K Voortgezette integraalrekening

Bladzijde 236

44 **a** $f(x) = \dfrac{x+2}{x^2-4x+8} = \dfrac{x-2+4}{x^2-4x+8} = \dfrac{\frac{1}{2}(2x-4)+4}{x^2-4x+8} = \dfrac{\frac{1}{2}(2x-4)}{x^2-4x+8} + \dfrac{4}{x^2-4x+8}$

$\int \dfrac{\frac{1}{2}(2x-4)}{x^2-4x+8}\,dx = \int \frac{1}{2} \cdot \dfrac{1}{x^2-4x+8}(2x-4)\,dx = \int \frac{1}{2} \cdot \dfrac{1}{x^2-4x+8}\,d(x^2-4x+8) = \frac{1}{2}\ln|x^2-4x+8| + c$

$\int \dfrac{4}{x^2-4x+8}\,dx = \int \dfrac{4}{(x-2)^2-4+8}\,dx = \int \dfrac{1}{\frac{1}{4}(x-2)^2+1}\,dx = \int \dfrac{1}{(\frac{1}{2}(x-2))^2+1}\,dx$

$= \int \dfrac{1}{(\frac{1}{2}x-1)^2+1}\,dx = 2\arctan(\frac{1}{2}x-1) + c$

Dus $F(x) = \frac{1}{2}\ln|x^2-4x+8| + 2\arctan(\frac{1}{2}x-1) + c$.

b $F(x) = \int 3x^2\sin(x^3)\,dx = \int \sin(x^3)\,dx^3 = -\cos(x^3) + c$

c $F(x) = \int \cos(x) \cdot \sqrt[3]{1+\sin(x)}\,dx = \int \sqrt[3]{1+\sin(x)}\,d\sin(x) = \int \sqrt[3]{1+u}\,du = \int (1+u)^{\frac{1}{3}}\,du$

$= \frac{3}{4}(1+u)^{1\frac{1}{3}} + c = \frac{3}{4}(1+u) \cdot \sqrt[3]{1+u} + c = \frac{3}{4}(1+\sin(x)) \cdot \sqrt[3]{1+\sin(x)} + c$

d $F(x) = \int (2x+4)\cos(2x)\,dx = \int (2x+4)\,d\frac{1}{2}\sin(2x) = (2x+4) \cdot \frac{1}{2}\sin(2x) - \int \frac{1}{2}\sin(2x)\,d(2x+4)$

$= (x+2)\sin(2x) - \int \sin(2x)\,dx = (x+2)\sin(2x) + \frac{1}{2}\cos(2x) + c$

e $F(x) = \int \dfrac{2x+1}{\sqrt{16-x^2}}\,dx = \int \left(\dfrac{2x}{\sqrt{16-x^2}} + \dfrac{1}{\sqrt{16-x^2}}\right)dx$

$= \int -\dfrac{1}{\sqrt{16-x^2}}\,d(16-x^2) + \int \dfrac{\frac{1}{4}}{\sqrt{1-(\frac{1}{4}x)^2}}\,dx$

$= \int -\dfrac{1}{\sqrt{u}}\,du + \arcsin(\frac{1}{4}x) = -2\sqrt{u} + \arcsin(\frac{1}{4}x) + c = -2\sqrt{16-x^2} + \arcsin(\frac{1}{4}x) + c$

f $F(x) = \int \dfrac{2x+4}{\sqrt{-x^2-4x-3}}\,dx = \int \dfrac{-1}{\sqrt{-x^2-4x-3}}\,d(-x^2-4x-3) = \int \dfrac{-1}{\sqrt{u}}\,du = -2\sqrt{u} + c$

$= -2\sqrt{-x^2-4x-3} + c$

g $f(x) = \dfrac{2x+7}{x^2+6x+8} = \dfrac{2x+7}{(x+2)(x+4)} = \dfrac{a}{x+2} + \dfrac{b}{x+4} = \dfrac{a(x+4)+b(x+2)}{(x+2)(x+4)} = \dfrac{(a+b)x+4a+2b}{(x+2)(x+4)}$

$\begin{cases} a+b=2 \\ 4a+2b=7 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 2a+2b=4 \\ 4a+2b=7 \end{cases}$ $-$

$-2a = -3$

$\left.\begin{array}{l} a = 1\frac{1}{2} \\ a+b=2 \end{array}\right\}$ $1\frac{1}{2}+b=2$

