{-4}} = a^{n+4}$

d $\frac{a^8}{a^0} = a^{8-0} = a^8$

e $(a^3)^{-2} = a^{-6}$

f $\frac{\left(\frac{1}{a^5}\right)}{a} = \frac{a^{-5}}{a^1} = a^{-5-1} = a^{-6}$

g $\frac{a}{a^{12}} = \frac{a^1}{a^{12}} = a^{1-12} = a^{-11}$

h $\frac{1}{a^8} \cdot (a^3)^n = a^{-8} \cdot a^{3n} = a^{-8+3n}$

i $\frac{\left(\frac{1}{a^n}\right)}{a^{-3}} = \frac{a^{-n}}{a^{-3}} = a^{-n--3} = a^{-n+3}$

Bladzijde 13

3

a $6a^{-5}b^3 = 6 \cdot \frac{1}{a^5} \cdot b^3 = \frac{6b^3}{a^5}$

b $\frac{1}{3}a^{-3} = \frac{1}{3} \cdot \frac{1}{a^3} = \frac{1}{3a^3}$

c $(5a^{-4}b^2)^{-1} = 5^{-1} \cdot (a^{-4})^{-1} \cdot (b^2)^{-1} = \frac{1}{5}a^4b^{-2} = \frac{a^4}{5b^2}$

d $\frac{3}{5}a^{-4} = \frac{3}{5} \cdot \frac{1}{a^4} = \frac{3}{5a^4}$

e $(\frac{1}{2}a)^{-3} = (2^{-1} \cdot a)^{-3} = 2^3 \cdot a^{-3} = 8 \cdot \frac{1}{a^3} = \frac{8}{a^3}$

f $\frac{1}{6}a^{-2}b^4 = \frac{1}{6} \cdot \frac{1}{a^2} \cdot b^4 = \frac{b^4}{6a^2}$

g $-4 \cdot (3a)^{-2} = -4 \cdot \frac{1}{(3a)^2} = -4 \cdot \frac{1}{9a^2} = -\frac{4}{9a^2}$

h $(3a)^{-2}b^{-3} = \frac{1}{(3a)^2} \cdot \frac{1}{b^3} = \frac{1}{9a^2b^3}$

i $(\frac{3}{8}a^{-1}b)^{-2} = (\frac{3}{8})^{-2} \cdot (a^{-1})^{-2} \cdot b^{-2} = \frac{1}{(\frac{3}{8})^2} \cdot a^2 \cdot \frac{1}{b^2} = \frac{64}{9} \cdot a^2 \cdot \frac{1}{b^2} = \frac{64a^2}{9b^2}$

4 Uit $(\sqrt[3]{x})^3 = x$ en $(x^{\frac{1}{3}})^3 = x$ volgt $x^{\frac{1}{3}} = \sqrt[3]{x}$.

Bladzijde 14

5

a $a \cdot \sqrt[3]{a} = a^1 \cdot a^{\frac{1}{3}} = a^{1\frac{1}{3}}$

b $\frac{1}{\sqrt{a}} = \frac{1}{a^{\frac{1}{2}}} = a^{-\frac{1}{2}}$

c $\frac{1}{a} = \frac{1}{a^1} = a^{-1}$

d $\frac{1}{a^3} = a^{-3}$

e $a^2 \cdot \sqrt{a} = a^2 \cdot a^{\frac{1}{2}} = a^{2\frac{1}{2}}$

f $\sqrt[3]{\frac{1}{a^2}} = \sqrt[3]{a^{-2}} = a^{-\frac{2}{3}}$

g $\sqrt[3]{a^{12}} = a^{\frac{12}{3}} = a^4$

h $\frac{1}{a^4} \cdot \sqrt[3]{a} = a^{-4} \cdot a^{\frac{1}{3}} = a^{-4+\frac{1}{3}} = a^{-3\frac{2}{3}}$

i $\frac{a^3}{\sqrt[3]{a}} = \frac{a^3}{a^{\frac{1}{3}}} = a^{3-\frac{1}{3}} = a^{2\frac{2}{3}}$

6

a $8\sqrt{2} = 2^3 \cdot 2^{\frac{1}{2}} = 2^{3\frac{1}{2}}$

b $\frac{1}{3}\sqrt{3} = 3^{-1} \cdot 3^{\frac{1}{2}} = 3^{-1+\frac{1}{2}} = 3^{-\frac{1}{2}}$

c $\frac{4\sqrt{2}}{\sqrt[3]{2}} = \frac{2^2 \cdot 2^{\frac{1}{2}}}{2^{\frac{1}{3}}} = 2^{2+\frac{1}{2}-\frac{1}{3}} = 2^{2\frac{1}{6}}$

d $\frac{1}{100}\sqrt{10} = 10^{-2} \cdot 10^{\frac{1}{2}} = 10^{-2+\frac{1}{2}} = 10^{-1\frac{1}{2}}$

e $\frac{1}{8} \cdot \sqrt[3]{\frac{1}{4}} = \frac{1}{2^3} \cdot \sqrt[3]{2^{-2}} = 2^{-3} \cdot 2^{-\frac{2}{3}} = 2^{-3-\frac{2}{3}} = 2^{-3\frac{2}{3}}$

f $10 \cdot \sqrt[3]{0{,}1} = 10^1 \cdot \sqrt[3]{10^{-1}} = 10^1 \cdot 10^{-\frac{1}{3}} = 10^{1-\frac{1}{3}} = 10^{\frac{2}{3}}$

7 **a** $5a^{3\frac{1}{3}} = 5a^3 \cdot a^{\frac{1}{3}} = 5a^3 \cdot \sqrt[3]{a}$

b $\frac{1}{2}a^{-\frac{1}{4}}b = \frac{b}{2a^{\frac{1}{4}}} = \frac{b}{2 \cdot \sqrt[4]{a}}$

c $3a^{-\frac{2}{3}} = \frac{3}{a^{\frac{2}{3}}} = \frac{3}{\sqrt[3]{a^2}}$

d $\frac{2}{3}a^{-3} \cdot b^{1\frac{1}{2}} = \frac{2b^{1\frac{1}{2}}}{3a^3} = \frac{2b\sqrt{b}}{3a^3}$

e $\frac{1}{5}a^{-\frac{1}{2}} \cdot b^{\frac{1}{3}} = \frac{b^{\frac{1}{3}}}{5a^{\frac{1}{2}}} = \frac{\sqrt[3]{b}}{5\sqrt{a}}$

f $(5a)^{-\frac{1}{2}} = \frac{1}{(5a)^{\frac{1}{2}}} = \frac{1}{\sqrt{5a}}$

8 **a** $\frac{x^6}{x^2 \cdot \sqrt{x}} = \frac{x^6}{x^2 \cdot x^{\frac{1}{2}}} = \frac{x^6}{x^{2\frac{1}{2}}} = x^{6-2\frac{1}{2}} = x^{3\frac{1}{2}}$

b $x \cdot \sqrt[7]{x^3} = x^1 \cdot x^{\frac{3}{7}} = x^{1\frac{3}{7}}$

c $\frac{x}{\sqrt[5]{x}} = \frac{x^1}{x^{\frac{1}{5}}} = x^{1-\frac{1}{5}} = x^{\frac{4}{5}}$

d $x^4 \cdot \sqrt{x} = x^4 \cdot x^{\frac{1}{2}} = x^{4\frac{1}{2}}$

e $\frac{\sqrt[3]{x}}{\sqrt{x}} = \frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = x^{\frac{1}{3}-\frac{1}{2}} = x^{-\frac{1}{6}}$

f $\frac{x^4 \cdot \sqrt[5]{x}}{x^5 \cdot \sqrt[4]{x}} = \frac{x^4 \cdot x^{\frac{1}{5}}}{x^5 \cdot x^{\frac{1}{4}}} = \frac{x^{4\frac{1}{5}}}{x^{5\frac{1}{4}}} = x^{4\frac{1}{5}-5\frac{1}{4}} = x^{-1\frac{1}{20}}$

9 De getallen als macht van 2 geschreven zijn: 2^0, 2^1, 2^{-1}, 2^2, 2^{-3}, 2^5, 2^{-8}, 2^{13}, 2^{-21}, 2^{34}, ...
Het tiende getal is dus 2^{34}.

10 **a** $y = (2x^2)^3 \cdot \frac{2}{x^{10}} = 8x^6 \cdot 2x^{-10} = 16x^{-4}$
Dus $y = 16x^{-4}$.

b $y = \frac{3}{x} \cdot \sqrt[4]{x^3} = 3x^{-1} \cdot x^{\frac{3}{4}} = 3x^{-1+\frac{3}{4}} = 3x^{-\frac{1}{4}}$
Dus $y = 3x^{-\frac{1}{4}}$.

c $y = 3(\frac{1}{3}x^2)^{-2} \cdot 6x^2 = 3 \cdot (\frac{1}{3})^{-2} \cdot (x^2)^{-2} \cdot 6x^2 = 3 \cdot 9 \cdot x^{-4} \cdot 6x^2 = 162x^{-2}$
Dus $y = 162x^{-2}$.

d $y = \frac{5}{3x\sqrt{x}} = \frac{5}{3} \cdot \frac{1}{x \cdot x^{\frac{1}{2}}} = 1\frac{2}{3} \cdot \frac{1}{x^{1\frac{1}{2}}} = 1\frac{2}{3}x^{-1\frac{1}{2}}$
Dus $y = 1\frac{2}{3}x^{-1\frac{1}{2}}$.

11 **a** $y = \frac{5}{x\sqrt{x}} = \frac{5}{x^{1\frac{1}{2}}} = 5x^{-1\frac{1}{2}}$
Dus $y = 5x^{-1\frac{1}{2}}$.

b $y = 5x\sqrt{x^3} = 5x^1 \cdot x^{\frac{3}{2}} = 5x^{1+1\frac{1}{2}} = 5x^{2\frac{1}{2}}$
Dus $y = 5x^{2\frac{1}{2}}$.

c $y = \frac{5}{x^3} \cdot 2\sqrt{x} = 5 \cdot x^{-3} \cdot 2x^{\frac{1}{2}} = 10x^{-3+\frac{1}{2}} = 10x^{-2\frac{1}{2}}$
Dus $y = 10x^{-2\frac{1}{2}}$.

d $y = 72x(\frac{1}{4}x\sqrt{x})^3 = 72x \cdot (\frac{1}{4})^3 \cdot (x^{1\frac{1}{2}})^3 = 72x \cdot \frac{1}{64} \cdot x^{4\frac{1}{2}} = 1\frac{1}{8}x^{5\frac{1}{2}}$
Dus $y = 1\frac{1}{8}x^{5\frac{1}{2}}$.

Bladzijde 15

12 **a** $(x^{\frac{2}{3}})^{\frac{3}{2}} = 10^{\frac{3}{2}}$

b $(x^{\frac{2}{3}})^{\frac{3}{2}} = 10^{\frac{3}{2}}$
$x^1 = 10^1 \cdot 10^{\frac{1}{2}}$
$x = 10\sqrt{10}$

13 **a** $x^{\frac{2}{3}} = 9$
$x = 9^{\frac{3}{2}}$
$x = 9\sqrt{9} = 27$

b $8x^{-1\frac{1}{2}} = 1$
$x^{-1\frac{1}{2}} = \frac{1}{8}$
$x = (\frac{1}{8})^{-\frac{2}{3}}$
$x = (2^{-3})^{-\frac{2}{3}}$
$x = 2^2 = 4$

c $5 - 2x^{-3} = 4$
$-2x^{-3} = -1$
$x^{-3} = \frac{1}{2}$
$x = (\frac{1}{2})^{-\frac{1}{3}}$
$x = (2^{-1})^{-\frac{1}{3}}$
$x = 2^{\frac{1}{3}} = \sqrt[3]{2}$

d $\sqrt{(2x)^3} = \frac{1}{8}$
$(2x)^3 = (\frac{1}{8})^2$
$8x^3 = \dfrac{1}{8^2}$
$x^3 = \dfrac{1}{8^3}$
$x = \frac{1}{8}$

Bladzijde 16

14 **a** $\frac{1}{3}x^{1\frac{1}{2}} = 2\frac{2}{3} - \frac{1}{3}x^{1\frac{1}{2}}$
$\frac{2}{3}x^{1\frac{1}{2}} = 2\frac{2}{3}$
$x^{\frac{3}{2}} = 4$
$x = 4^{\frac{2}{3}} = \sqrt[3]{16}$

b $3 \cdot \sqrt[4]{(2x)^{-1}} = 6$
$\sqrt[4]{(2x)^{-1}} = 2$
$(2x)^{-\frac{1}{4}} = 2$
$2x = 2^{-4}$
$2x = \frac{1}{16}$
$x = \frac{1}{32}$

c $(x + 9)^{-1\frac{1}{2}} = \frac{8}{27}$
$x + 9 = (\frac{8}{27})^{-\frac{2}{3}}$
$x + 9 = ((\frac{2}{3})^3)^{-\frac{2}{3}}$
$x + 9 = (\frac{2}{3})^{-2} = \dfrac{1}{(\frac{2}{3})^2} = \dfrac{1}{\frac{4}{9}} = \frac{9}{4}$
$x = -6\frac{3}{4}$

d $\frac{1}{3}(x^2 - 3)^{2\frac{1}{2}} = 81$
$(x^2 - 3)^{2\frac{1}{2}} = 243$
$x^2 - 3 = 243^{\frac{2}{5}} = (3^5)^{\frac{2}{5}} = 3^2 = 9$
$x^2 = 12$
$x = \sqrt{12} \vee x = -\sqrt{12}$
$x = 2\sqrt{3} \vee x = -2\sqrt{3}$

15 $(x^2 + 10)^{1\frac{1}{2}} = 3x^2(x^2 + 10)^{\frac{1}{2}}$
$(x^2 + 10)^1 \cdot (x^2 + 10)^{\frac{1}{2}} = 3x^2(x^2 + 10)^{\frac{1}{2}}$
$(x^2 + 10)^{\frac{1}{2}} = 0 \vee x^2 + 10 = 3x^2$
$x^2 + 10 = 0 \vee 2x^2 = 10$
$x^2 = -10 \vee x^2 = 5$
$x = \sqrt{5} \vee x = -\sqrt{5}$

16 $y = 27x^3$
$27x^3 = y$
$x^3 = \frac{1}{27}y$
$x = (\frac{1}{27}y)^{\frac{1}{3}}$
$x = (\frac{1}{27})^{\frac{1}{3}} \cdot y^{\frac{1}{3}}$
$x = (3^{-3})^{\frac{1}{3}} \cdot y^{\frac{1}{3}}$
$x = 3^{-1} \cdot y^{\frac{1}{3}}$
$x = \frac{1}{3} \cdot y^{\frac{1}{3}}$
Dus $x = \frac{1}{3}y^{\frac{1}{3}}$.

