$MN = 25$ geeft $r + s = 25$

$$\begin{cases} r+s=25 \\ r-s=20 \end{cases} +$$
$$2r = 45$$

$$\left.\begin{array}{l} r = 22\tfrac{1}{2} \\ r+s=25 \end{array}\right\} 22\tfrac{1}{2} + s = 25$$
$$s = 2\tfrac{1}{2}$$

Dus $r = 22\frac{1}{2}$ en $s = 2\frac{1}{2}$.

37 M op k en $x_M = -3$ geeft $y_M = -1\frac{1}{3} \cdot -3 + 4 = 8$.
Dus $M(-3, 8)$.
N op k en $y_N = -4$ geeft $-1\frac{1}{3}x_N + 4 = -4$

$$-1\tfrac{1}{3}x_N = -8$$
$$x_N = 6$$

Dus $N(6, -4)$.
$d(M, N) = \sqrt{(6 - -3)^2 + (-4 - 8)^2} = \sqrt{81 + 144} = \sqrt{225} = 15$, $r_1 = 3$ en $r_2 = 4$.
$d(c_1, c_2) = d(M, N) - r_1 - r_2 = 15 - 3 - 4 = 8$
$d(c_1, l) = d(c_2, l) = 4$
$d(M, l) = r_1 + d(c_1, l) = 3 + 4 = 7$
$d(N, l) = r_2 + d(c_2, l) = 4 + 4 = 8$
k: $y = -1\frac{1}{3}x + 4$ oftewel k: $1\frac{1}{3}x + y = 4$ oftewel k: $4x + 3y = 12$
$l \perp k$ geeft l: $3x - 4y = c$

$$d(M, l) = 7 \text{ geeft } \frac{|3 \cdot -3 - 4 \cdot 8 - c|}{\sqrt{3^2 + 4^2}} = 7$$
$$\frac{|-41 - c|}{\sqrt{25}} = 7$$
$$\frac{|-41 - c|}{5} = 7$$
$$|-41 - c| = 35$$
$$-41 - c = 35 \vee -41 - c = -35$$
$$c = -76 \vee c = -6$$

Dus l_1: $3x - 4y = -76$ en l_2: $3x - 4y = -6$.

$$d(N, l_1) = \frac{|3 \cdot 6 - 4 \cdot -4 + 76|}{5} = \frac{|110|}{5} = \frac{110}{5} = 22, \text{ dus } l_1 \text{ voldoet niet.}$$

$$d(N, l_2) = \frac{|3 \cdot 6 - 4 \cdot -4 + 6|}{5} = \frac{|40|}{5} = \frac{40}{5} = 8, \text{ dus } l_2 \text{ voldoet.}$$

Een vergelijking van l is dus $3x - 4y = -6$.

8 Goniometrische functies

38 De draaiingshoek van A is $30° = \frac{1}{6}\pi$ rad.

De draaiingshoek van B is $\frac{1}{6}\pi + 2$ rad.

De draaiingshoek van C is $\frac{1}{6}\pi + 4$ rad.

$A(\cos(\frac{1}{6}\pi), \sin(\frac{1}{6}\pi)) \approx A(0{,}87; 0{,}5)$

$B(\cos(\frac{1}{6}\pi + 2), \sin(\frac{1}{6}\pi + 2)) \approx B(-0{,}82; 0{,}58)$

$C(\cos(\frac{1}{6}\pi + 4), \sin(\frac{1}{6}\pi + 4)) \approx C(-0{,}19; -0{,}98)$

Bladzijde 181

39 **a** $A(\cos(\frac{2}{3}\pi), \sin(\frac{2}{3}\pi)) = A(-\frac{1}{2}, \frac{1}{2}\sqrt{3})$

b $C(\cos(-\frac{1}{6}\pi), \sin(-\frac{1}{6}\pi)) = C(\frac{1}{2}\sqrt{3}, -\frac{1}{2})$

c $\cos(\beta) = -\frac{1}{2}\sqrt{2}$ geeft $\beta = -\frac{3}{4}\pi$

d De lengte van de langste cirkelboog BC is $1\frac{1}{4}\pi - -\frac{1}{6}\pi = 1\frac{5}{12}\pi$.