$b = \frac{1}{2}$

$f(x) = \dfrac{1\frac{1}{2}}{x+2} + \dfrac{\frac{1}{2}}{x+4}$ geeft $F(x) = 1\frac{1}{2}\ln|x+2| + \frac{1}{2}\ln|x+4| + c$

45 **a** $f(x) = 4\sin^2(x)\cos(x)$ geeft

$f'(x) = 8\sin(x)\cos(x) \cdot \cos(x) + 4\sin^2(x) \cdot -\sin(x) = 8\sin(x)\cos^2(x) - 4\sin^3(x)$

$f(x) = 0$ geeft $4\sin^2(x) \cdot \cos(x) = 0$

$\sin(x) = 0 \vee \cos(x) = 0$

$x = 0 \vee x = \pi \vee x = 2\pi \vee x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi$

$f'(0) = 8 \cdot 0 \cdot 1^2 - 4 \cdot 0 = 0$

$f'(\pi) = 8 \cdot 0 \cdot (-1)^2 - 4 \cdot 0 = 0$

$f'(2\pi) = 8 \cdot 0 \cdot 1^2 - 4 \cdot 0 = 0$

Dus de grafiek raakt de x-as in $(0, 0)$, $(\pi, 0)$ en $(2\pi, 0)$.

b Stel $k: y = ax + b$ met $a = f'(\frac{1}{2}\pi) = 8 \cdot 1 \cdot 0 - 4 \cdot 1 = -4$.

$\left.\begin{array}{l} y = -4x + b \\ f(\frac{1}{2}\pi) = 0, \text{ dus } A(\frac{1}{2}\pi, 0) \end{array}\right\}$ $-4 \cdot \frac{1}{2}\pi + b = 0$

$b = 2\pi$

Dus $k: y = -4x + 2\pi$.

c $O(V)=\int\limits_{0}^{\frac{1}{2}\pi} 4\sin^2(x)\cos(x)\,\mathrm{d}x=\int\limits_{x=0}^{x=\frac{1}{2}\pi} 4\sin^2(x)\,\mathrm{d}\sin(x)=\left[\tfrac{4}{3}\sin^3(x)\right]_0^{\frac{1}{2}\pi}=\tfrac{4}{3}\cdot 1-\tfrac{4}{3}\cdot 0=1\tfrac{1}{3}$

46 **a** $f(x)=\tfrac{2}{3}$ geeft $\dfrac{2x}{x^2-4}=\tfrac{2}{3}$

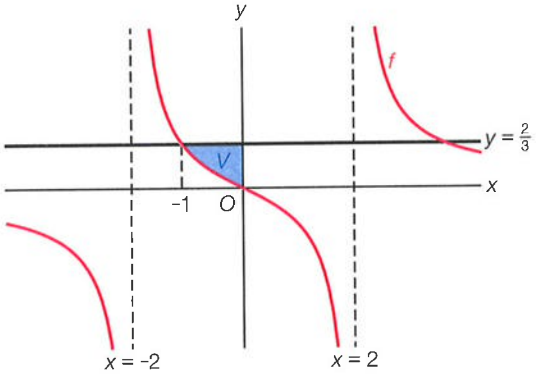
$2x^2-8=6x$

$2x^2-6x-8=0$

$x^2-3x-4=0$

$(x+1)(x-4)=0$

$x=-1 \vee x=4$



$$O(V)=\int\limits_{-1}^{0}\left(\tfrac{2}{3}-\frac{2x}{x^2-4}\right)\mathrm{d}x$$

$$\int\frac{2x}{x^2-4}\,\mathrm{d}x=\int\frac{1}{x^2-4}\,2x\,\mathrm{d}x=\int\frac{1}{x^2-4}\,\mathrm{d}(x^2-4)=\ln|x^2-4|+c$$

Dus $O(V)=\left[\tfrac{2}{3}x-\ln|x^2-4|\right]_{-1}^{0}=0-\ln(4)-(-\tfrac{2}{3}-\ln(3))=-\ln(4)+\tfrac{2}{3}+\ln(3)=\tfrac{2}{3}+\ln(\tfrac{3}{4})$.

b $\int\limits_{p}^{p+4} f(x)\,\mathrm{d}x=\int\limits_{p}^{p+4}\frac{2x}{x^2-4}\,\mathrm{d}x=\left[\ln(x^2-4)\right]_p^{p+4}=\ln((p+4)^2-4)-\ln(p^2-4)$

$=\ln(p^2+8p+16-4)-\ln(p^2-4)=\ln\left(\dfrac{p^2+8p+12}{p^2-4}\right)$

Omdat $p^2-4>0$ als $p>2$, kan de modulus wegblijven.