Bladzijde 17

17 **a** $y = 5x^{1\frac{1}{2}}$

$5x^{1\frac{1}{2}} = y$

$x^{\frac{3}{2}} = 0{,}2y$

$x = (0{,}2y)^{\frac{2}{3}}$

$x = 0{,}2^{\frac{2}{3}} \cdot y^{\frac{2}{3}}$

$x \approx 0{,}34 \cdot y^{0{,}67}$

Dus $x = 0{,}34y^{0{,}67}$.

b $y = 0{,}1x^{-1\frac{2}{3}}$

$0{,}1x^{-1\frac{2}{3}} = y$

$x^{-\frac{5}{3}} = 10y$

$x = (10y)^{-\frac{3}{5}}$

$x = 10^{-\frac{3}{5}} \cdot y^{-\frac{3}{5}}$

$x \approx 0{,}25 \cdot y^{-0{,}6}$

Dus $x = 0{,}25y^{-0{,}6}$.

c $y = 125x^{-2\frac{1}{2}}$

$125x^{-2\frac{1}{2}} = y$

$x^{-\frac{5}{2}} = 0{,}008y$

$x = (0{,}008y)^{-\frac{2}{5}}$

$x = 0{,}008^{-\frac{2}{5}} \cdot y^{-\frac{2}{5}}$

$x \approx 6{,}90 \cdot y^{-0{,}4}$

Dus $x = 6{,}90y^{-0{,}4}$.

18 **a** $y = 15x \cdot \sqrt[3]{x} = 15x \cdot x^{\frac{1}{3}} = 15x^{1\frac{1}{3}}$

$y = 15x^{1\frac{1}{3}}$ geeft $15x^{1\frac{1}{3}} = y$

$x^{\frac{4}{3}} = \frac{1}{15}y$

$x = (\frac{1}{15}y)^{\frac{3}{4}}$

$x = (\frac{1}{15})^{\frac{3}{4}} \cdot y^{\frac{3}{4}}$

$x \approx 0{,}13 \cdot y^{0{,}75}$

Dus $x = 0{,}13y^{0{,}75}$.

b $y = \dfrac{12}{x \cdot \sqrt[4]{x}} = \dfrac{12}{x \cdot x^{\frac{1}{4}}} = \dfrac{12}{x^{1\frac{1}{4}}} = 12x^{-1\frac{1}{4}}$

$y = 12x^{-1\frac{1}{4}}$ geeft $12x^{-1\frac{1}{4}} = y$

$x^{-\frac{5}{4}} = \frac{1}{12}y$

$x = (\frac{1}{12}y)^{-\frac{4}{5}}$

$x = (\frac{1}{12})^{-\frac{4}{5}} \cdot y^{-\frac{4}{5}}$

$x \approx 7{,}30 \cdot y^{-0{,}8}$

Dus $x = 7{,}30y^{-0{,}8}$.

c $y = \dfrac{6}{x^2 \cdot \sqrt[5]{x^3}} = \dfrac{6}{x^2 \cdot x^{\frac{3}{5}}} = \dfrac{6}{x^{2\frac{3}{5}}} = 6x^{-2\frac{3}{5}}$

$y = 6x^{-2\frac{3}{5}}$ geeft $6x^{-2\frac{3}{5}} = y$

$x^{-\frac{13}{5}} = \frac{1}{6}y$

$x = (\frac{1}{6}y)^{-\frac{5}{13}}$

$x = (\frac{1}{6})^{-\frac{5}{13}} \cdot y^{-\frac{5}{13}}$

$x \approx 1{,}99 \cdot y^{-0{,}38}$

Dus $x = 1{,}99y^{-0{,}38}$.

19 **a** $K = 15q^{-1{,}6}$ geeft $15q^{-1{,}6} = K$

$q^{-\frac{8}{5}} = \frac{1}{15}K$

$q = (\frac{1}{15}K)^{-\frac{5}{8}}$

$q = (\frac{1}{15})^{-\frac{5}{8}} \cdot K^{-\frac{5}{8}}$

$q \approx 5{,}43 \cdot K^{-0{,}625}$

Dus $q = 5{,}43K^{-0{,}625}$.

b $v = 25t\sqrt{t} = 25t^{1\frac{1}{2}}$

$v = 25t^{1\frac{1}{2}}$ geeft $25t^{1\frac{1}{2}} = v$

$t^{\frac{3}{2}} = 0{,}04v$

$t = (0{,}04v)^{\frac{2}{3}}$

$t = (0{,}04)^{\frac{2}{3}} \cdot v^{\frac{2}{3}}$

$t \approx 0{,}12 \cdot v^{0{,}67}$

Dus $t = 0{,}12v^{0{,}67}$.

20 $F = \dfrac{m\sqrt{m}}{m\sqrt{m} - 1}$

$F(m\sqrt{m} - 1) = m\sqrt{m}$

$m\sqrt{m} \cdot F - F = m\sqrt{m}$

$m\sqrt{m} \cdot F - m\sqrt{m} = F$

$m\sqrt{m}(F - 1) = F$

$m\sqrt{m} = \dfrac{F}{F-1}$

$m^{\frac{3}{2}} = \dfrac{F}{F-1}$

$m = \left(\dfrac{F}{F-1}\right)^{\frac{2}{3}}$

Bladzijde 18

21 **a** $h_0 = 0{,}6$ en $d_0 = 1000$ geeft

$h = 0{,}6 \cdot \left(\dfrac{1000}{d}\right)^{0{,}25} = 0{,}6 \cdot (1000d^{-1})^{0{,}25} = 0{,}6 \cdot 1000^{0{,}25} \cdot d^{-0{,}25} \approx 3{,}37 \cdot d^{-0{,}25}$

Dus $h = 3{,}37d^{-0{,}25}$.

b $d = 300$ geeft $h = 3{,}37 \cdot 300^{-0{,}25} \approx 0{,}8$

De waterhoogte bij een waterdiepte van 300 meter is 8 dm.

c $h = 3{,}37d^{-0{,}25}$ geeft $3{,}37d^{-0{,}25} = h$

$$d^{-\frac{1}{4}} = \frac{1}{3{,}37}h$$

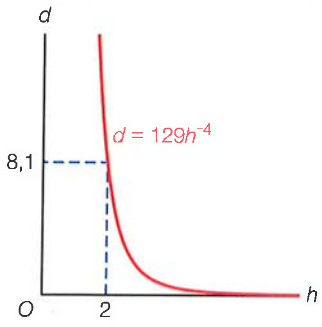
$$d = (\tfrac{1}{3{,}37}h)^{-4}$$

$$d = (\tfrac{1}{3{,}37})^{-4} \cdot h^{-4}$$

$$d \approx 129 \cdot h^{-4}$$

Dus $d = 129h^{-4}$.

d $h = 2$ geeft $d = 129 \cdot 2^{-4} \approx 8{,}1$



$h > 2$ geeft $d < 8{,}1$

Dus bij waterdieptes minder dan 8,1 meter is de golf meer dan 2 meter hoog.

22 **a** $h = 1{,}5$ geeft $D = 1014(1 - 0{,}0226 \cdot 1{,}5)^{5{,}26} \approx 846$

Dus op een hoogte van 1,5 km is de luchtdruk 846 mbar.

b $D = 1014(1 - 0{,}0226h)^{5{,}26}$ geeft $1014(1 - 0{,}0226h)^{5{,}26} = D$

$$(1 - 0{,}0226h)^{5{,}26} = \frac{1}{1014}D$$

$$1 - 0{,}0226h = (\tfrac{1}{1014}D)^{\frac{1}{5{,}26}}$$

$$-0{,}0226h = (\tfrac{1}{1014})^{\frac{1}{5{,}26}} \cdot D^{\frac{1}{5{,}26}} - 1$$

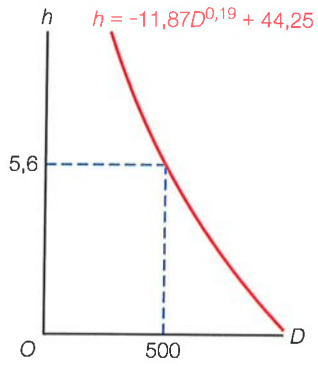
$$h = \frac{(\frac{1}{1014})^{\frac{1}{5{,}26}}}{-0{,}0226} \cdot D^{\frac{1}{5{,}26}} - \frac{1}{-0{,}0226}$$

$$h \approx -11{,}87 \cdot D^{0{,}19} + 44{,}25$$

Dus $h = -11{,}87D^{0{,}19} + 44{,}25$.

c 0,5 bar = 500 mbar

$D = 500$ geeft $h = -11{,}87 \cdot 500^{0{,}19} + 44{,}25 \approx 5{,}6$



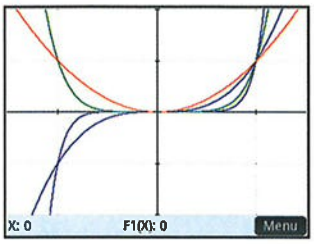
$D < 500$ geeft $h > 5{,}6$

Bij hoogten hoger dan 5,6 km is de luchtdruk minder dan 500 mbar = 0,5 bar.

5.2 Machtsfuncties en wortelfuncties

Bladzijde 20

23 **a**



b (0, 0) en (1, 1)

c Van de grafieken van f en h ligt geen enkel punt onder de x-as.

Bladzijde 22

24 **a** $y = -\frac{1}{2}x^3$

↓ translatie (-3, -5)

$y = -\frac{1}{2}(x + 3)^3 - 5$

↓ verm. x-as, -3

$y = -3(-\frac{1}{2}(x + 3)^3 - 5)$

oftewel $y = 1\frac{1}{2}(x + 3)^3 + 15$

b Het punt van symmetrie is (-3, 15).

25 **a** $y = 0{,}3x^4$

↓ translatie (-5, 6)

$y = 0{,}3(x + 5)^4 + 6$

↓ verm. x-as, -4

$y = -4(0{,}3(x + 5)^4 + 6)$

oftewel $y = -1{,}2(x + 5)^4 - 24$

De top is (-5, -24).

b $y = 0{,}3x^4$

↓ verm. x-as, -4

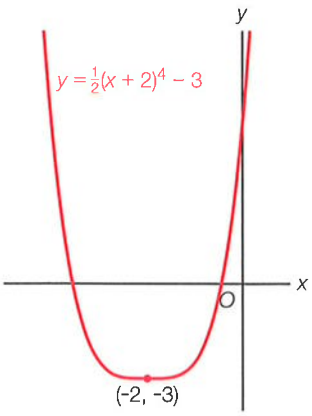
$y = -1{,}2x^4$

↓ translatie (-5, 6)

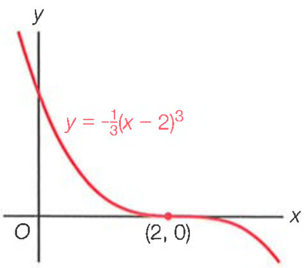
$y = -1{,}2(x + 5)^4 + 6$

De top is (-5, 6).

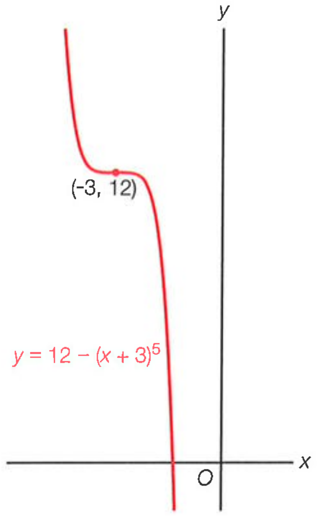
26 **a**



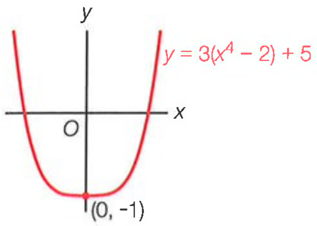
b



c



d $y = 3(x^4 - 2) + 5 = 3x^4 - 6 + 5 = 3x^4 - 1$



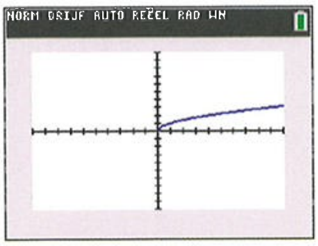
27 **a** $f(x) = \frac{1}{2}(x - 3)^5 + 7$
↓ translatie (1, 2)
$y = \frac{1}{2}(x - 1 - 3)^5 + 7 + 2$
oftewel $y = \frac{1}{2}(x - 4)^5 + 9$
↓ verm. x-as, -1
$y = -\frac{1}{2}(x - 4)^5 - 9$
Het punt van symmetrie is (4, -9).

b $g(x) = -2\frac{1}{2}(x + 4)^6 - 1$
↓ verm. x-as, -4
$y = 10(x + 4)^6 + 4$
↓ translatie (-5, -4)
$h(x) = 10(x + 5 + 4)^6 + 4 - 4$
oftewel $h(x) = 10(x + 9)^6$
min. is $h(-9) = 0$

28 **a** Voor f geldt $y = (x - 5)^3 + 5$, dus voor f^{inv} geldt $x = (y - 5)^3 + 5$.
$x = (y - 5)^3 + 5$ geeft $(y - 5)^3 + 5 = x$
$(y - 5)^3 = x - 5$
$y - 5 = \sqrt[3]{x - 5}$
$y = \sqrt[3]{x - 5} + 5$
Dus $g(x) = \sqrt[3]{x - 5} + 5$ is de inverse van f.