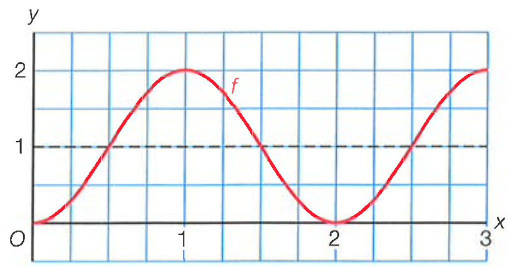
40 **a** $f(x) = 1 - \sin(\pi x + \frac{1}{2}\pi) = 1 - \sin(\pi(x + \frac{1}{2}))$

evenwichtsstand 1

amplitude 1

periode $\dfrac{2\pi}{\pi} = 2$

$-1 < 0$, dus grafiek dalend door het punt $(1\frac{1}{2}, 1)$.



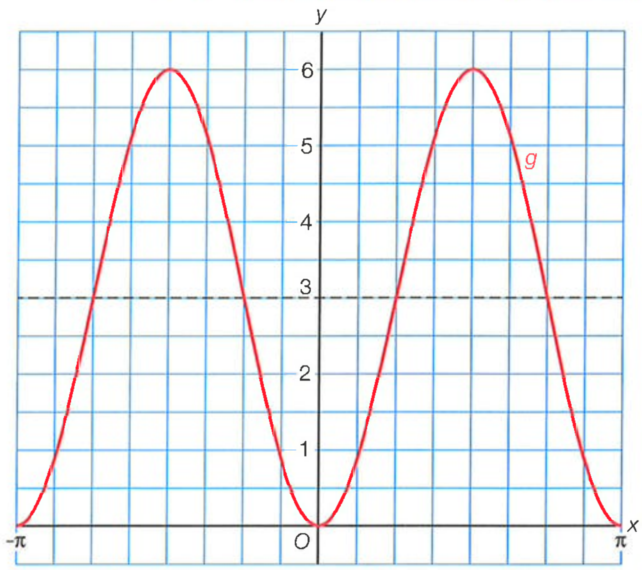
b $g(x) = 3 + 3\cos(2x - \pi) = 3 + 3\cos(2(x - \frac{1}{2}\pi))$

evenwichtsstand 3

amplitude 3

periode $\dfrac{2\pi}{2} = \pi$

$3 > 0$, dus $(\frac{1}{2}\pi, 6)$ is een hoogste punt.



41 **a** $a = \dfrac{20 + -10}{2} = 5$

$b = 20 - 5 = 15$

Stijgend door de evenwichtsstand bij $t = 3$ en bij $t = 8$, dus periode $= 8 - 3 = 5$,

dus $c = \dfrac{2\pi}{5} = \frac{2}{5}\pi$.

Dalend door de evenwichtsstand bij $t = 3 - \frac{1}{2} \cdot$ periode $= 3 - \frac{1}{2} \cdot 5 = \frac{1}{2}$, dus $d = \frac{1}{2}$.

Dus $N = 5 - 15\sin(\frac{2}{5}\pi(t - \frac{1}{2}))$.

b Er is een hoogste punt bij $t = 3 + \frac{1}{4} \cdot$ periode $= 3 + \frac{1}{4} \cdot 5 = 4\frac{1}{4}$.

Dus $N = 5 + 15\cos(\frac{2}{5}\pi(t - 4\frac{1}{4}))$.

42 **a** Stel $y = a + b\sin(c(x - d))$.

$a = \dfrac{5 + -1}{2} = 2$

$b = 5 - 2 = 3$

periode $= 4\pi$, dus $c = \dfrac{2\pi}{4\pi} = \frac{1}{2}$.

Stijgend door de evenwichtsstand bij $x = \frac{1}{2}\pi$, dus $d = \frac{1}{2}\pi$.