$\int\limits_{p}^{p+4} f(x)\,\mathrm{d}x=\ln(2)$

$\ln\left(\dfrac{p^2+8p+12}{p^2-4}\right)=\ln(2)$

$\dfrac{p^2+8p+12}{p^2-4}=2$

$p^2+8p+12=2p^2-8$

$p^2-8p-20=0$

$(p+2)(p-10)=0$

$p=-2 \quad \vee \quad p=10$

vold. niet

Dus voor $p=10$.

47 **a** $f(x) = \dfrac{x^3 + 10x}{x^2 + 1}$ geeft

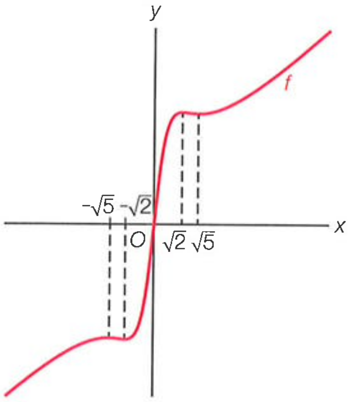
$f'(x) = \dfrac{(x^2 + 1) \cdot (3x^2 + 10) - (x^3 + 10x) \cdot 2x}{(x^2 + 1)^2} = \dfrac{3x^4 + 10x^2 + 3x^2 + 10 - 2x^4 - 20x^2}{(x^2 + 1)^2} = \dfrac{x^4 - 7x^2 + 10}{(x^2 + 1)^2}$

$f'(x) = 0$ geeft $x^4 - 7x^2 + 10 = 0$

$(x^2 - 2)(x^2 - 5) = 0$

$x^2 = 2 \vee x^2 = 5$

$x = \sqrt{2} \vee x = -\sqrt{2} \vee x = \sqrt{5} \vee x = -\sqrt{5}$



$f(\sqrt{2}) = \dfrac{(\sqrt{2})^3 + 10 \cdot \sqrt{2}}{(\sqrt{2})^2 + 1} = \dfrac{2\sqrt{2} + 10\sqrt{2}}{2 + 1} = \dfrac{12\sqrt{2}}{3} = 4\sqrt{2}$

$f(-\sqrt{2}) = \dfrac{(-\sqrt{2})^3 + 10 \cdot -\sqrt{2}}{(-\sqrt{2})^2 + 1} = \dfrac{-2\sqrt{2} - 10\sqrt{2}}{2 + 1} = \dfrac{-12\sqrt{2}}{3} = -4\sqrt{2}$

$f(\sqrt{5}) = \dfrac{(\sqrt{5})^3 + 10 \cdot \sqrt{5}}{(\sqrt{5})^2 + 1} = \dfrac{5\sqrt{5} + 10\sqrt{5}}{5 + 1} = \dfrac{15\sqrt{5}}{6} = 2\tfrac{1}{2}\sqrt{5}$

$f(-\sqrt{5}) = \dfrac{(-\sqrt{5})^3 + 10 \cdot -\sqrt{5}}{(-\sqrt{5})^2 + 1} = \dfrac{-5\sqrt{5} - 10\sqrt{5}}{5 + 1} = \dfrac{-15\sqrt{5}}{6} = -2\tfrac{1}{2}\sqrt{5}$

De toppen zijn $(-\sqrt{5}, -2\tfrac{1}{2}\sqrt{5})$, $(-\sqrt{2}, -4\sqrt{2})$, $(\sqrt{2}, 4\sqrt{2})$ en $(\sqrt{5}, 2\tfrac{1}{2}\sqrt{5})$.

b Stel $k: y = ax + b$ met $a = f'(1) = \dfrac{1 - 7 + 10}{(1 + 1)^2} = 1$.