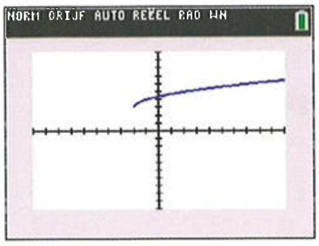
b $f(x) = g(x)$ geeft $(x - 5)^3 + 5 = \sqrt[3]{x - 5} + 5$
$(x - 5)^3 = \sqrt[3]{x - 5}$
$(x - 5)^9 = x - 5$
$(x - 5)^8 \cdot (x - 5) = x - 5$
$x - 5 = 0 \vee (x - 5)^8 = 1$
$x = 5 \vee x - 5 = 1 \vee x - 5 = -1$
$x = 5 \vee x = 6 \vee x = 4$
De snijpunten zijn (4, 4), (5, 5) en (6, 6).

29 **a**



Het domein is [0, →⟩ en het bereik is [0, →⟩.

b

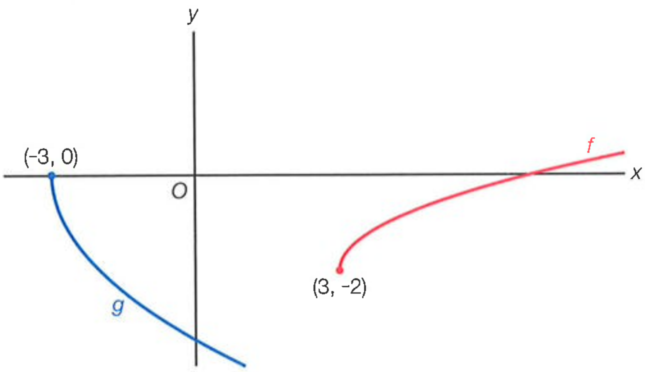


De grafiek van $y = \sqrt{x+2} + 3$ ontstaat uit de grafiek van $y = \sqrt{x}$ bij de translatie (−2, 3).

Bladzijde 23

30 **a** De grafiek van $f(x) = \sqrt{x-3} - 2$ ontstaat uit die van $y = \sqrt{x}$ bij de translatie (3, −2).
De grafiek van $g(x) = -2\sqrt{x+3}$ ontstaat uit die van $y = \sqrt{x}$ bij de vermenigvuldiging ten opzichte van de x-as met −2 en de translatie (−3, 0).

b



c $D_f = [3, \rightarrow\rangle$, $B_f = [-2, \rightarrow\rangle$, $D_g = [-3, \rightarrow\rangle$ en $B_g = \langle\leftarrow, 0]$

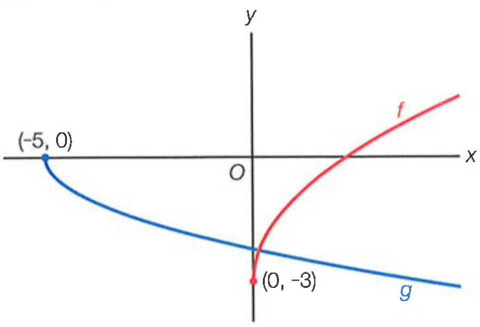
Bladzijde 24

31 **a**

$y = \sqrt{x}$
↓ verm. x-as, 2
$y = 2\sqrt{x}$
↓ translatie (0, −3)
$f(x) = 2\sqrt{x} - 3$

$y = \sqrt{x}$
↓ verm. x-as, −1
$y = -\sqrt{x}$
↓ translatie (−5, 0)
$g(x) = -\sqrt{x+5}$

b



c $D_f = [0, \rightarrow\rangle$, $B_f = [-3, \rightarrow\rangle$, $D_g = [-5, \rightarrow\rangle$ en $B_g = \langle\leftarrow, 0]$

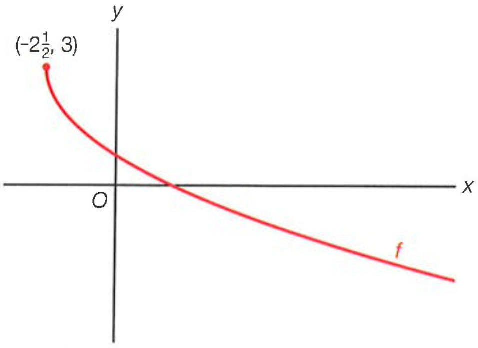
32 a randpunt (-5, 3), $D_f = [-5, \rightarrow\rangle$ en $B_f = [3, \rightarrow\rangle$
b randpunt (-3, -7), $D_g = [-3, \rightarrow\rangle$ en $B_g = [-7, \rightarrow\rangle$
c randpunt (-1, 0), $D_h = [-1, \rightarrow\rangle$ en $B_h = \langle\leftarrow, 0]$
d randpunt (0, 1), $D_k = [0, \rightarrow\rangle$ en $B_k = [1, \rightarrow\rangle$
e randpunt (1, -1), $D_l = [1, \rightarrow\rangle$ en $B_l = \langle\leftarrow, -1]$
f randpunt (0, -3), $D_m = [0, \rightarrow\rangle$ en $B_m = [-3, \rightarrow\rangle$

33 a Voor het domein van $f(x) = 5 - \sqrt{2x - 6}$ moet gelden $2x - 6 \geq 0$
$2x \geq 6$
$x \geq 3$
Dus het domein is $D_f = [3, \rightarrow\rangle$.
b De uitkomst van een wortel is minstens 0, dus de uitkomst van $f(x)$ is hoogstens 5.
Dus het bereik is $B_f = \langle\leftarrow, 5]$.

Bladzijde 26

34 a $8 - 4x \geq 0$
$-4x \geq -8$
$x \leq 2$
Dus $D_f = \langle\leftarrow, 2]$, $B_f = [3, \rightarrow\rangle$ en randpunt (2, 3).
b $4x - 8 \geq 0$
$4x \geq 8$
$x \geq 2$
Dus $D_g = [2, \rightarrow\rangle$, $B_g = [3, \rightarrow\rangle$ en randpunt (2, 3).
c $2x + 6 \geq 0$
$2x \geq -6$
$x \geq -3$
Dus $D_h = [-3, \rightarrow\rangle$, $B_h = \langle\leftarrow, 5]$ en randpunt (-3, 5).
d $x \geq 0$
Dus $D_k = [0, \rightarrow\rangle$, $B_k = \langle\leftarrow, 3]$ en randpunt (0, 3).

35 a $2x + 5 \geq 0$
$2x \geq -5$
$x \geq -2\frac{1}{2}$
$D_f = [-2\frac{1}{2}, \rightarrow\rangle$
randpunt $(-2\frac{1}{2}, 3)$



$B_f = \langle\leftarrow, 3]$

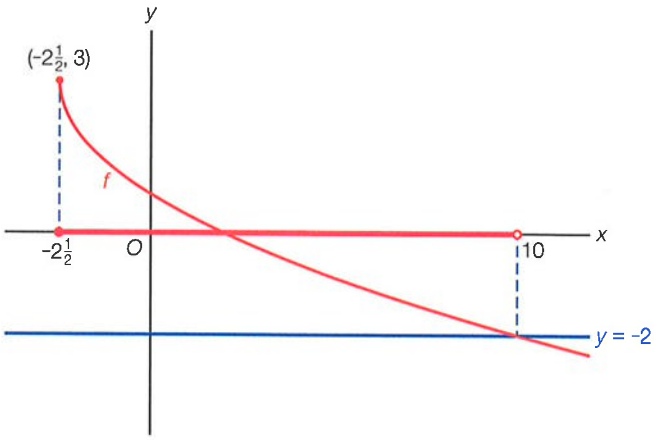
b $f(x) = -2$ geeft $3 - \sqrt{2x+5} = -2$

$\sqrt{2x+5} = 5$

$2x + 5 = 25$

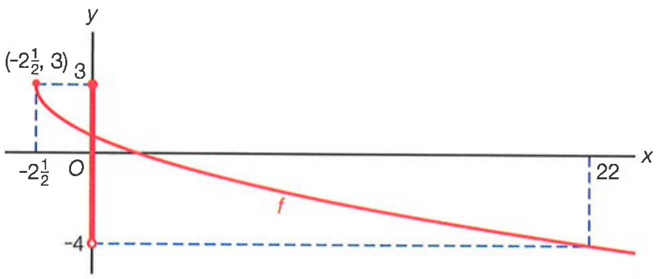
$2x = 20$

$x = 10$



$f(x) > -2$ geeft $-2\frac{1}{2} \leq x < 10$

c $f(22) = -4$



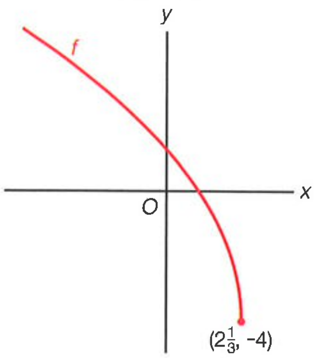
$x < 22$ geeft $-4 < f(x) \leq 3$

36 **a** $7 - 3x \geq 0$

$-3x \geq -7$

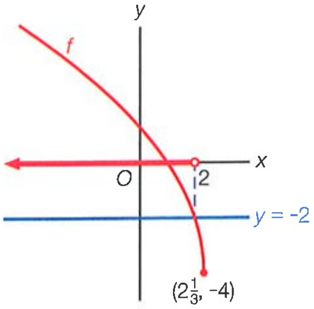
$x \leq 2\frac{1}{3}$ dus $D_f = \langle \leftarrow, 2\frac{1}{3}]$

randpunt $(2\frac{1}{3}, -4)$



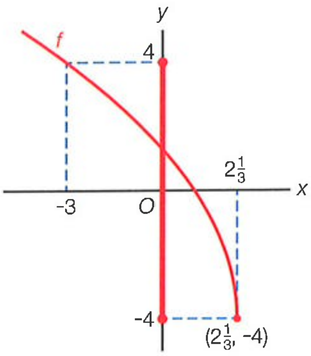
$B_f = [-4, \rightarrow\rangle$

b $f(x) = -2$ geeft $2\sqrt{7-3x} - 4 = -2$
$2\sqrt{7-3x} = 2$
$\sqrt{7-3x} = 1$
$7 - 3x = 1$
$-3x = -6$
$x = 2$



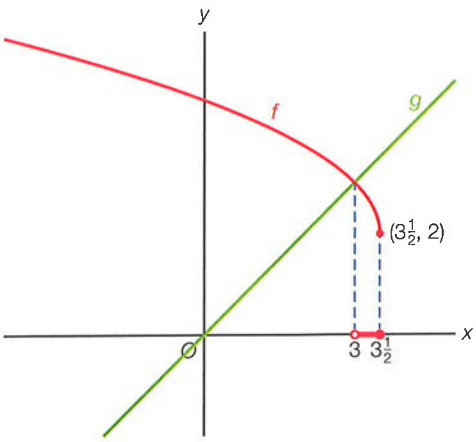
$f(x) > -2$ geeft $x < 2$

c $f(-3) = 4$



$x \geq -3$ geeft $-4 \leq f(x) \leq 4$

37 Stel $f(x) = 2 + \sqrt{7-2x}$ en $g(x) = x$.
$7 - 2x \geq 0$
$-2x \geq -7$
$x \leq 3\frac{1}{2}$
Dus $D_f = \langle\leftarrow, 3\frac{1}{2}]$ en het randpunt van de grafiek van f is $(3\frac{1}{2}, 2)$.
$f(x) = g(x)$ geeft $2 + \sqrt{7-2x} = x$
$\sqrt{7-2x} = x - 2$
kwadrateren geeft
$7 - 2x = x^2 - 4x + 4$
$-x^2 + 2x + 3 = 0$
$x^2 - 2x - 3 = 0$
$(x+1)(x-3) = 0$
$x = -1 \vee x = 3$
vold. niet vold.



$2 + \sqrt{7-2x} < x$ geeft $3 < x \leq 3\frac{1}{2}$

38 Er geldt $D_f = [-8, \rightarrow\rangle$, $B_f = \langle\leftarrow, 3]$ en $g = f^{\text{inv}}$, dus $D_g = B_f = \langle\leftarrow, 3]$ en $B_g = D_f = [-8, \rightarrow\rangle$.

39 $5 + ax = 0$

$ax = -5$

$x = -\frac{5}{a}$

Dus het randpunt van de grafiek van f is $\left(-\frac{5}{a}, 4\right)$.

$\left(-\frac{5}{a}, 4\right)$ op de lijn $y = 2x - 1$ geeft $2 \cdot -\frac{5}{a} - 1 = 4$

$-\frac{10}{a} = 5$

$a = -2$

40 (5, 3) op de grafiek van $f(x) = a\sqrt{x + b}$ geeft $a\sqrt{5 + b} = 3$

$a = \frac{3}{\sqrt{b + 5}}$

(13, 9) op de grafiek van $f(x) = a\sqrt{x + b}$ geeft $a\sqrt{13 + b} = 9$

$a = \frac{9}{\sqrt{b + 13}}$

$a = \frac{3}{\sqrt{b + 5}}$ en $a = \frac{9}{\sqrt{b + 13}}$ geeft $\frac{3}{\sqrt{b + 5}} = \frac{9}{\sqrt{b + 13}}$

$3\sqrt{b + 13} = 9\sqrt{b + 5}$

$\sqrt{b + 13} = 3\sqrt{b + 5}$

kwadrateren geeft

$b + 13 = 9(b + 5)$

$b + 13 = 9b + 45$

$-8b = 32$

$b = -4$ vold.