Dus $y = 2 + 3\sin(\frac{1}{2}(x - \frac{1}{2}\pi))$.

Bijvoorbeeld:

$y = \sin(x) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\sin(x) \xrightarrow{\text{verm. } y\text{-as, } 2} y = 3\sin(\frac{1}{2}x) \xrightarrow{\text{translatie } (\frac{1}{2}\pi,\ 2)} y = 2 + 3\sin(\frac{1}{2}(x - \frac{1}{2}\pi))$

b Een hoogste punt is $(1\frac{1}{2}\pi, 5)$, dus $y = 2 + 3\cos(\frac{1}{2}(x - 1\frac{1}{2}\pi))$.

Bijvoorbeeld:

$y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\cos(x) \xrightarrow{\text{verm. } y\text{-as, } 2} y = 3\cos(\frac{1}{2}x) \xrightarrow{\text{translatie } (1\frac{1}{2}\pi,\ 2)} y = 2 + 3\cos(\frac{1}{2}(x - 1\frac{1}{2}\pi))$

43 **a** $\sin^2(x + \frac{1}{3}\pi) = 1$

$\sin(x + \frac{1}{3}\pi) = 1 \vee \sin(x + \frac{1}{3}\pi) = -1$

$x + \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot 2\pi \vee x + \frac{1}{3}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$

$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi$

b $\cos(2x - \frac{1}{3}\pi) \cdot \sin(2x - \frac{1}{3}\pi) = 0$

$\cos(2x - \frac{1}{3}\pi) = 0 \vee \sin(2x - \frac{1}{3}\pi) = 0$

$2x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi \vee 2x - \frac{1}{3}\pi = k \cdot \pi$

$2x = \frac{5}{6}\pi + k \cdot \pi \vee 2x = \frac{1}{3}\pi + k \cdot \pi$

$x = \frac{5}{12}\pi + k \cdot \frac{1}{2}\pi \vee x = \frac{1}{6}\pi + k \cdot \frac{1}{2}\pi$

c $\tan(2x - \frac{1}{4}\pi) = \tan(\frac{1}{2}x + \frac{1}{3}\pi)$

$2x - \frac{1}{4}\pi = \frac{1}{2}x + \frac{1}{3}\pi + k \cdot \pi$

$1\frac{1}{2}x = \frac{7}{12}\pi + k \cdot \pi$

$x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$

d $\sin(\frac{1}{2}\pi x + \frac{1}{4}\pi) \cdot (\cos(\pi x) - 1) = 0$

$\sin(\frac{1}{2}\pi x + \frac{1}{4}\pi) = 0 \vee \cos(\pi x) - 1 = 0$

$\frac{1}{2}\pi x + \frac{1}{4}\pi = k \cdot \pi \vee \cos(\pi x) = 1$

$\frac{1}{2}\pi x = -\frac{1}{4}\pi + k \cdot \pi \vee \pi x = k \cdot 2\pi$

$x = -\frac{1}{2} + k \cdot 2 \vee x = k \cdot 2$

e $\tan^2(\pi x) = \tan(\pi x)$

$\tan(\pi x) = 0 \vee \tan(\pi x) = 1$

$\pi x = k \cdot \pi \vee \pi x = \frac{1}{4}\pi + k \cdot \pi$

$x = k \cdot 1 \vee x = \frac{1}{4} + k \cdot 1$

f $\sin(2x + \frac{1}{4}\pi) = \sin(x - \frac{1}{3}\pi)$

$2x + \frac{1}{4}\pi = x - \frac{1}{3}\pi + k \cdot 2\pi \vee 2x + \frac{1}{4}\pi = \pi - (x - \frac{1}{3}\pi) + k \cdot 2\pi$

$x = -\frac{7}{12}\pi + k \cdot 2\pi \vee 2x + \frac{1}{4}\pi = \pi - x + \frac{1}{3}\pi + k \cdot 2\pi$