$\left.\begin{array}{l} y = x + b \\ f(1) = 5\tfrac{1}{2}, \text{ dus } A(1, 5\tfrac{1}{2}) \end{array}\right\} \begin{array}{l} 1 + b = 5\tfrac{1}{2} \\ b = 4\tfrac{1}{2} \end{array}$

Dus $k: y = x + 4\tfrac{1}{2}$.

c $f(x) = p$ heeft precies één oplossing voor $p < -4\sqrt{2} \vee -2\tfrac{1}{2}\sqrt{5} < p < 2\tfrac{1}{2}\sqrt{5} \vee p > 4\sqrt{2}$.

d $x^2 + 1 \,/\, x^3 + 10x \,\backslash\, x$

$\underline{x^3 + x} \; -$

$9x$

$f(x) = x + \dfrac{9x}{x^2 + 1}$

$\int \dfrac{9x}{x^2 + 1}\,\mathrm{d}x = \int 4\tfrac{1}{2} \cdot \dfrac{1}{x^2 + 1} \cdot 2x\,\mathrm{d}x = \int 4\tfrac{1}{2} \cdot \dfrac{1}{x^2 + 1}\,\mathrm{d}(x^2 + 1) = 4\tfrac{1}{2}\ln|x^2 + 1| + c$

$O(V) = \int_0^3 f(x)\,\mathrm{d}x = \int_0^3 \left(x + \dfrac{9x}{x^2 + 1}\right)\mathrm{d}x = \left[\tfrac{1}{2}x^2 + 4\tfrac{1}{2}\ln(x^2 + 1)\right]_0^3$

$= 4\tfrac{1}{2} + 4\tfrac{1}{2}\ln(10) - 0 = 4\tfrac{1}{2} + 4\tfrac{1}{2}\ln(10)$

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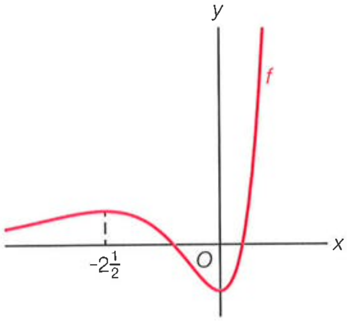
48 **a** $f(x) = (2x^2 + x - 1)e^x$ geeft $f'(x) = (4x + 1) \cdot e^x + (2x^2 + x - 1) \cdot e^x = (2x^2 + 5x)e^x$

$f'(x) = 0$ geeft $(2x^2 + 5x)e^x = 0$

$2x^2 + 5x = 0$

$x(2x + 5) = 0$

$x = 0 \vee x = -2\frac{1}{2}$



$f(0) = -1$ en $f(-2\frac{1}{2}) = 9e^{-2\frac{1}{2}} = \dfrac{9}{e^{2\frac{1}{2}}} = \dfrac{9}{e^2 \cdot \sqrt{e}}$

Dus de toppen zijn $(0, -1)$ en $\left(-2\frac{1}{2}, \dfrac{9}{e^2 \cdot \sqrt{e}}\right)$.

b $f(x) = 0$ geeft $(2x^2 + x - 1)e^x = 0$

$2x^2 + x - 1 = 0$

$D = 1^2 - 4 \cdot 2 \cdot -1 = 9$

$x = \dfrac{-1+3}{4} = \frac{1}{2} \vee x = \dfrac{-1-3}{4} = -1$

$$\int (2x^2 + x - 1)e^x\,dx = \int (2x^2 + x - 1)\,de^x = (2x^2 + x - 1)e^x - \int e^x\,d(2x^2 + x - 1)$$
$$= (2x^2 + x - 1)e^x - \int (4x + 1)e^x\,dx = (2x^2 + x - 1)e^x - \int (4x + 1)\,de^x$$
$$= (2x^2 + x - 1)e^x - (4x + 1)e^x + \int e^x\,d(4x + 1) = (2x^2 + x - 1)e^x - (4x + 1)e^x + \int 4e^x\,dx$$
$$= (2x^2 + x - 1)e^x - (4x + 1)e^x + 4e^x = (2x^2 - 3x + 2)e^x$$

$$O(V) = -\int_{-1}^{\frac{1}{2}} f(x)\,dx = -\left[(2x^2 - 3x + 2)e^x\right]_{-1}^{\frac{1}{2}} = -(\tfrac{1}{2} - 1\tfrac{1}{2} + 2)e^{\frac{1}{2}} + (2 + 3 + 2)e^{-1} = -e^{\frac{1}{2}} + 7e^{-1} = \frac{7}{e} - \sqrt{e}$$

c $-\displaystyle\int_{-1}^{p} f(x)\,dx = \frac{1}{2}\left(\frac{7}{e} - \sqrt{e}\right)$ geeft $-\left[(2x^2 - 3x + 2)e^x\right]_{-1}^{p} = \dfrac{7}{2e} - \frac{1}{2}\sqrt{e}$

$-(2p^2 - 3p + 2)e^p + \dfrac{7}{e} = \dfrac{7}{2e} - \frac{1}{2}\sqrt{e}$

$-(2p^2 - 3p + 2)e^p = -\dfrac{7}{2e} - \frac{1}{2}\sqrt{e}$

$(2p^2 - 3p + 2)e^p = \dfrac{7}{2e} + \frac{1}{2}\sqrt{e}$

Voer in $y_1 = (2x^2 - 3x + 2)e^x$ en $y_2 = \dfrac{7}{2e} + \frac{1}{2}\sqrt{e}$.