$b = -4$ en $a = \frac{3}{\sqrt{b + 5}}$ geeft $a = \frac{3}{\sqrt{-4 + 5}} = 3$

Dus $a = 3$ en $b = -4$.

41 **a** $y = 2\sqrt{x}$ geeft $2\sqrt{x} = y$

kwadrateren geeft

$4x = y^2$

$x = \frac{1}{4}y^2$

b $y = \sqrt{x - 2}$ geeft $\sqrt{x - 2} = y$

kwadrateren geeft

$x - 2 = y^2$

$x = y^2 + 2$

Uit $y = \sqrt{x - 2}$ volgt $x = y^2 + 2$.

c $y = 2\sqrt{x - 2}$ geeft $2\sqrt{x - 2} = y$

kwadrateren geeft

$4(x - 2) = y^2$

$x - 2 = \frac{1}{4}y^2$

$x = \frac{1}{4}y^2 + 2$

Uit $y = 2\sqrt{x - 2}$ volgt $x = \frac{1}{4}y^2 + 2$.

Bladzijde 27

42 **a** $F = 3\sqrt{2t-1}$ geeft $3\sqrt{2t-1} = F$

kwadrateren geeft

$9(2t-1) = F^2$

$2t - 1 = \frac{1}{9}F^2$

$2t = \frac{1}{9}F^2 + 1$

$t = \frac{1}{18}F^2 + \frac{1}{2}$

b $A = 5 + \sqrt{4-3B}$ geeft $5 + \sqrt{4-3B} = A$

$\sqrt{4-3B} = A - 5$

kwadrateren geeft

$4 - 3B = (A-5)^2$

$4 - 3B = A^2 - 10A + 25$

$-3B = A^2 - 10A + 21$

$B = -\frac{1}{3}A^2 + 3\frac{1}{3}A - 7$

c $2x\sqrt{y} - 5 = 0$ geeft $2x\sqrt{y} = 5$

kwadrateren geeft

$4x^2y = 25$

$y = \dfrac{25}{4x^2}$

d $R\sqrt{q} - \sqrt{R} = 6$ geeft $R\sqrt{q} = 6 + \sqrt{R}$

kwadrateren geeft

$R^2q = (6 + \sqrt{R})^2$

$R^2q = 36 + 12\sqrt{R} + R$

$q = \dfrac{36 + 12\sqrt{R} + R}{R^2}$

Bladzijde 28

43 **a** $T = -27{,}4$ geeft $v = 331\sqrt{1 - \dfrac{27{,}4}{273}} = 313{,}9...$

$T = 38{,}6$ geeft $v = 331\sqrt{1 + \dfrac{38{,}6}{273}} = 353{,}6...$

Het verschil is $353{,}6... - 313{,}9... \approx 40$ m/s.

b $v = 340$ geeft $331\sqrt{1 + \dfrac{T}{273}} = 340$

$\sqrt{1 + \dfrac{T}{273}} = \dfrac{340}{331}$

kwadrateren geeft

$1 + \dfrac{T}{273} = \left(\dfrac{340}{331}\right)^2$

$\dfrac{T}{273} = \left(\dfrac{340}{331}\right)^2 - 1$

$T = 273 \cdot \left(\dfrac{340}{331}\right)^2 - 273 \approx 15$

Bij een temperatuur van 15 °C is de geluidssnelheid 340 m/s.

c $T_0 = 25$ en $h = 2{,}5$ geeft $T = 25 - 6{,}5 \cdot 2{,}5 = 8{,}75$

$T = 8{,}75$ geeft $v = 331\sqrt{1 + \dfrac{8{,}75}{273}} = 336{,}2...$ m/s $= 1210{,}5...$ km/uur

De snelheid is 1211 km/uur.

d $T = -1$ en $h = 2$ geeft $T_0 - 6{,}5 \cdot 2 = -1$

$T_0 = 12$

Dus $T = 12 - 6{,}5h$.

Dit geeft $v = 331\sqrt{1 + \frac{12 - 6{,}5h}{273}} = 331\sqrt{1 + \frac{12}{273} - \frac{6{,}5}{273}h} \approx 331\sqrt{-0{,}0238h + 1{,}0440}$.

Dus $v = 331\sqrt{-0{,}0238h + 1{,}0440}$.

e $v = 331\sqrt{1 + \frac{T}{273}}$ geeft $331\sqrt{1 + \frac{T}{273}} = v$

$\sqrt{1 + \frac{T}{273}} = \frac{v}{331}$

kwadrateren geeft

$1 + \frac{T}{273} = \left(\frac{v}{331}\right)^2$

$\frac{T}{273} = \left(\frac{v}{331}\right)^2 - 1$

$T = 273 \cdot \left(\frac{v}{331}\right)^2 - 273$

$T = \frac{273}{109\,561}v^2 - 273$

$T = T_0 - 6{,}5h$ en $T = \frac{273}{109\,561}v^2 - 273$ geeft $T_0 - 6{,}5h = \frac{273}{109\,561}v^2 - 273$

$-6{,}5h = \frac{273}{109\,561}v^2 - T_0 - 273$

$h = -\frac{273}{109\,561 \cdot 6{,}5}v^2 + \frac{1}{6{,}5}T_0 + \frac{273}{6{,}5}$

$h \approx -0{,}0004v^2 + 0{,}1538T_0 + 42$

Dus $h = -0{,}0004v^2 + 0{,}1538T_0 + 42$.

44 Voor f geldt $y = a + \sqrt{bx + c}$, dus voor f^{inv} geldt $x = a + \sqrt{by + c}$.

$x = a + \sqrt{by + c}$ geeft $a + \sqrt{by + c} = x$

$\sqrt{by + c} = x - a$

kwadrateren geeft

$by + c = (x - a)^2$

$by + c = x^2 - 2ax + a^2$

$by = x^2 - 2ax + a^2 - c$

$y = \frac{1}{b}x^2 - \frac{2a}{b}x + \frac{a^2 - c}{b}$

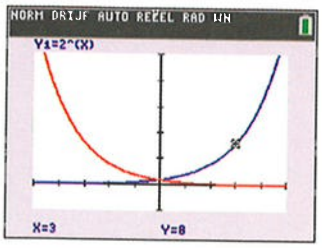
$f^{\text{inv}}(x) = \frac{1}{2}x^2 - 3x$ geeft $\frac{1}{b} = \frac{1}{2} \wedge -\frac{2a}{b} = -3 \wedge \frac{a^2 - c}{b} = 0$

Dus $b = 2 \wedge -a = -3 \wedge a^2 - c = 0$ oftewel $a = 3$, $b = 2$ en $c = 9$.

5.3 Exponentiële functies

Bladzijde 30

45 **a**



De grafieken zijn elkaars gespiegelde in de y-as.

b $f(-10) = 2^{-10} \approx 9{,}77 \cdot 10^{-4}$
$f(-20) = 2^{-20} \approx 9{,}54 \cdot 10^{-7}$
$f(-100) = 2^{-100} \approx 7{,}89 \cdot 10^{-31}$
c Voor elke x is $2^x > 0$, dus er is geen origineel te vinden waarvan het beeld 0 is.
d $B_f = B_g = \langle 0, \rightarrow\rangle$

Bladzijde 32

46 **a** $y = 3^x$
$\downarrow$ translatie (-2, -1)
$y = 3^{x+2} - 1$
De asymptoot is de lijn $y = -1$.
b $y = 0{,}5^x$
$\downarrow$ verm. x-as, 2
$y = 2 \cdot 0{,}5^x$
$\downarrow$ translatie (0, 3)
$y = 2 \cdot 0{,}5^x + 3$
De asymptoot is de lijn $y = 3$.
c $y = 2^x$
$\downarrow$ verm. y-as, $\frac{1}{3}$
$y = 2^{3x}$
$\downarrow$ verm. x-as, 3
$y = 3 \cdot 2^{3x}$
$\downarrow$ translatie (0, 4)
$y = 3 \cdot 2^{3x} + 4$
De asymptoot is de lijn $y = 4$.
d $y = 0{,}8^x$
$\downarrow$ verm. x-as, -1
$y = -0{,}8^x$
$\downarrow$ translatie (-1, 10)
$y = 10 - 0{,}8^{x+1}$
$\downarrow$ verm. y-as, 2,5
$y = 10 - 0{,}8^{0{,}4x+1}$
De asymptoot is de lijn $y = 10$.

47 **a** Het bereik is $\langle -6, \rightarrow\rangle$ en de asymptoot is de lijn $y = -6$.
b Het bereik is $\langle \leftarrow, 5\rangle$ en de asymptoot is de lijn $y = 5$.
c Het bereik is $\langle \leftarrow, 1000\rangle$ en de asymptoot is de lijn $y = 1000$.
d Het bereik is $\langle \leftarrow, 1000\rangle$ en de asymptoot is de lijn $y = 1000$.

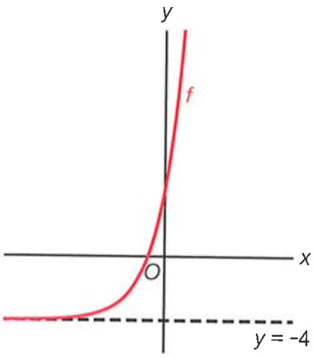
48 **a** $y = 3^x$
$\downarrow$ spiegelen in de x-as
$y = -3^x$
$\downarrow$ translatie (0, -1)
$y = -3^x - 1$
b $y = 3^x$
$\downarrow$ translatie (2, 5)
$y = 3^{x-2} + 5$
$\downarrow$verm. y-as, $\frac{1}{4}$
$y = 3^{4x-2} + 5$
c $y = 3^x$
$\downarrow$ translatie (4, -5)
$y = 3^{x-4} - 5$
$\downarrow$ verm. x-as, 3
$y = 3(3^{x-4} - 5)$
oftewel $y = 3^{x-3} - 15$

d $y = 3^x$
$\downarrow$ verm. x-as, 3
$y = 3 \cdot 3^x$
$\downarrow$ translatie (4, -5)
$y = 3 \cdot 3^{x-4} - 5$
oftewel $y = 3^{x-3} - 5$

49 **a** $y = 2^x$
$\downarrow$ translatie (-3, -4)
$f(x) = 2^{x+3} - 4$

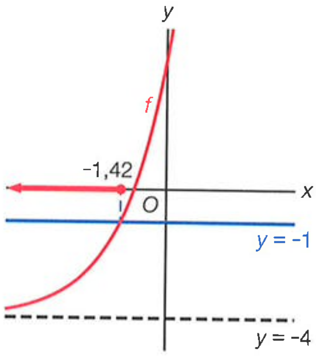
b $B_f = \langle -4, \rightarrow \rangle$
De horizontale asymptoot is de lijn $y = -4$.

c



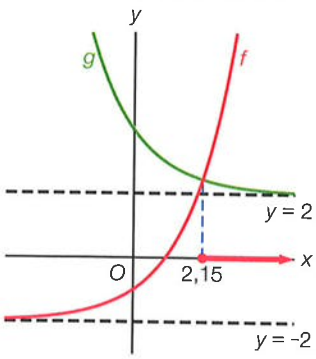
d $f(3) = 60$
$x \leq 3$ geeft $-4 < f(x) \leq 60$

e Voer in $y_2 = -1$.
De optie snijpunt geeft $x \approx -1{,}42$.



$f(x) \leq -1$ geeft $x \leq -1{,}42$

50 **a** Voer in $y_1 = 2^x - 2$ en $y_2 = (\frac{1}{2})^{x-1} + 2$.
De optie snijpunt geeft $x \approx 2{,}15$.



$f(x) \geq g(x)$ geeft $x \geq 2{,}15$

b $B_f = \langle -2, \rightarrow \rangle$, dus $f(x) = p$ heeft geen oplossingen voor $p \leq -2$.

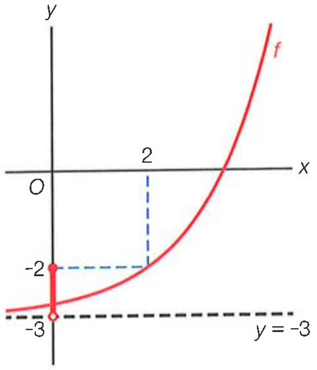
Bladzijde 33

51 **a** $y = 2^x$
$\downarrow$ translatie (2, -3)
$f(x) = 2^{x-2} - 3$
$B_f = \langle -3, \rightarrow \rangle$

$y = 0{,}5^x$
$\downarrow$ verm. x-as 4
$y = 4 \cdot 0{,}5^x$
$\downarrow$ translatie (3, -1)
$g(x) = 4 \cdot 0{,}5^{x-3} - 1$
$B_g = \langle -1, \rightarrow \rangle$

$B_f = \langle -3, \rightarrow \rangle$ en $B_g = \langle -1, \rightarrow \rangle$, dus $f(x) = p$ heeft één oplossing voor $p > -3$ en $g(x) = p$ heeft geen oplossingen voor $p \leq -1$.
Dus voor $-3 < p \leq -1$ heeft $f(x) = p$ één oplossing en $g(x) = p$ geen oplossingen.

b $f(2) = -2$



$x \leq 2$ geeft $-3 < f(x) \leq -2$

c $f(1) = -2\frac{1}{2}$ en $g(1) = 15$, dus $A(1, -2\frac{1}{2})$ en $B(1, 15)$.
$AB = y_B - y_A = 15 - -2\frac{1}{2} = 17\frac{1}{2}$

d Voer in $y_1 = 2^{x-2} - 3$, $y_2 = 4 \cdot 0{,}5^{x-3} - 1$ en $y_3 = 5$.
De optie snijpunt met y_1 en y_3 geeft $x = 5$, dus $P(5, 5)$.
De optie snijpunt met y_2 en y_3 geeft $x = 2{,}415...$, dus $Q(2{,}415...; 5)$.
$PQ = x_P - x_Q = 5 - 2{,}415... \approx 2{,}585$

52 **a** $8 \cdot 4^x = 2^3 \cdot (2^2)^x = 2^3 \cdot 2^{2x} = 2^{2x+3}$

b Bij de herleiding van $(2^2)^x$ tot 2^{2x} is de regel $(a^p)^q = a^{pq}$ gebruikt.
Bij de herleiding van $2^3 \cdot 2^{2x}$ tot 2^{2x+3} is de regel $a^p \cdot a^q = a^{p+q}$ gebruikt.