$x = -\frac{7}{12}\pi + k \cdot 2\pi \vee 3x = 1\frac{1}{12}\pi + k \cdot 2\pi$

$x = -\frac{7}{12}\pi + k \cdot 2\pi \vee x = \frac{13}{36}\pi + k \cdot \frac{2}{3}\pi$

Bladzijde 182

44 **a** $\sin^2(\frac{1}{2}x + \frac{1}{4}\pi) = \frac{1}{2}$

$\sin(\frac{1}{2}x + \frac{1}{4}\pi) = \frac{1}{2}\sqrt{2} \vee \sin(\frac{1}{2}x + \frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$

$\frac{1}{2}x + \frac{1}{4}\pi = \frac{1}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x + \frac{1}{4}\pi = \frac{3}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x + \frac{1}{4}\pi = 1\frac{1}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x + \frac{1}{4}\pi = -\frac{1}{4}\pi + k \cdot 2\pi$

$\frac{1}{2}x = k \cdot 2\pi \vee \frac{1}{2}x = \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{1}{2}\pi + k \cdot 2\pi$

$x = k \cdot 4\pi \vee x = \pi + k \cdot 4\pi \vee x = 2\pi + k \cdot 4\pi \vee x = -\pi + k \cdot 4\pi$

x in $[0, 2\pi]$ geeft $x = 0 \vee x = \pi \vee x = 2\pi$

b $\tan^2(x) = 3$

$\tan(x) = \sqrt{3} \vee \tan(x) = -\sqrt{3}$

$x = \frac{1}{3}\pi + k \cdot \pi \vee x = \frac{2}{3}\pi + k \cdot \pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = \frac{2}{3}\pi \vee x = 1\frac{2}{3}\pi$

c $\cos^2(x) - \frac{1}{2}\cos(x) = 0$

$\cos(x)(\cos(x) - \frac{1}{2}) = 0$

$\cos(x) = 0 \vee \cos(x) = \frac{1}{2}$

$x = \frac{1}{2}\pi + k \cdot \pi \vee x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi \vee x = \frac{1}{3}\pi \vee x = 1\frac{2}{3}\pi$

d $\cos(2x) - \cos(\frac{1}{2}x) = 0$

$\cos(2x) = \cos(\frac{1}{2}x)$

$2x = \frac{1}{2}x + k \cdot 2\pi \vee 2x = -\frac{1}{2}x + k \cdot 2\pi$

$1\frac{1}{2}x = k \cdot 2\pi \vee 2\frac{1}{2}x = k \cdot 2\pi$

$x = k \cdot 1\frac{1}{3}\pi \vee x = k \cdot \frac{4}{5}\pi$

x in $[0, 2\pi]$ geeft $x = 0 \vee x = 1\frac{1}{3}\pi \vee x = \frac{4}{5}\pi \vee x = 1\frac{3}{5}\pi$

45 **a** $\cos(\frac{2}{3}\pi(x-1)) = \frac{1}{2}\sqrt{3}$

$\frac{2}{3}\pi(x-1) = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{2}{3}\pi(x-1) = -\frac{1}{6}\pi + k \cdot 2\pi$

$\frac{2}{3}\pi x - \frac{2}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{2}{3}\pi x - \frac{2}{3}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$

$\frac{2}{3}\pi x = \frac{5}{6}\pi + k \cdot 2\pi \vee \frac{2}{3}\pi x = \frac{1}{2}\pi + k \cdot 2\pi$

$x = 1\frac{1}{4} + k \cdot 3 \vee x = \frac{3}{4} + k \cdot 3$

x in $[0, 5]$ geeft $x = 1\frac{1}{4} \vee x = 4\frac{1}{4} \vee x = \frac{3}{4} \vee x = 3\frac{3}{4}$

b $\sin(\frac{1}{2}\pi x) = \sin(\frac{1}{2}\pi(x-1))$

$\frac{1}{2}\pi x = \frac{1}{2}\pi(x-1) + k \cdot 2\pi \vee \frac{1}{2}\pi x = \pi - \frac{1}{2}\pi(x-1) + k \cdot 2\pi$