De optie snijpunt geeft $x = -0{,}1130...$

Dus $p \approx -0{,}113$.

49 **a** $F(x) = \int x^3 \ln(x)\,dx = \int \ln(x)\,d(\frac{1}{4}x^4) = \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4\,d\ln(x) = \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4 \cdot \dfrac{1}{x}\,dx$

$= \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^3\,dx = \frac{1}{4}x^4 \ln(x) - \frac{1}{16}x^4 + c$

b $F(x) = \int x \ln(x^3)\,dx = \int 3x \ln(x)\,dx = \int 3\ln(x)\,d\frac{1}{2}x^2 = 3\ln(x) \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2\,d\,3\ln(x)$

$= 1\frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x^2 \cdot \dfrac{3}{x}\,dx = 1\frac{1}{2}x^2 \ln(x) - \int 1\frac{1}{2}x\,dx = 1\frac{1}{2}x^2 \ln(x) - \frac{3}{4}x^2 + c$

c $F_n(x) = \int x^n \ln(x)\,dx = \int \ln(x)\,d\dfrac{1}{n+1}x^{n+1} = \dfrac{1}{n+1}x^{n+1}\ln(x) - \int \dfrac{1}{n+1}x^{n+1}\,d\ln(x)$

$= \dfrac{1}{n+1}x^{n+1}\ln(x) - \int \dfrac{1}{n+1}x^{n+1} \cdot \dfrac{1}{x}\,dx = \dfrac{1}{n+1}x^{n+1}\ln(x) - \int \dfrac{1}{n+1}x^n\,dx$

$= \dfrac{1}{n+1}x^{n+1}\ln(x) - \dfrac{1}{(n+1)^2}x^{n+1} + c$

d Er geldt $x > 0$, omdat anders x^m niet bestaat.

Uit c volgt $\int x\ln(x)\,dx = \frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + c$.

$F_m(x) = \int x\ln(x^m)\,dx = \int mx\ln(x)\,dx = \frac{1}{2}mx^2\ln(x) - \frac{1}{4}mx^2 + c$

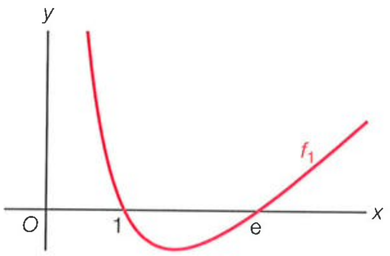
50 **a** $f_1(x) = 2\ln^2(x) - 2\ln(x)$

$f_1(x) = 0$ geeft $2\ln^2(x) - 2\ln(x) = 0$

$2\ln(x)(\ln(x) - 1) = 0$

$\ln(x) = 0 \vee \ln(x) = 1$

$x = 1 \vee x = \mathrm{e}$



$\int 2\ln^2(x)\,dx = 2x\ln^2(x) - \int 2x\,d\ln^2(x) = 2x\ln^2(x) - \int 2x \cdot 2\ln(x) \cdot \frac{1}{x}dx$

$= 2x\ln^2(x) - \int 4\ln(x)\,dx$

$= 2x\ln^2(x) - 4(x\ln(x) - x) + c = 2x\ln^2(x) - 4x\ln(x) + 4x + c$

$\int 2\ln(x)\,dx = 2(x\ln(x) - x) + c = 2x\ln(x) - 2x + c$

$\int_1^{\mathrm{e}} f_1(x)\,dx = \int_1^{\mathrm{e}} (2\ln^2(x) - 2\ln(x))\,dx = \left[2x\ln^2(x) - 4x\ln(x) + 4x - (2x\ln(x) - 2x)\right]_1^{\mathrm{e}}$

$= \left[2x\ln^2(x) - 6x\ln(x) + 6x\right]_1^{\mathrm{e}} = 2\mathrm{e} - 6\mathrm{e} + 6\mathrm{e} - (0 - 0 + 6) = 2\mathrm{e} - 6$

De oppervlakte is $-(2\mathrm{e} - 6) = 6 - 2\mathrm{e}$.

b $f_p(x) = 0$ geeft $2\ln^2(x) - 2p\ln(x) = 0$

$2\ln(x)(\ln(x) - p) = 0$

$\ln(x) = 0 \vee \ln(x) = p$

$x = 1 \vee x = \mathrm{e}^p$

Uit a volgt $F_p(x) = 2x\ln^2(x) - 4x\ln(x) + 4x - 2px\ln(x) + 2px + c$.