53 **a** $y = 15 \cdot 2^{3x+2} = 15 \cdot 2^{3x} \cdot 2^2 = 15 \cdot (2^3)^x \cdot 4 = 60 \cdot 8^x$
Dus $y = 60 \cdot 8^x$.

b $y = 50 \cdot 2^{3x-1} = 50 \cdot 2^{3x} \cdot 2^{-1} = 50 \cdot (2^3)^x \cdot \frac{1}{2} = 25 \cdot 8^x$
Dus $y = 25 \cdot 8^x$.

c $y = 260 \cdot 4^{1\frac{1}{2}x-1} = 260 \cdot (2^2)^{1\frac{1}{2}x-1} = 260 \cdot 2^{3x-2} = 260 \cdot (2^3)^x \cdot 2^{-2} = 260 \cdot 8^x \cdot \frac{1}{4} = 65 \cdot 8^x$
Dus $y = 65 \cdot 8^x$.

d $y = 8 \cdot 4^{-2x-1} = 8 \cdot 4^{-2x} \cdot 4^{-1} = 8 \cdot (4^{-2})^x \cdot \frac{1}{4} = 2 \cdot (\frac{1}{16})^x$
Dus $y = 2 \cdot (\frac{1}{16})^x$.

Bladzijde 34

54 **a** $y = 2^x \cdot 2^{2x-3} = 2^x \cdot 2^{2x} \cdot 2^{-3} = 2^{3x} \cdot \frac{1}{8} = \frac{1}{8} \cdot (2^3)^x = \frac{1}{8} \cdot 8^x$
Dus $y = \frac{1}{8} \cdot 8^x$.

b $y = 63 \cdot 3^{\frac{1}{2}x-2} = 63 \cdot 3^{\frac{1}{2}x} \cdot 3^{-2} = 63 \cdot (3^{\frac{1}{2}})^x \cdot \frac{1}{9} = 7 \cdot (\sqrt{3})^x$
Dus $y = 7 \cdot (\sqrt{3})^x$.

c $y = 50\,000 \cdot 100^{-x-2\frac{1}{2}} = 50\,000 \cdot 100^{-x} \cdot 100^{-2\frac{1}{2}} = 50\,000 \cdot (100^{-1})^x \cdot 10^{-5} = \frac{1}{2} \cdot (\frac{1}{100})^x$
Dus $y = \frac{1}{2} \cdot (\frac{1}{100})^x$.

d $y = \dfrac{3}{4^{2x-1}} = 3 \cdot 4^{-2x+1} = 3 \cdot 4^{-2x} \cdot 4^1 = 3 \cdot (4^{-2})^x \cdot 4 = 12 \cdot (\frac{1}{16})^x$
Dus $y = 12 \cdot (\frac{1}{16})^x$.

55 $f(x) = 5 - 2^{\frac{1}{2}x + 4}$
$\downarrow$ translatie (12, -8)
$h(x) = 5 - 2^{\frac{1}{2}(x-12)+4} - 8 = -3 - 2^{\frac{1}{2}x - 6 + 4} = -3 - 2^{\frac{1}{2}x} \cdot 2^{-2} = -3 - (2^{\frac{1}{2}})^x \cdot \frac{1}{4} = -3 - \frac{1}{4} \cdot (\sqrt{2})^x$
Dus $a = -3$, $b = -\frac{1}{4}$ en $g = \sqrt{2}$.

56 $f(x) = 2^{x-3}$
$\downarrow$ translatie (-5, 0)
$g(x) = 2^{x+5-3} = 2^{x-3} \cdot 2^5 = 32 \cdot 2^{x-3}$
De vermenigvuldiging ten opzichte van de x-as met 32 toepassen op de grafiek van f geeft de grafiek van g.
Dus $a = 32$.

57 $4\sqrt{2} = 2^2 \cdot 2^{\frac{1}{2}} = 2^{2\frac{1}{2}}$
$2^{x-1} = 4\sqrt{2}$
$2^{x-1} = 2^{2\frac{1}{2}}$
$x - 1 = 2\frac{1}{2}$
$x = 3\frac{1}{2}$

Bladzijde 35

58
a $2^{x+1} = 64$
$2^{x+1} = 2^6$
$x + 1 = 6$
$x = 5$
b $2^{x-3} = \frac{1}{8}$
$2^{x-3} = 2^{-3}$
$x - 3 = -3$
$x = 0$
c $3^{2x} = 3$
$3^{2x} = 3^1$
$2x = 1$
$x = \frac{1}{2}$
d $(\frac{1}{2})^x = 1$
$(\frac{1}{2})^x = (\frac{1}{2})^0$
$x = 0$
e $2^x = \frac{1}{4}\sqrt{2}$
$2^x = 2^{-2} \cdot 2^{\frac{1}{2}}$
$2^x = 2^{-1\frac{1}{2}}$
$x = -1\frac{1}{2}$
f $5^{x+6} = (\frac{1}{5})^x$
$5^{x+6} = (5^{-1})^x$
$5^{x+6} = 5^{-x}$
$x + 6 = -x$
$2x = -6$
$x = -3$

59
a $3^{2x+1} = 27\sqrt{3}$
$3^{2x+1} = 3^3 \cdot 3^{\frac{1}{2}}$
$3^{2x+1} = 3^{3\frac{1}{2}}$
$2x + 1 = 3\frac{1}{2}$
$2x = 2\frac{1}{2}$
$x = 1\frac{1}{4}$
b $10^{2x+1} = 0{,}01$
$10^{2x+1} = 10^{-2}$
$2x + 1 = -2$
$2x = -3$
$x = -1\frac{1}{2}$
c $3^x - 2 = 25$
$3^x = 27$
$3^x = 3^3$
$x = 3$
d $5 \cdot 2^x = 80$
$2^x = 16$
$2^x = 2^4$
$x = 4$
e $10 \cdot 3^x = 270$
$3^x = 27$
$3^x = 3^3$
$x = 3$
f $3 \cdot 8^{2-x} = 48$
$8^{2-x} = 16$
$(2^3)^{2-x} = 2^4$
$2^{6-3x} = 2^4$
$6 - 3x = 4$
$-3x = -2$
$x = \frac{2}{3}$

60 **a** $3 \cdot 2^x + 4 = 28$
$3 \cdot 2^x = 24$
$2^x = 8$
$2^x = 2^3$
$x = 3$

b $5^{2x-6} = 0{,}04$
$5^{2x-6} = \frac{1}{25}$
$5^{2x-6} = 5^{-2}$
$2x - 6 = -2$
$2x = 4$
$x = 2$

c $3 \cdot 7^{3x+1} = 147$
$7^{3x+1} = 49$
$7^{3x+1} = 7^2$
$3x + 1 = 2$
$3x = 1$
$x = \frac{1}{3}$

d $32^{x-2} = \frac{1}{16}$
$(2^5)^{x-2} = 2^{-4}$
$2^{5x-10} = 2^{-4}$
$5x - 10 = -4$
$5x = 6$
$x = 1\frac{1}{5}$

e $5 \cdot 4^{x-1} = 2\frac{1}{2}$
$4^{x-1} = \frac{1}{2}$
$(2^2)^{x-1} = 2^{-1}$
$2^{2x-2} = 2^{-1}$
$2x - 2 = -1$
$2x = 1$
$x = \frac{1}{2}$

f $8 \cdot 2^x = 4^{x+1}$
$2^3 \cdot 2^x = (2^2)^{x+1}$
$2^{3+x} = 2^{2x+2}$
$3 + x = 2x + 2$
$-x = -1$
$x = 1$

61 $f(x) = 3^{x+2} - 5$
$\downarrow$ translatie (3, 7)
$y = 3^{x-1} + 2$
$\downarrow$ verm. y-as, b
$g(x) = 3^{\frac{1}{b}x-1} + 2$

$\left.\begin{array}{l} g(x) = 3^{\frac{1}{b}x-1} + 2 \\ \text{door } A(-15, 83) \end{array}\right\}$ $3^{\frac{-15}{b}-1} + 2 = 83$
$3^{\frac{-15}{b}-1} = 81$
$3^{\frac{-15}{b}-1} = 3^4$
$\frac{-15}{b} - 1 = 4$
$\frac{-15}{b} = 5$
$b = -3$

62 **a** $2^{x+1} = 2^x \cdot 2^1 = 2 \cdot 2^x$, dus uit $2^{x+1} + 2^x = 48$ volgt $2 \cdot 2^x + 2^x = 48$.
b $2 \cdot 2^x + 2^x = 2 \cdot 2^x + 1 \cdot 2^x = 3 \cdot 2^x$, dus uit $2 \cdot 2^x + 2^x = 48$ volgt $3 \cdot 2^x = 48$.
$3 \cdot 2^x = 48$
$2^x = 16$
$2^x = 2^4$
$x = 4$

63 **a** $3^{x+2} + 3^x = 810$
$3^x \cdot 3^2 + 3^x = 810$
$9 \cdot 3^x + 3^x = 810$
$10 \cdot 3^x = 810$
$3^x = 81$
$3^x = 3^4$
$x = 4$

b $2^{x-1} + 2^{x+1} = 10$
$2^x \cdot 2^{-1} + 2^x \cdot 2^1 = 10$
$\frac{1}{2} \cdot 2^x + 2 \cdot 2^x = 10$
$2\frac{1}{2} \cdot 2^x = 10$
$2^x = 4$
$2^x = 2^2$
$x = 2$

c $2^{x+3} - 2^x = \frac{7}{8}$
$2^x \cdot 2^3 - 2^x = \frac{7}{8}$
$8 \cdot 2^x - 2^x = \frac{7}{8}$
$7 \cdot 2^x = \frac{7}{8}$
$2^x = \frac{1}{8}$
$2^x = 2^{-3}$
$x = -3$

d $3^{x+2} = 24 + 3^x$
$3^x \cdot 3^2 = 24 + 3^x$
$9 \cdot 3^x - 3^x = 24$
$8 \cdot 3^x = 24$
$3^x = 3$
$3^x = 3^1$
$x = 1$

e $3^x - 3^{x-1} = 2\sqrt{3}$
$3^x - 3^x \cdot 3^{-1} = 2\sqrt{3}$
$3^x - \frac{1}{3} \cdot 3^x = 2\sqrt{3}$
$\frac{2}{3} \cdot 3^x = 2\sqrt{3}$
$3^x = 3\sqrt{3}$
$3^x = 3^{1\frac{1}{2}}$
$x = 1\frac{1}{2}$

f $5^{x-1} + 5^{x-2} = 6\sqrt{5}$
$5^x \cdot 5^{-1} + 5^x \cdot 5^{-2} = 6\sqrt{5}$
$\frac{1}{5} \cdot 5^x + \frac{1}{25} \cdot 5^x = 6\sqrt{5}$
$\frac{6}{25} \cdot 5^x = 6\sqrt{5}$
$5^x = 25\sqrt{5}$
$5^x = 5^{2\frac{1}{2}}$
$x = 2\frac{1}{2}$

Bladzijde 36

64 **a** $3^{x+1} = 9^{x+2}$
$3^{x+1} = (3^2)^{x+2}$
$3^{x+1} = 3^{2x+4}$
$x + 1 = 2x + 4$
$-x = 3$
$x = -3$

b $3^{x+1} - 3^{x-1} = 8\sqrt{3}$
$3^x \cdot 3^1 - 3^x \cdot 3^{-1} = 8\sqrt{3}$
$3 \cdot 3^x - \frac{1}{3} \cdot 3^x = 8\sqrt{3}$
$2\frac{2}{3} \cdot 3^x = 8\sqrt{3}$
$3^x = 3\sqrt{3}$
$3^x = 3^{1\frac{1}{2}}$
$x = 1\frac{1}{2}$

c $3^{x^2} = (\frac{1}{3})^{x-6}$
$3^{x^2} = (3^{-1})^{x-6}$
$3^{x^2} = 3^{-x+6}$
$x^2 = -x + 6$
$x^2 + x - 6 = 0$
$(x - 2)(x + 3) = 0$
$x = 2 \vee x = -3$

d $5^{x^2+5} = 125^{x+1}$
$5^{x^2+5} = (5^3)^{x+1}$
$5^{x^2+5} = 5^{3x+3}$
$x^2 + 5 = 3x + 3$
$x^2 - 3x + 2 = 0$
$(x - 1)(x - 2) = 0$
$x = 1 \vee x = 2$

e $2^{x+2} - (\frac{1}{2})^{-x+1} = 28$
$2^x \cdot 2^2 - (2^{-1})^{-x+1} = 28$
$4 \cdot 2^x - 2^{x-1} = 28$
$4 \cdot 2^x - 2^x \cdot 2^{-1} = 28$
$4 \cdot 2^x - \frac{1}{2} \cdot 2^x = 28$
$3\frac{1}{2} \cdot 2^x = 28$
$2^x = 8$
$2^x = 2^3$
$x = 3$

f $4^{x^2+1} = 8^{x^2-1}$
$(2^2)^{x^2+1} = (2^3)^{x^2-1}$
$2^{2x^2+2} = 2^{3x^2-3}$
$2x^2 + 2 = 3x^2 - 3$
$-x^2 = -5$
$x^2 = 5$
$x = \sqrt{5} \vee x = -\sqrt{5}$

65 $f(x) = 2^x$
↓ translatie (3, 8)
$y = 2^{x-3} + 8$
↓verm. x-as, 4
$g(x) = 4 \cdot 2^{x-3} + 32$
$f(x) = g(x)$ geeft $2^x = 4 \cdot 2^{x-3} + 32$
$2^x = 4 \cdot 2^x \cdot 2^{-3} + 32$
$2^x = \frac{1}{2} \cdot 2^x + 32$
$\frac{1}{2} \cdot 2^x = 32$
$2^x = 64$
$2^x = 2^6$
$x = 6$

Het snijpunt van de grafieken van f en g is het punt (6, 64).