$\frac{1}{2}\pi x = \frac{1}{2}\pi x - \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{1}{2}\pi x = \pi - \frac{1}{2}\pi x + \frac{1}{2}\pi + k \cdot 2\pi$

$0 = -\frac{1}{2}\pi + k \cdot 2\pi \vee \pi x = 1\frac{1}{2}\pi + k \cdot 2\pi$

geen opl. $\qquad x = 1\frac{1}{2} + k \cdot 2$

x in $[0, 5]$ geeft $x = 1\frac{1}{2} \vee x = 3\frac{1}{2}$

c $\tan(\frac{1}{3}\pi x) + \sqrt{3} = 0$

$\tan(\frac{1}{3}\pi x) = -\sqrt{3}$

$\frac{1}{3}\pi x = \frac{2}{3}\pi + k \cdot \pi$

$x = 2 + k \cdot 3$

x in $[0, 5]$ geeft $x = 2 \vee x = 5$

d $\sin^2(\frac{1}{3}\pi x) = \frac{1}{2}\sqrt{3} \cdot \sin(\frac{1}{3}\pi x)$

$\sin(\frac{1}{3}\pi x) = 0 \vee \sin(\frac{1}{3}\pi x) = \frac{1}{2}\sqrt{3}$

$\frac{1}{3}\pi x = k \cdot \pi \vee \frac{1}{3}\pi x = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{1}{3}\pi x = \frac{2}{3}\pi + k \cdot 2\pi$

$x = k \cdot 3 \vee x = 1 + k \cdot 6 \vee x = 2 + k \cdot 6$

x in $[0, 5]$ geeft $x = 0 \vee x = 3 \vee x = 1 \vee x = 2$

46 **a** $f(x) = 2$ geeft $2 - 4\sin(2x - \frac{1}{3}\pi) = 2$

$4\sin(2x - \frac{1}{3}\pi) = 0$

$\sin(2x - \frac{1}{3}\pi) = 0$

$2x - \frac{1}{3}\pi = k \cdot \pi$

$2x = \frac{1}{3}\pi + k \cdot \pi$

$x = \frac{1}{6}\pi + k \cdot \frac{1}{2}\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{2}{3}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{2}{3}\pi$

De grafiek van f snijdt de lijn van de evenwichtsstand in de punten $(\frac{1}{6}\pi, 2)$, $(\frac{2}{3}\pi, 2)$, $(1\frac{1}{6}\pi, 2)$ en $(1\frac{2}{3}\pi, 2)$.

b Toppen als $\sin(2x - \frac{1}{3}\pi) = 1 \vee \sin(2x - \frac{1}{3}\pi) = -1$

$2x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$

$2x = \frac{5}{6}\pi + k \cdot 2\pi \vee 2x = 1\frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{5}{12}\pi + k \cdot \pi \vee x = \frac{11}{12}\pi + k \cdot \pi$

x in $[0, 2\pi]$ geeft $x = \frac{5}{12}\pi \vee x = 1\frac{5}{12}\pi \vee x = \frac{11}{12}\pi \vee x = 1\frac{11}{12}\pi$

De toppen van de grafiek zijn $(\frac{5}{12}\pi, -2)$, $(1\frac{5}{12}\pi, -2)$, $(\frac{11}{12}\pi, 6)$ en $(1\frac{11}{12}\pi, 6)$.