$\int_1^{\mathrm{e}^p} f_p(x)\,dx = \left[2x\ln^2(x) - 4x\ln(x) + 4x - 2px\ln(x) + 2px\right]_1^{\mathrm{e}^p}$

$= 2\mathrm{e}^p \cdot p^2 - 4\mathrm{e}^p \cdot p + 4\mathrm{e}^p - 2p\mathrm{e}^p \cdot p + 2p\mathrm{e}^p - (0 - 0 + 4 - 0 + 2p)$

$= 2p^2\mathrm{e}^p - 4p\mathrm{e}^p + 4\mathrm{e}^p - 2p^2\mathrm{e}^p + 2p\mathrm{e}^p - 4 - 2p$

$= -2p\mathrm{e}^p + 4\mathrm{e}^p - 4 - 2p = (4 - 2p)\mathrm{e}^p - 4 - 2p$

$f_p(x) \leq 0$ voor $1 \leq x \leq \mathrm{e}^p$, dus opp $= (2p - 4)\mathrm{e}^p + 2p + 4$.

opp $= 8$ geeft $(2p - 4)\mathrm{e}^p + 2p + 4 = 8$

$(2p - 4)\mathrm{e}^p + 2p - 4 = 0$

$(2p - 4)(\mathrm{e}^p + 1) = 0$

$2p - 4 = 0 \vee \mathrm{e}^p + 1 = 0$

$2p = 4$ geen opl.

$p = 2$

Dus voor $p = 2$.

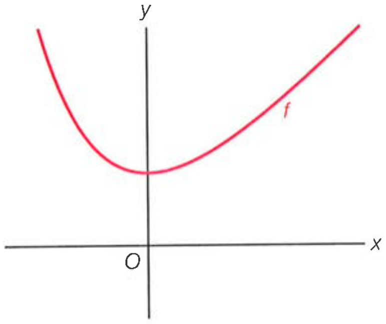
51 **a** $f(x) = x + e^{-x}$ geeft $f'(x) = 1 - e^{-x}$

$f'(x) = 0$ geeft $1 - e^{-x} = 0$

$e^{-x} = 1$

$-x = 0$

$x = 0$



min. is $f(0) = 1$

$B_f = [1, \rightarrow\rangle$

b $p > 0$

$O(V_p) = 6$ geeft $\int_{-p}^{p} (x + e^{-x})\,dx = 6$

$\left[\tfrac{1}{2}x^2 - e^{-x}\right]_{-p}^{p} = 6$

$\tfrac{1}{2}p^2 - e^{-p} - (\tfrac{1}{2}p^2 - e^{p}) = 6$

$\tfrac{1}{2}p^2 - e^{-p} - \tfrac{1}{2}p^2 + e^{p} = 6$

$e^{p} - 6 - e^{-p} = 0$

$(e^{p})^2 - 6\,e^{p} - 1 = 0$

Stel $e^{p} = u$.

$u^2 - 6u - 1 = 0$

$D = (-6)^2 - 4 \cdot 1 \cdot -1 = 40$

$u = \dfrac{6 + 2\sqrt{10}}{2} \vee u = \dfrac{6 - 2\sqrt{10}}{2}$

$u = 3 + \sqrt{10} \;\vee\; u = 3 - \sqrt{10}$

$e^{p} = 3 + \sqrt{10} \vee e^{p} = 3 - \sqrt{10}$

$p = \ln(3 + \sqrt{10})$ geen opl.