66 **a** $f(x) = g(x)$ geeft $3^{x+1} - 4 = 6 - 3^{x-1}$
$3^x \cdot 3 - 4 = 6 - 3^x \cdot 3^{-1}$
$3 \cdot 3^x - 4 = 6 - \frac{1}{3} \cdot 3^x$
$3\frac{1}{3} \cdot 3^x = 10$
$3^x = 3$
$x = 1$

$f(x) \leq g(x)$ geeft $x \leq 1$

b $f(2\frac{1}{2}) = 3^{3\frac{1}{2}} - 4 = 3^3 \cdot 3^{\frac{1}{2}} - 4 = 27\sqrt{3} - 4$, dus $A(2\frac{1}{2}, 27\sqrt{3} - 4)$
$g(2\frac{1}{2}) = 6 - 3^{1\frac{1}{2}} = 6 - 3 \cdot 3^{\frac{1}{2}} = 6 - 3\sqrt{3}$, dus $B(2\frac{1}{2}, 6 - 3\sqrt{3})$
$AB = y_A - y_B = 27\sqrt{3} - 4 - (6 - 3\sqrt{3}) = 30\sqrt{3} - 10$

c $f(x) - g(x) = 80$ geeft $3^{x+1} - 4 - (6 - 3^{x-1}) = 80$
$3^x \cdot 3 - 4 - 6 + 3^x \cdot 3^{-1} = 80$
$3 \cdot 3^x - 10 + \frac{1}{3} \cdot 3^x = 80$
$3\frac{1}{3} \cdot 3^x = 90$
$3^x = 27$
$3^x = 3^3$
$x = 3$

d $g(x) - f(x) = 6 - 3^{x-1} - (3^{x+1} - 4) = 6 - 3^x \cdot 3^{-1} - 3^x \cdot 3 + 4 = 6 - \frac{1}{3} \cdot 3^x - 3 \cdot 3^x + 4 = 10 - 3\frac{1}{3} \cdot 3^x$
Het bereik van $g(x) - f(x)$ is $\langle \leftarrow, 10 \rangle$.
Dus de vergelijking $g(x) - f(x) = p$ heeft geen oplossingen voor $p \geq 10$.

5.4 Logaritmen

Bladzijde 38

67
a $2^3 = 8$
b $2^{-2} = \frac{1}{4}$
c $2^{\frac{1}{2}} = \sqrt{2}$
d $3^2 = 9$
e $3^{-3} = \frac{1}{27}$
f $3^{\frac{1}{5}} = \sqrt[5]{3}$

68
a ${}^5\log(125) = {}^5\log(5^3) = 3$
b ${}^{10}\log(0,1) = {}^{10}\log(10^{-1}) = -1$
c ${}^2\log(4) = {}^2\log(2^2) = 2$
d ${}^7\log(49) = {}^7\log(7^2) = 2$
e ${}^2\log(\sqrt{2}) = {}^2\log(2^{\frac{1}{2}}) = \frac{1}{2}$
f ${}^2\log(0,5) = {}^2\log(2^{-1}) = -1$
g ${}^4\log(0,25) = {}^4\log(4^{-1}) = -1$
h ${}^4\log(4) = {}^4\log(4^1) = 1$
i ${}^4\log(1) = {}^4\log(4^0) = 0$

Bladzijde 39

69
a ${}^2\log(64\sqrt{2}) = {}^2\log(2^6 \cdot 2^{\frac{1}{2}}) = {}^2\log(2^{6\frac{1}{2}}) = 6\frac{1}{2}$
b ${}^3\log(\frac{1}{9}\sqrt{3}) = {}^3\log(3^{-2} \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{-1\frac{1}{2}}) = -1\frac{1}{2}$
c ${}^3\log(3^{21,5}) = 21,5$
d ${}^5\log(\frac{1}{125}) = {}^5\log(5^{-3}) = -3$
e ${}^{\frac{1}{3}}\log(\frac{1}{27}) = {}^{\frac{1}{3}}\log((\frac{1}{3})^3) = 3$
f ${}^{\frac{1}{2}}\log(\frac{1}{4}) = {}^{\frac{1}{2}}\log((\frac{1}{2})^2) = 2$
g ${}^2\log(\frac{1}{32} \cdot \sqrt[3]{2}) = {}^2\log(2^{-5} \cdot 2^{\frac{1}{3}}) = {}^2\log(2^{-4\frac{2}{3}}) = -4\frac{2}{3}$
h ${}^5\log(1) = {}^5\log(5^0) = 0$
i ${}^3\log(81 \cdot \sqrt[5]{27}) = {}^3\log(3^4 \cdot \sqrt[5]{3^3}) = {}^3\log(3^4 \cdot 3^{\frac{3}{5}}) = {}^3\log(3^{4\frac{3}{5}}) = 4\frac{3}{5}$

70
a ${}^3\log(9) = {}^3\log(3^2) = 2$
Dus ${}^3\log(x) = 2$ geeft $x = 9$.
b ${}^5\log(\frac{1}{25}) = {}^5\log(5^{-2}) = -2$
Dus ${}^5\log(x) = -2$ geeft $x = \frac{1}{25}$.

71
a $x = {}^5\log(0,2) = {}^5\log(\frac{1}{5}) = {}^5\log(5^{-1}) = -1$
b ${}^9\log(x) = \frac{1}{2}$
$x = 9^{\frac{1}{2}} = \sqrt{9} = 3$
c ${}^x\log(1000) = 3$
$x^3 = 1000$
$x = 10$

72 **a** $^3\log(x+2)=2$
$x+2=3^2$
$x+2=9$
$x=7$

b $1+{}^{\frac{1}{2}}\log(x)=4$
${}^{\frac{1}{2}}\log(x)=3$
$x=(\frac{1}{2})^3$
$x=\frac{1}{8}$

c $^3\log(2x+1)=4$
$2x+1=3^4$
$2x+1=81$
$2x=80$
$x=40$

d $5+{}^4\log(x)=3$
$^4\log(x)=-2$
$x=4^{-2}$
$x=\frac{1}{16}$

e ${}^{\frac{1}{2}}\log(x-1)=3$
$x-1=(\frac{1}{2})^3$
$x-1=\frac{1}{8}$
$x=1\frac{1}{8}$

f $^2\log(x^2-4)=5$
$x^2-4=2^5$
$x^2-4=32$
$x^2=36$
$x=6 \vee x=-6$

73 **a** $4\cdot{}^3\log(x)=2$
$^3\log(x)=\frac{1}{2}$
$x=3^{\frac{1}{2}}$
$x=\sqrt{3}$

b $^3\log(4x-1)=-2$
$4x-1=3^{-2}$
$4x-1=\frac{1}{9}$
$4x=1\frac{1}{9}$
$x=\frac{5}{18}$

c $3+{}^2\log(x)=-1$
$^2\log(x)=-4$
$x=2^{-4}$
$x=\frac{1}{16}$

d $^5\log(3x+2)=1$
$3x+2=5^1$
$3x+2=5$
$3x=3$
$x=1$

e $^3\log(0{,}4x-5)=2$
$0{,}4x-5=3^2$
$0{,}4x-5=9$
$0{,}4x=14$
$x=35$

f $4+2\cdot{}^2\log(x)=7$
$2\cdot{}^2\log(x)=3$
$^2\log(x)=1\frac{1}{2}$
$x=2^{1\frac{1}{2}}$
$x=2^1\cdot 2^{\frac{1}{2}}$
$x=2\sqrt{2}$

74 Voer in $y_1=2^x$ en $y_2=30$.
De optie snijpunt geeft $x\approx 4{,}91$.

Bladzijde 40

75 **a** $2^{x-1}=15$
$x-1={}^2\log(15)$
$x=1+{}^2\log(15)$

b $1+2^x=15$
$2^x=14$
$x={}^2\log(14)$

c $4+3^{x+1}=25$
$3^{x+1}=21$
$x+1={}^3\log(21)$
$x=-1+{}^3\log(21)$

d $14-2^{x+3}=2$
$-2^{x+3}=-12$
$2^{x+3}=12$
$x+3={}^2\log(12)$
$x=-3+{}^2\log(12)$

e $7+4^{2x}=12$
$4^{2x}=5$
$2x={}^4\log(5)$
$x=\frac{1}{2}\cdot{}^4\log(5)$

f $3\cdot 5^{2x+1}=60$
$5^{2x+1}=20$
$2x+1={}^5\log(20)$
$2x=-1+{}^5\log(20)$
$x=-\frac{1}{2}+\frac{1}{2}\cdot{}^5\log(20)$

g $3^{x+2}+3^x=600$
$3^x\cdot 3^2+3^x=600$
$9\cdot 3^x+3^x=600$
$10\cdot 3^x=600$
$3^x=60$
$x={}^3\log(60)$

h $2^{1+2x}=4^x+6$
$2^1\cdot 2^{2x}=4^x+6$
$2\cdot(2^2)^x=4^x+6$
$2\cdot 4^x=4^x+6$
$4^x=6$
$x={}^4\log(6)$

76 **a** $4^x = 2^{x+2} - 3$
$(2^2)^x = 2^x \cdot 2^2 - 3$
$(2^x)^2 = 4 \cdot 2^x - 3$

b 2^x bij de vergelijking $(2^x)^2 = 4 \cdot 2^x - 3$ vervangen door u geeft $u^2 = 4u - 3$.

c $u^2 = 4u - 3$
$u^2 - 4u + 3 = 0$
$(u-1)(u-3) = 0$
$u = 1 \vee u = 3$

d $u = 1$ geeft $2^x = 1$ oftewel $x = 0$.
$u = 3$ geeft $2^x = 3$ oftewel $x = {}^2\log(3)$.

77 **a** $9^x - 3^{x+1} = 4$
$(3^2)^x - 3^x \cdot 3^1 = 4$
$(3^x)^2 - 3 \cdot 3^x - 4 = 0$
Stel $3^x = u$.
$u^2 - 3u - 4 = 0$
$(u+1)(u-4) = 0$
$u = -1 \vee u = 4$
$3^x = -1 \vee 3^x = 4$
$x = {}^3\log(4)$

b $4^x = 2^x + 42$
$(2^2)^x = 2^x + 42$
$(2^x)^2 - 2^x - 42 = 0$
Stel $2^x = u$.
$u^2 - u - 42 = 0$
$(u+6)(u-7) = 0$
$u = -6 \vee u = 7$
$2^x = -6 \vee 2^x = 7$
$x = {}^2\log(7)$

c $2^x = 24 - 2^{2x-1}$
$2^x = 24 - 2^{2x} \cdot 2^{-1}$
$2^x = 24 - \frac{1}{2} \cdot (2^x)^2$
$\frac{1}{2} \cdot (2^x)^2 + 2^x - 24 = 0$
$(2^x)^2 + 2 \cdot 2^x - 48 = 0$
Stel $2^x = u$.
$u^2 + 2u - 48 = 0$
$(u-6)(u+8) = 0$
$u = 6 \vee u = -8$
$2^x = 6 \vee 2^x = -8$
$x = {}^2\log(6)$

d $9^x = 5 \cdot 3^x + 6$
$(3^2)^x = 5 \cdot 3^x + 6$
$(3^x)^2 - 5 \cdot 3^x - 6 = 0$
Stel $3^x = u$.
$u^2 - 5u - 6 = 0$
$(u+1)(u-6) = 0$
$u = -1 \vee u = 6$
$3^x = -1 \vee 3^x = 6$
$x = {}^3\log(6)$

78 **a** $5^{x-1} + 5^{2x-1} = 4$
$5^x \cdot 5^{-1} + 5^{2x} \cdot 5^{-1} = 4$
$\frac{1}{5} \cdot 5^x + \frac{1}{5} \cdot (5^x)^2 = 4$
$5^x + (5^x)^2 = 20$
$(5^x)^2 + 5^x - 20 = 0$
Stel $5^x = u$.
$u^2 + u - 20 = 0$
$(u-4)(u+5) = 0$
$u = 4 \vee u = -5$
$5^x = 4 \vee 5^x = -5$
$x = {}^5\log(4)$

b $3^x - 2 = 8 \cdot (\frac{1}{3})^x$
$3^x - 2 = 8 \cdot \frac{1}{3^x}$
$(3^x)^2 - 2 \cdot 3^x = 8$
$(3^x)^2 - 2 \cdot 3^x - 8 = 0$
Stel $3^x = u$.
$u^2 - 2u - 8 = 0$
$(u+2)(u-4) = 0$
$u = -2 \vee u = 4$
$3^x = -2 \vee 3^x = 4$
$x = {}^3\log(4)$

c $2^x = 6 - 5 \cdot (\frac{1}{2})^x$
$2^x = 6 - 5 \cdot \frac{1}{2^x}$
$(2^x)^2 = 6 \cdot 2^x - 5$
$(2^x)^2 - 6 \cdot 2^x + 5 = 0$
Stel $2^x = u$.
$u^2 - 6u + 5 = 0$
$(u-1)(u-5) = 0$
$u = 1 \vee u = 5$
$2^x = 1 \vee 2^x = 5$
$x = 0 \vee x = {}^2\log(5)$

d $3^x + 2 \cdot (\frac{1}{3})^{x-2} = 9$
$3^x + 2 \cdot (\frac{1}{3})^x \cdot (\frac{1}{3})^{-2} = 9$
$3^x + 18 \cdot \frac{1}{3^x} = 9$
$(3^x)^2 + 18 = 9 \cdot 3^x$
$(3^x)^2 - 9 \cdot 3^x + 18 = 0$
Stel $3^x = u$.
$u^2 - 9u + 18 = 0$
$(u-3)(u-6) = 0$
$u = 3 \vee u = 6$
$3^x = 3 \vee 3^x = 6$
$x = 1 \vee x = {}^3\log(6)$

Bladzijde 41

79 $3 \cdot 27^x + 2 \cdot (\frac{1}{3})^x = 7 \cdot 3^x$

$3 \cdot (3^3)^x + 2 \cdot \frac{1}{3^x} = 7 \cdot 3^x$

$3 \cdot (3^x)^3 + 2 \cdot \frac{1}{3^x} = 7 \cdot 3^x$

$3 \cdot (3^x)^4 + 2 = 7 \cdot (3^x)^2$

$3 \cdot (3^x)^4 - 7 \cdot (3^x)^2 + 2 = 0$

Stel $3^x = u$.