c $f(x) = 2 - 4\sin(x)$ geeft $2 - 4\sin(2x - \frac{1}{3}\pi) = 2 - 4\sin(x)$

$4\sin(2x - \frac{1}{3}\pi) = 4\sin(x)$

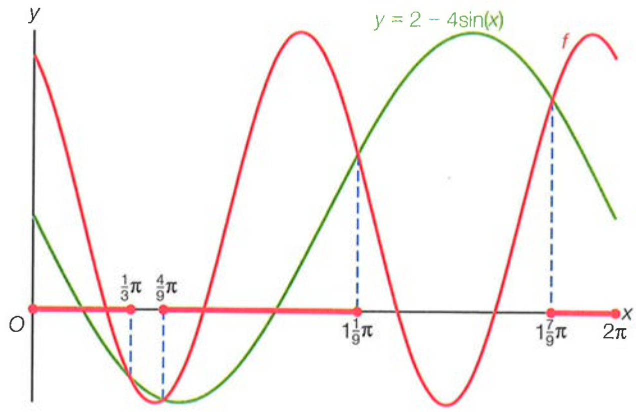
$\sin(2x - \frac{1}{3}\pi) = \sin(x)$

$2x - \frac{1}{3}\pi = x + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \pi - x + k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot 2\pi \vee 3x = 1\frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = \frac{4}{9}\pi + k \cdot \frac{2}{3}\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{3}\pi \vee x = \frac{4}{9}\pi \vee x = 1\frac{1}{9}\pi \vee x = 1\frac{7}{9}\pi$



$f(x) \geq 2 - 4\sin(x)$ geeft $0 \leq x \leq \frac{1}{3}\pi \vee \frac{4}{9}\pi \leq x \leq 1\frac{1}{9}\pi \vee 1\frac{7}{9}\pi \leq x \leq 2\pi$

47 **a** $-\cos(2\pi x - \frac{1}{4}\pi) = \cos(2\pi x - \frac{1}{4}\pi + \pi) = \sin(2\pi x - \frac{1}{4}\pi + \pi + \frac{1}{2}\pi) = \sin(2\pi x + 1\frac{1}{4}\pi)$

b $-\sin(2(x + 1\frac{1}{4}\pi)) = -\sin(2x + 2\frac{1}{2}\pi) = \sin(2x + 2\frac{1}{2}\pi + \pi) = \cos(2x + 2\frac{1}{2}\pi + \pi - \frac{1}{2}\pi) =$
$\cos(2x + 3\pi) = \cos(2x + \pi)$

c $\sin(2x + \frac{2}{3}\pi) - 2\sin(2x - \frac{1}{3}\pi) = \cos(2x + \frac{2}{3}\pi - \frac{1}{2}\pi) - 2\cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi) =$
$\cos(2x + \frac{1}{6}\pi) + 2\cos(2x - \frac{5}{6}\pi + \pi) = \cos(2x + \frac{1}{6}\pi) + 2\cos(2x + \frac{1}{6}\pi) = 3\cos(2x + \frac{1}{6}\pi)$

d $\dfrac{\cos(2x - \frac{1}{3}\pi)}{\sin(2x + \frac{2}{3}\pi)} = \dfrac{\sin(2x - \frac{1}{3}\pi + \frac{1}{2}\pi)}{\cos(2x + \frac{2}{3}\pi - \frac{1}{2}\pi)} = \dfrac{\sin(2x + \frac{1}{6}\pi)}{\cos(2x + \frac{1}{6}\pi)} = \tan(2x + \frac{1}{6}\pi)$

48 **a** $f(x) = x^2 \cdot \sin(3x - \frac{1}{2}\pi)$ geeft $f'(x) = 2x \cdot \sin(3x - \frac{1}{2}\pi) + x^2 \cdot \cos(3x - \frac{1}{2}\pi) \cdot 3$
$= 2x\sin(3x - \frac{1}{2}\pi) + 3x^2\cos(3x - \frac{1}{2}\pi)$

b $f(x) = \dfrac{x}{2 + \cos(x)}$ geeft $f'(x) = \dfrac{(2 + \cos(x)) \cdot 1 - x \cdot -\sin(x)}{(2 + \cos(x))^2} = \dfrac{2 + \cos(x) + x\sin(x)}{(2 + \cos(x))^2}$

c $f(x) = x^2 \cdot \cos^3(x)$ geeft $f'(x) = 2x \cdot \cos^3(x) + x^2 \cdot 3\cos^2(x) \cdot -\sin(x) = 2x\cos^3(x) - 3x^2\sin(x)\cos^2(x)$