$p < 0$

$O(V_p) = 6$ geeft $\int_{p}^{-p} (x + e^{-x})\,dx = 6$

$\left[\tfrac{1}{2}x^2 - e^{-x}\right]_{p}^{-p} = 6$

$\tfrac{1}{2}p^2 - e^{p} - (\tfrac{1}{2}p^2 - e^{-p}) = 6$

$\tfrac{1}{2}p^2 - e^{p} - \tfrac{1}{2}p^2 + e^{-p} = 6$

$-e^{p} - 6 + e^{-p} = 0$

$(e^{p})^2 + 6\,e^{p} - 1 = 0$

Stel $e^{p} = u$.

$u^2 + 6u - 1 = 0$

$D = 6^2 - 4 \cdot 1 \cdot -1 = 40$

$u = \dfrac{-6 + 2\sqrt{10}}{2} \vee u = \dfrac{-6 - 2\sqrt{10}}{2}$

$u = -3 + \sqrt{10} \;\vee\; u = -3 - \sqrt{10}$

$e^{p} = -3 + \sqrt{10} \vee e^{p} = -3 - \sqrt{10}$

$p = \ln(-3 + \sqrt{10})$ geen opl.

Dus voor $p = \ln(3 + \sqrt{10})$ en voor $p = \ln(-3 + \sqrt{10})$.

c $I = \pi\int_{-1}^{0} (f(x))^2\,dx = \pi\int_{-1}^{0} (x + e^{-x})^2\,dx = \pi\int_{-1}^{0} (x^2 + 2x\,e^{-x} + e^{-2x})\,dx$

$\int 2x\,e^{-x}\,dx = \int 2x\,d\,{-e^{-x}} = -2x\,e^{-x} + \int e^{-x}\,d\,2x = -2x\,e^{-x} + \int 2e^{-x}\,dx = -2x\,e^{-x} - 2\,e^{-x} + c$

$I = \pi\left[\tfrac{1}{3}x^3 - 2x\,e^{-x} - 2\,e^{-x} - \tfrac{1}{2}e^{-2x}\right]_{-1}^{0}$

$= \pi(0 - 0 - 2 - \tfrac{1}{2}) - \pi(-\tfrac{1}{3} + 2e - 2e - \tfrac{1}{2}e^2)$

$= \pi(-2\tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{2}e^2) = (\tfrac{1}{2}e^2 - 2\tfrac{1}{6})\pi$

52 **a** $F(x) = \int \dfrac{1}{e^{x} + 1}\,dx = \int \dfrac{e^{-x}}{1 + e^{-x}}\,dx = \int \dfrac{-1}{1 + e^{-x}}\,d(1 + e^{-x}) = \int -\dfrac{1}{u}\,du = -\ln|u| + c = -\ln(1 + e^{-x}) + c$

b $F(x) = \int \dfrac{\ln(\sin(x))}{\cos^2(x)}\,dx = \int \ln(\sin(x))\,d\tan(x) = \tan(x) \cdot \ln(\sin(x)) - \int \tan(x)\,d\ln(\sin(x))$

$= \tan(x) \cdot \ln(\sin(x)) - \int \tan(x) \cdot \dfrac{1}{\sin(x)} \cdot \cos(x)\,dx$

$= \tan(x) \cdot \ln(\sin(x)) - \int \dfrac{\sin(x)}{\cos(x)} \cdot \dfrac{1}{\sin(x)} \cdot \cos(x)\,dx$

$= \tan(x) \cdot \ln(\sin(x)) - \int 1\,dx$

$= \tan(x) \cdot \ln(\sin(x)) - x + c$

c $\sqrt{2x+1} = u$
$2x + 1 = u^2$
$2x = u^2 - 1$
$x = \frac{1}{2}u^2 - \frac{1}{2}$
$F(x) = \int x\sqrt{2x+1}\,dx = \int(\frac{1}{2}u^2 - \frac{1}{2}) \cdot u\,d(\frac{1}{2}u^2 - \frac{1}{2}) = \int(\frac{1}{2}u^2 - \frac{1}{2}) \cdot u \cdot u\,du = \int(\frac{1}{2}u^4 - \frac{1}{2}u^2)\,du$
$= \frac{1}{10}u^5 - \frac{1}{6}u^3 + c = \frac{1}{10}(2x+1)^2 \cdot \sqrt{2x+1} - \frac{1}{6}(2x+1)\sqrt{2x+1} + c$