$3u^4 - 7u^2 + 2 = 0$

$D = (-7)^2 - 4 \cdot 3 \cdot 2 = 25$

$u^2 = \frac{7+5}{6} \vee u^2 = \frac{7-5}{6}$

$u^2 = 2 \vee u^2 = \frac{1}{3}$

$u = \sqrt{2} \vee u = -\sqrt{2} \vee u = \sqrt{\frac{1}{3}} \vee u = -\sqrt{\frac{1}{3}}$

$3^x = \sqrt{2} \vee 3^x = -\sqrt{2} \vee 3^x = \sqrt{\frac{1}{3}} \vee 3^x = -\sqrt{\frac{1}{3}}$

$x = {}^3\log(\sqrt{2}) \vee 3^x = 3^{-\frac{1}{2}}$

$x = {}^3\log(\sqrt{2}) \vee x = -\frac{1}{2}$

80 Voor f geldt $y = 2^x$, dus voor f^{inv} geldt $x = 2^y$.

$x = 2^y$ geeft $2^y = x$

$y = {}^2\log(x)$

Dus $g(x) = {}^2\log(x)$ is de inverse van f.

Bladzijde 42

81 **a** $ax + b = 0$ geeft $x = -\frac{b}{a}$.

De verticale asymptoot van de grafiek is de lijn $x = -\frac{b}{a}$.

b ${}^3\log(2x + 5) = 2$

$2x + 5 = 3^2$

$2x = 4$

$x = 2$

${}^3\log(2x + 5) \leq 2$ geeft $-2\frac{1}{2} < x \leq 2$

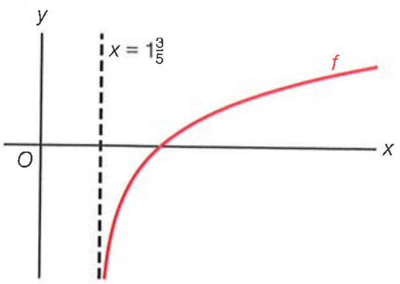
82 **a** $5x - 8 > 0$

$5x > 8$

$x > 1\frac{3}{5}$

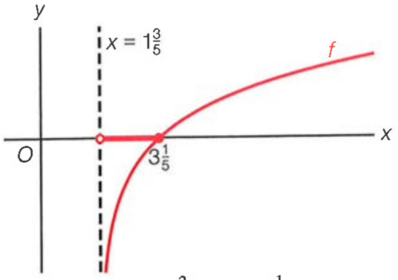
$D_f = \langle 1\frac{3}{5}, \rightarrow \rangle$ en de verticale asymptoot van de grafiek van f is de lijn $x = 1\frac{3}{5}$.

b



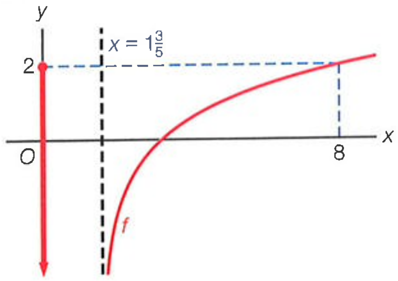
c $f(x) = 0$ geeft $-3 + {}^2\log(5x - 8) = 0$

$${}^2\log(5x - 8) = 3$$
$$5x - 8 = 2^3$$
$$5x = 16$$
$$x = 3\tfrac{1}{5}$$



$f(x) \leq 0$ geeft $1\frac{3}{5} < x \leq 3\frac{1}{5}$

d $f(8) = -3 + {}^2\log(32) = -3 + 5 = 2$



Voor $x \leq 8$ is $f(x) \leq 2$.

83 **a** $f(x) = 5$ geeft ${}^{\frac{1}{2}}\log(x + 3) = 5$

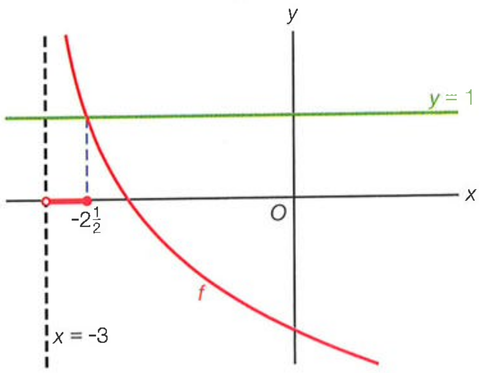
$$x + 3 = (\tfrac{1}{2})^5$$
$$x = -2\tfrac{31}{32}$$

b $f(-1) = {}^{\frac{1}{2}}\log(2) = -1$ en $g(-1) = {}^3\log(6)$

$AB = {}^3\log(6) - -1 = {}^3\log(6) + 1$

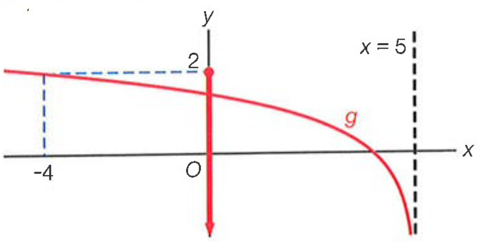
c $f(x) = 1$ geeft ${}^{\frac{1}{2}}\log(x + 3) = 1$

$$x + 3 = (\tfrac{1}{2})^1$$
$$x = -2\tfrac{1}{2}$$



$f(x) \geq 1$ geeft $-3 < x \leq -2\frac{1}{2}$

d $g(-4) = {}^3\log(9) = 2$



Voor $x \geq -4$ is $g(x) \leq 2$.

84 $A(4, 1)$ op de grafiek van f geeft ${}^2\log(4 + c) + d = 1$.
$B(7, 3)$ op de grafiek van f geeft ${}^2\log(7 + c) + d = 3$.
${}^2\log(4 + c) + d = 1$ geeft ${}^2\log(4 + c) = 1 - d$
$4 + c = 2^{1-d}$
$c = 2^{1-d} - 4$
${}^2\log(7 + c) + d = 3$ geeft ${}^2\log(7 + c) = 3 - d$
$7 + c = 2^{3-d}$
$c = 2^{3-d} - 7$
$c = 2^{1-d} - 4$ en $c = 2^{3-d} - 7$ geeft $2^{1-d} - 4 = 2^{3-d} - 7$
$2 \cdot 2^{-d} - 4 = 2^3 \cdot 2^{-d} - 7$
$2 \cdot 2^{-d} - 4 = 8 \cdot 2^{-d} - 7$
$-6 \cdot 2^{-d} = -3$
$2^{-d} = \frac{1}{2}$
$d = 1$
$d = 1$ geeft $c = 2^{1-1} - 4 = 2^0 - 4 = 1 - 4 = -3$
Dus $c = -3$ en $d = 1$.

Diagnostische toets

Bladzijde 46

1 **a** $(3a^{-5}b^4)^{-2} = 3^{-2} \cdot a^{10} \cdot b^{-8} = \frac{1}{9} \cdot a^{10} \cdot \frac{1}{b^8} = \frac{a^{10}}{9b^8}$

b $(\frac{2}{3}a^{-2}b)^{-2} = (\frac{2}{3})^{-2} \cdot a^4 \cdot b^{-2} = \frac{1}{(\frac{2}{3})^2} \cdot a^4 \cdot \frac{1}{b^2} = \frac{1}{\frac{4}{9}} \cdot a^4 \cdot \frac{1}{b^2} = \frac{9}{4} \cdot a^4 \cdot \frac{1}{b^2} = \frac{9a^4}{4b^2}$

c $3a^{1\frac{1}{3}}b^{-3} = 3a \cdot \sqrt[3]{a} \cdot \frac{1}{b^3} = \frac{3a \cdot \sqrt[3]{a}}{b^3}$

d $(a^{-\frac{1}{4}})^3 = a^{-\frac{3}{4}} = \frac{1}{a^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{a^3}}$

e $a^{-2}b^{\frac{1}{5}} = \frac{1}{a^2} \cdot \sqrt[5]{b} = \frac{\sqrt[5]{b}}{a^2}$

f $7a^{-\frac{1}{3}}b^{\frac{3}{5}} = 7 \cdot \frac{1}{a^{\frac{1}{3}}} \cdot \sqrt[5]{b^3} = \frac{7 \cdot \sqrt[5]{b^3}}{\sqrt[3]{a}}$

2 **a** $\frac{\sqrt{x}}{x^2} = \frac{x^{\frac{1}{2}}}{x^2} = x^{\frac{1}{2}-2} = x^{-1\frac{1}{2}}$

b $x^2 \cdot \sqrt[3]{x} = x^2 \cdot x^{\frac{1}{3}} = x^{2\frac{1}{3}}$

c $\frac{1}{\sqrt[3]{x^2}} = \frac{1}{x^{\frac{2}{3}}} = x^{-\frac{2}{3}}$

3 **a** $y = 3x^{-3}(\frac{1}{2}x^2)^3 = 3 \cdot x^{-3} \cdot \frac{1}{8}x^6 = \frac{3}{8}x^3$
Dus $y = \frac{3}{8}x^3$.

b $y = \frac{1}{5x^2 \cdot \sqrt{x}} = \frac{1}{5} \cdot \frac{1}{x^{2\frac{1}{2}}} = \frac{1}{5}x^{-2\frac{1}{2}}$
Dus $y = \frac{1}{5}x^{-2\frac{1}{2}}$.

c $y = \frac{12}{5x^{-3}} \cdot \sqrt[5]{x^3} = \frac{12}{5} \cdot \frac{1}{x^{-3}} \cdot x^{\frac{3}{5}} = 2\frac{2}{5} \cdot x^3 \cdot x^{\frac{3}{5}} = 2\frac{2}{5}x^{3\frac{3}{5}}$
Dus $y = 2\frac{2}{5}x^{3\frac{3}{5}}$.

4 **a** $x^2 \cdot \sqrt[3]{x} = 128$
$x^2 \cdot x^{\frac{1}{3}} = 128$
$x^{2\frac{1}{3}} = 128$
$x = (2^7)^{\frac{3}{7}}$
$x = 2^3$
$x = 8$

b $(2x + 3)^{-\frac{2}{3}} = \frac{4}{9}$
$2x + 3 = (\frac{4}{9})^{-\frac{3}{2}}$
$2x + 3 = ((\frac{2}{3})^2)^{-\frac{3}{2}}$
$2x + 3 = (\frac{2}{3})^{-3}$
$2x + 3 = \dfrac{1}{(\frac{2}{3})^3}$
$2x + 3 = \dfrac{1}{\frac{8}{27}}$
$2x + 3 = \frac{27}{8}$
$2x = \frac{3}{8}$
$x = \frac{3}{16}$

c $112 - 2x^{-4} = 5x^{-4}$
$-7x^{-4} = -112$
$x^{-4} = 16$
$x = 16^{-\frac{1}{4}}$
$x = (2^4)^{-\frac{1}{4}}$
$x = \frac{1}{2}$

5 **a** $y = 0{,}02x^{-1\frac{3}{5}}$
$0{,}02x^{-1\frac{3}{5}} = y$
$x^{-\frac{8}{5}} = 50y$
$x = (50y)^{-\frac{5}{8}}$
$x = 50^{-\frac{5}{8}} \cdot y^{-\frac{5}{8}}$
$x \approx 0{,}09 \cdot y^{-0{,}63}$
Dus $x = 0{,}09y^{-0{,}63}$.

b $y = \frac{1}{4}x^2 \cdot \sqrt[3]{x} = \frac{1}{4}x^2 \cdot x^{\frac{1}{3}} = \frac{1}{4}x^{2\frac{1}{3}}$
$\frac{1}{4}x^{2\frac{1}{3}} = y$
$x^{\frac{7}{3}} = 4y$
$x = (4y)^{\frac{3}{7}}$
$x = 4^{\frac{3}{7}} \cdot y^{\frac{3}{7}}$
$x \approx 1{,}81 \cdot y^{0{,}43}$
Dus $x = 1{,}81y^{0{,}43}$.

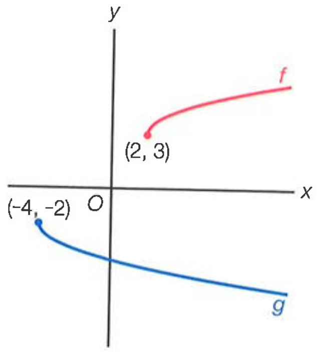
c $y = \dfrac{20}{x^2 \cdot \sqrt{x}} = \dfrac{20}{x^{2\frac{1}{2}}} = 20x^{-2\frac{1}{2}}$
$20x^{-2\frac{1}{2}} = y$
$x^{-\frac{5}{2}} = 0{,}05y$
$x = (0{,}05y)^{-\frac{2}{5}}$
$x = 0{,}05^{-\frac{2}{5}} \cdot y^{-\frac{2}{5}}$
$x \approx 3{,}31 \cdot y^{-0{,}4}$
Dus $x = 3{,}31y^{-0{,}4}$.