d $f(x) = x^2 - \dfrac{1}{\tan^2(x)} = x^2 - \tan^{-2}(x)$ geeft $f'(x) = 2x + 2\tan^{-3}(x) \cdot (1 + \tan^2(x)) = 2x + \dfrac{2 + 2\tan^2(x)}{\tan^3(x)}$

e $f(x) = \sqrt{2\sin(x) + 3\cos(x)}$ geeft $f'(x) = \dfrac{1}{2\sqrt{2\sin(x) + 3\cos(x)}} \cdot (2\cos(x) - 3\sin(x)) = \dfrac{2\cos(x) - 3\sin(x)}{2\sqrt{2\sin(x) + 3\cos(x)}}$

f $f(x) = \dfrac{5}{4\sin(x) + 3\cos(x)}$ geeft $f'(x) = \dfrac{(4\sin(x) + 3\cos(x)) \cdot 0 - 5 \cdot (4\cos(x) - 3\sin(x))}{(4\sin(x) + 3\cos(x))^2} = \dfrac{15\sin(x) - 20\cos(x)}{(4\sin(x) + 3\cos(x))^2}$

49 **a** $f(x) = \dfrac{\cos(x)}{2 + \sin(x)}$ geeft $f'(x) = \dfrac{(2 + \sin(x)) \cdot -\sin(x) - \cos(x) \cdot \cos(x)}{(2 + \sin(x))^2}$
$= \dfrac{-2\sin(x) - \sin^2(x) - \cos^2(x)}{(2 + \sin(x))^2}$
$= \dfrac{-2\sin(x) - (\sin^2(x) + \cos^2(x))}{(2 + \sin(x))^2}$
$= \dfrac{-1 - 2\sin(x)}{(2 + \sin(x))^2}$

b Stel k: $y = ax + b$ met $a = f'(\frac{1}{2}\pi) = \dfrac{-1 - 2 \cdot 1}{(2 + 1)^2} = \dfrac{-3}{9} = -\frac{1}{3}$.

$\left.\begin{array}{l} y = -\frac{1}{3}x + b \\ f(\frac{1}{2}\pi) = 0, \text{ dus } A(\frac{1}{2}\pi, 0) \end{array}\right\} -\frac{1}{3} \cdot \frac{1}{2}\pi + b = 0$
$b = \frac{1}{6}\pi$

Dus k: $y = -\frac{1}{3}x + \frac{1}{6}\pi$.

c $f'(x) = 0$ geeft $\dfrac{-1 - 2\sin(x)}{(2 + \sin(x))^2} = 0$

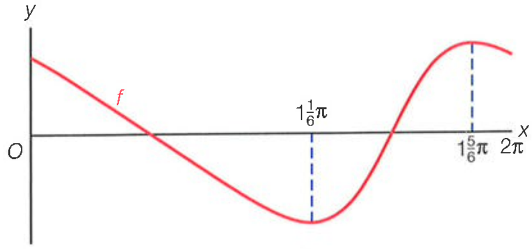
$-1 - 2\sin(x) = 0$

$2\sin(x) = -1$

$\sin(x) = -\frac{1}{2}$

$x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = 1\frac{1}{6}\pi$ (vold.) $\vee$ $x = 1\frac{5}{6}\pi$ (vold.)



min. is $f(1\frac{1}{6}\pi) = \dfrac{-\frac{1}{2}\sqrt{3}}{2 - \frac{1}{2}} = \dfrac{-\frac{1}{2}\sqrt{3}}{1\frac{1}{2}} = -\frac{1}{3}\sqrt{3}$.

max. is $f(1\frac{5}{6}\pi) = \dfrac{\frac{1}{2}\sqrt{3}}{2 - \frac{1}{2}} = \dfrac{\frac{1}{2}\sqrt{3}}{1\frac{1}{2}} = \frac{1}{3}\sqrt{3}$.

Dus $B_f = [-\frac{1}{3}\sqrt{3}, \frac{1}{3}\sqrt{3}]$.