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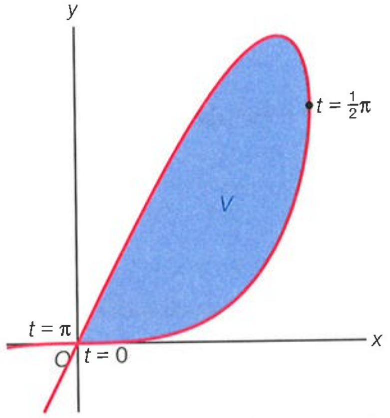
53 **a** $x = 11$ geeft $t^2 - 4 = 11$
$t^2 = 15$
$t = \sqrt{15} \vee t = -\sqrt{15}$
$y(\sqrt{15}) = \sqrt{15}\ln(4 + \sqrt{15})$
$y(-\sqrt{15}) = -\sqrt{15}\ln(4 - \sqrt{15}) = \sqrt{15}\ln((4 - \sqrt{15})^{-1}) = \sqrt{15}\ln\left(\frac{1}{4 - \sqrt{15}}\right)$
$= \sqrt{15}\ln\left(\frac{1}{4-\sqrt{15}} \cdot \frac{4+\sqrt{15}}{4+\sqrt{15}}\right) = \sqrt{15}\ln\left(\frac{4+\sqrt{15}}{16-15}\right) = \sqrt{15}\ln(4+\sqrt{15})$
Dus $y(\sqrt{15}) = y(-\sqrt{15})$ en dus snijdt K zichzelf in het punt $(11, \sqrt{15}\ln(4 + \sqrt{15}))$.
Dit snijpunt ligt op de lijn $x = 11$, dus K snijdt zichzelf op de lijn $x = 11$.

b $\int y\,dx = \int t\ln(t+4)\,d(t^2-4) = \int t\ln(t+4) \cdot 2t\,dt = \int 2t^2\ln(t+4)\,dt$
De GR geeft $\int_{-\sqrt{15}}^{\sqrt{15}} 2x^2\ln(x+4)\,dx = 67{,}636...$
Dus oppervlakte $\approx 67{,}64$.

54 **a** $x = 0$ geeft $4\sin(t) = 0$
$t = k \cdot \pi$
$t = \frac{1}{2}\pi$ geeft het punt $(4, 4)$ en $t = \frac{1}{4}\pi$ geeft het punt $(2\sqrt{2}, 2\sqrt{2} - 2)$.
Dus V wordt in positieve richting omlopen.
$O(V) = \int_{t=\pi}^{t=0} y\,dx = \int_{t=\pi}^{t=0} (4\sin(t) - 2\sin(2t))\,d\,4\sin(t) = \int_{\pi}^{0}(4\sin(t) - 4\sin(t)\cos(t)) \cdot 4\cos(t)\,dt$
$= \int_{\pi}^{0}(16\sin(t)\cos(t) - 16\sin(t)\cos^2(t))\,dt = \int_{\pi}^{0}16\sin(t)\cos(t)\,dt - \int_{\pi}^{0}16\sin(t)\cos^2(t)\,dt$
$= \int_{\pi}^{0}8\sin(2t)\,dt + \int_{t=\pi}^{t=0} 16\cos^2(t)\,d\cos(t) = [-4\cos(2t)]_{\pi}^{0} + [5\frac{1}{3}\cos^3(t)]_{\pi}^{0}$
$= -4\cos(0) + 4\cos(2\pi) + 5\frac{1}{3}\cos^3(0) - 5\frac{1}{3}\cos^3(\pi) = -4 + 4 + 5\frac{1}{3} \cdot 1^3 - 5\frac{1}{3} \cdot (-1)^3 = 10\frac{2}{3}$

b x is maximaal 4 voor $t = \frac{1}{2}\pi$.



$$I(L) = \pi \int_{t=\pi}^{t=\frac{1}{2}\pi} y^2 \, dx - \pi \int_{t=0}^{t=\frac{1}{2}\pi} y^2 \, dx = \pi \int_{t=\pi}^{t=\frac{1}{2}\pi} y^2 \, dx + \pi \int_{t=\frac{1}{2}\pi}^{t=0} y^2 \, dx = \pi \int_{t=\pi}^{t=0} y^2 \, dx$$

$$= \pi \int_{t=\pi}^{t=0} (4\sin(t) - 2\sin(2t))^2 \, d\,4\sin(t) = \pi \int_{\pi}^{0} (4\sin(t) - 2\sin(2t))^2 \cdot 4\cos(t) \, dt$$

De GR geeft $\pi \int_{\pi}^{0} (4\sin(x) - 2\sin(2x))^2 \cdot 4\cos(x) \, dx = 157{,}913...$

Dus $I(L) \approx 157{,}91$.