6 $f(x) = \frac{1}{3}(x+2)^4 - 6$
↓ verm. x-as, -4
$y = -1\frac{1}{3}(x+2)^4 + 24$
↓ translatie (3, −15)
$g(x) = -1\frac{1}{3}(x-3+2)^4 + 24 - 15$
oftewel $g(x) = -1\frac{1}{3}(x-1)^4 + 9$
max. is $g(1) = 9$

7 **a** $y = \sqrt{x}$
↓ translatie (2, 3)
$f(x) = 3 + \sqrt{x-2}$

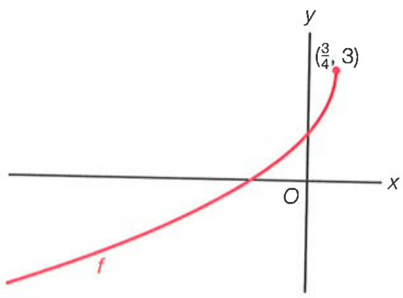
$y = \sqrt{x}$
↓ verm. x-as, −1
$y = -\sqrt{x}$
↓ translatie (−4, −2)
$g(x) = -2 - \sqrt{x+4}$

b



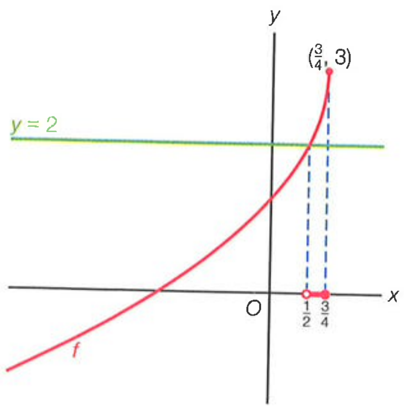
c $D_f = [2, \rightarrow\rangle$, $B_f = [3, \rightarrow\rangle$, $D_g = [-4, \rightarrow\rangle$ en $B_g = \langle\leftarrow, -2]$

8 **a** $3 - 4x \geq 0$
$-4x \geq -3$
$x \leq \frac{3}{4}$
$D_f = \langle\leftarrow, \frac{3}{4}]$ en het randpunt is $(\frac{3}{4}, 3)$.



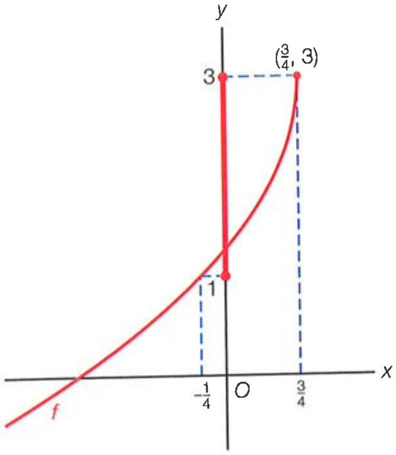
$B_f = \langle\leftarrow, 3]$

b $f(x) = 2$ geeft $3 - \sqrt{3-4x} = 2$
$\sqrt{3-4x} = 1$
$3 - 4x = 1$
$-4x = -2$
$x = \frac{1}{2}$



$f(x) > 2$ geeft $\frac{1}{2} < x \leq \frac{3}{4}$

c $f(-\frac{1}{4}) = 1$



Voor $x \geq -\frac{1}{4}$ is $1 \leq f(x) \leq 3$.

9 **a** $N = 2\sqrt{-5t+1}$

$2\sqrt{-5t+1} = N$

kwadrateren geeft

$4(-5t+1) = N^2$

$-20t + 4 = N^2$

$-20t = N^2 - 4$

$t = -\frac{1}{20}N^2 + \frac{1}{5}$

b $2x\sqrt{y} - 6\sqrt{x} = 1$

$2x\sqrt{y} = 6\sqrt{x} + 1$

kwadrateren geeft

$4x^2y = (6\sqrt{x}+1)^2$

$4x^2y = 36x + 12\sqrt{x} + 1$

$y = \dfrac{36x + 12\sqrt{x} + 1}{4x^2}$

Bladzijde 47

10 **a** $y = 2^x$

↓ verm. x-as, 0,3

$y = 0{,}3 \cdot 2^x$

↓ translatie (-2, -5)

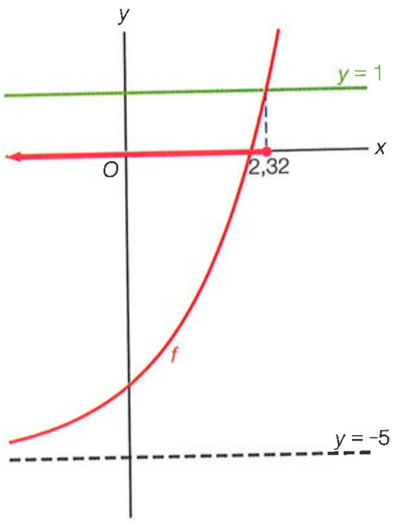
$f(x) = 0{,}3 \cdot 2^{x+2} - 5$

b $B_f = \langle -5, \rightarrow \rangle$

De horizontale asymptoot is de lijn $y = -5$.

c Voer in $y_1 = 0{,}3 \cdot 2^{x+2} - 5$ en $y_2 = 1$.

De optie snijpunt geeft $x \approx 2{,}32$.



$f(x) \leq 1$ geeft $x \leq 2{,}32$

11 **a** $y = 2^x \cdot 2^{-4x+3} = 2^x \cdot 2^{-4x} \cdot 2^3 = 2^{-3x} \cdot 8 = 8 \cdot (2^{-3})^x = 8 \cdot (\frac{1}{8})^x$
Dus $y = 8 \cdot (\frac{1}{8})^x$.

b $y = 108 \cdot 3^{4x-3} = 108 \cdot 3^{4x} \cdot 3^{-3} = 108 \cdot (3^4)^x \cdot \frac{1}{27} = 4 \cdot 81^x$
Dus $y = 4 \cdot 81^x$.

c $y = \frac{250}{5^{-2x+3}} = \frac{250}{5^{-2x} \cdot 5^3} = \frac{250}{5^{-2x} \cdot 125} = 2 \cdot 5^{2x} = 2 \cdot (5^2)^x = 2 \cdot 25^x$
Dus $y = 2 \cdot 25^x$.

12 **a** $5^{x-1} = 125\sqrt{5}$
$5^{x-1} = 5^3 \cdot 5^{\frac{1}{2}}$
$5^{x-1} = 5^{3\frac{1}{2}}$
$x - 1 = 3\frac{1}{2}$
$x = 4\frac{1}{2}$

b $3^{2x-5} = (\frac{1}{27})^x$
$3^{2x-5} = (3^{-3})^x$
$3^{2x-5} = 3^{-3x}$
$2x - 5 = -3x$
$5x = 5$
$x = 1$

c $2 \cdot 4^{2x-1} - 3 = 61$
$2 \cdot 4^{2x-1} = 64$
$4^{2x-1} = 32$
$(2^2)^{2x-1} = 2^5$
$2^{4x-2} = 2^5$
$4x - 2 = 5$
$4x = 7$
$x = 1\frac{3}{4}$

d $9^{x-1} = 27^{x+1}$
$(3^2)^{x-1} = (3^3)^{x+1}$
$3^{2x-2} = 3^{3x+3}$
$2x - 2 = 3x + 3$
$-x = 5$
$x = -5$

e $2^{x+2} + 2^{x-1} = 36$
$2^x \cdot 2^2 + 2^x \cdot 2^{-1} = 36$
$4 \cdot 2^x + \frac{1}{2} \cdot 2^x = 36$
$4\frac{1}{2} \cdot 2^x = 36$
$2^x = 8$
$2^x = 2^3$
$x = 3$

f $2^{x^2} = (\frac{1}{8})^x$
$2^{x^2} = (2^{-3})^x$
$2^{x^2} = 2^{-3x}$
$x^2 = -3x$
$x^2 + 3x = 0$
$x(x + 3) = 0$
$x = 0 \vee x = -3$

13 **a** $f(x) = g(x)$ geeft $2^{x+2} - 3 = 6 - 2^{x-1}$
$2^x \cdot 2^2 - 3 = 6 - 2^x \cdot 2^{-1}$
$4 \cdot 2^x - 3 = 6 - \frac{1}{2} \cdot 2^x$
$4\frac{1}{2} \cdot 2^x = 9$
$2^x = 2$
$x = 1$
$f(x) \geq g(x)$ geeft $x \geq 1$

b $g(4) = 6 - 2^3 = 6 - 8 = -2$
Voor $x \leq 4$ is $-2 \leq g(x) < 6$.

c $f(x) + g(x) = 2^{x+2} - 3 + 6 - 2^{x-1} = 2^x \cdot 2^2 - 3 + 6 - 2^x \cdot 2^{-1} = 4 \cdot 2^x + 3 - \frac{1}{2} \cdot 2^x = 3\frac{1}{2} \cdot 2^x + 3$
Het bereik van $f(x) + g(x)$ is $\langle 3, \rightarrow \rangle$.
Dus de vergelijking $f(x) + g(x) = p$ heeft geen oplossing voor $p \leq 3$.

14 **a** ${}^3\log(3\sqrt{3}) = {}^3\log(3^1 \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{1\frac{1}{2}}) = 1\frac{1}{2}$

b ${}^2\log(\frac{1}{4}\sqrt{8}) = {}^2\log(2^{-2} \cdot \sqrt{2^3}) = {}^2\log(2^{-2} \cdot 2^{1\frac{1}{2}}) = {}^2\log(2^{-\frac{1}{2}}) = -\frac{1}{2}$

c ${}^2\log(\frac{1}{16} \cdot \sqrt[3]{2}) = {}^2\log(2^{-4} \cdot 2^{\frac{1}{3}}) = {}^2\log(2^{-3\frac{2}{3}}) = -3\frac{2}{3}$

15 **a** ${}^4\log(2x - 3) = 2$
$2x - 3 = 4^2$
$2x - 3 = 16$
$2x = 19$
$x = 9\frac{1}{2}$

b ${}^{\frac{1}{2}}\log(x - 3) = -4$
$x - 3 = (\frac{1}{2})^{-4}$
$x - 3 = 16$
$x = 19$

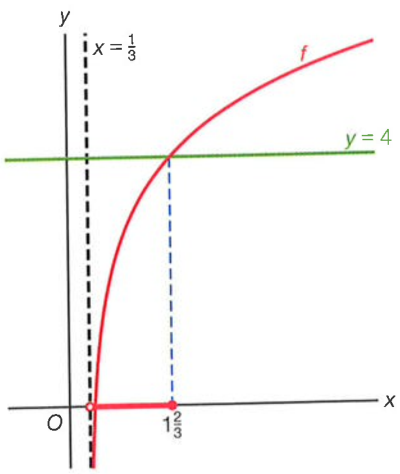
c $5 + 3 \cdot {}^{2}\log(x) = 20$
$3 \cdot {}^{2}\log(x) = 15$
${}^{2}\log(x) = 5$
$x = 2^5$
$x = 32$

16 **a** $7^{x-3} = 20$
$x - 3 = {}^{7}\log(20)$
$x = 3 + {}^{7}\log(20)$

b $4^x - 2^{x+4} = 80$
$(2^2)^x - 2^x \cdot 2^4 = 80$
$(2^x)^2 - 16 \cdot 2^x - 80 = 0$
Stel $2^x = u$.
$u^2 - 16u - 80 = 0$
$(u + 4)(u - 20) = 0$
$u = -4 \lor u = 20$
$2^x = -4 \lor 2^x = 20$
$x = {}^{2}\log(20)$

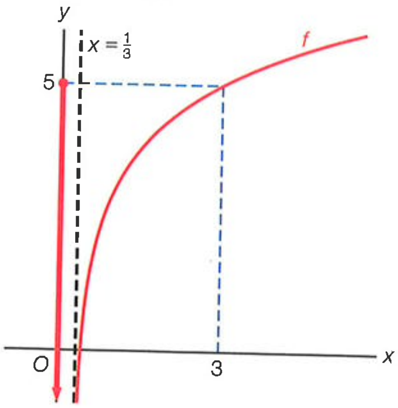
c $5^{x+1} - 6 \cdot (\frac{1}{5})^{x-1} = 25$
$5^x \cdot 5 - 6 \cdot (\frac{1}{5})^x \cdot (\frac{1}{5})^{-1} = 25$
$5 \cdot 5^x - 30 \cdot \frac{1}{5^x} = 25$
$5 \cdot (5^x)^2 - 30 = 25 \cdot 5^x$
$(5^x)^2 - 5 \cdot 5^x - 6 = 0$
Stel $5^x = u$.
$u^2 - 5u - 6 = 0$
$(u + 1)(u - 6) = 0$
$u = -1 \lor u = 6$
$5^x = -1 \lor 5^x = 6$
$x = {}^{5}\log(6)$

17 **a** $3x - 1 > 0$ geeft $x > \frac{1}{3}$
$D_f = \langle \frac{1}{3}, \rightarrow \rangle$ en de verticale asymptoot van de grafiek is de lijn $x = \frac{1}{3}$.
$f(x) = 4$ geeft ${}^{2}\log(3x - 1) + 2 = 4$
${}^{2}\log(3x - 1) = 2$
$3x - 1 = 2^2$
$3x - 1 = 4$
$3x = 5$
$x = 1\frac{2}{3}$



$f(x) \leq 4$ geeft $\frac{1}{3} < x \leq 1\frac{2}{3}$

b $f(3) = {}^2\log(8) + 2 = 5$



Voor $x \leq 3$ is $f(x) \leq 5$.