4 Vergelijkingen en herleidingen

Bladzijde 190

37 **a** $\begin{cases} 2x + 3y = 58 \\ 5x - 2y = 12 \end{cases} \begin{vmatrix} 2 \\ 3 \end{vmatrix}$ geeft $\begin{cases} 4x + 6y = 116 \\ 15x - 6y = 36 \end{cases}$ +

$19x = 152$

$\left.\begin{array}{l} x = 8 \\ 5x - 2y = 12 \end{array}\right\}$ $5 \cdot 8 - 2y = 12$

$40 - 2y = 12$

$-2y = -28$

$y = 14$

Dus $(x, y) = (8, 14)$.

b $2x + y = 13$ geeft $y = -2x + 13$

Substitutie van $y = -2x + 13$ in $x^2 + y^2 = 58$ geeft $x^2 + (-2x + 13)^2 = 58$

$x^2 + 4x^2 - 52x + 169 = 58$

$5x^2 - 52x + 111 = 0$

$D = (-52)^2 - 4 \cdot 5 \cdot 111 = 484$

$x = \frac{52 + 22}{10} = 7\frac{2}{5} \vee x = \frac{52 - 22}{10} = 3$

$x = 3$ geeft $y = -2 \cdot 3 + 13 = 7$

$x = 7\frac{2}{5}$ geeft $y = -2 \cdot 7\frac{2}{5} + 13 = -1\frac{4}{5}$

Dus $(x, y) = (3, 7) \vee (x, y) = (7\frac{2}{5}, -1\frac{4}{5})$.

c $\begin{cases} 0{,}4x - 0{,}32y = 2 \\ 0{,}6x - 0{,}28y = 5 \end{cases} \begin{vmatrix} 6 \\ 4 \end{vmatrix}$ geeft $\begin{cases} 2{,}4x - 1{,}92y = 12 \\ 2{,}4x - 1{,}12y = 20 \end{cases}$ −

$-0{,}8y = -8$

$\left.\begin{array}{l} y = 10 \\ 0{,}4x - 0{,}32y = 2 \end{array}\right\}$ $0{,}4x - 0{,}32 \cdot 10 = 2$

$0{,}4x - 3{,}2 = 2$

$0{,}4x = 5{,}2$

$x = 13$

Dus $(x, y) = (13, 10)$.

d $x + 5y = 17$ geeft $x = -5y + 17$

Substitutie van $x = -5y + 17$ in $(2x - 1)^2 + (3y - 1)^2 = 73$ geeft

$(2(-5y + 17) - 1)^2 + (3y - 1)^2 = 73$

$(-10y + 34 - 1)^2 + (3y - 1)^2 = 73$

$(-10y + 33)^2 + (3y - 1)^2 = 73$

$100y^2 - 660y + 1089 + 9y^2 - 6y + 1 = 73$

$109y^2 - 666y + 1017 = 0$

$D = (-666)^2 - 4 \cdot 109 \cdot 1017 = 144$

$y = \frac{666 + 12}{218} = 3\frac{12}{109} \vee y = \frac{666 - 12}{218} = 3$

$y = 3$ geeft $x = -5 \cdot 3 + 17 = 2$

$y = 3\frac{12}{109}$ geeft $x = -5 \cdot 3\frac{12}{109} + 17 = 1\frac{49}{109}$

Dus $(x, y) = (2, 3) \vee (x, y) = (1\frac{49}{109}, 3\frac{12}{109})$.

38 **a** De grafiek gaat door $(0, 0)$.

$f(0) = 0$ geeft $0 + 0 + 0 + d = 0$, dus $d = 0$.

b $f(x) = ax^3 + bx^2 + cx$ geeft $f'(x) = 3ax^2 + 2bx + c$

$f'(0) = 0$ geeft $0 + 0 + c = 0$, dus $c = 0$.

c $f(x) = ax^3 + bx^2$

De grafiek gaat door de punten $(6, 4)$ en $(12, 0)$.

$f(6) = 4$ geeft $216a + 36b = 4$ oftewel $54a + 9b = 1$.

$f(12) = 0$ geeft $1728a + 144b = 0$ oftewel $b = -12a$.

Substitutie van $b = -12a$ in $54a + 9b = 1$ geeft $54a + 9 \cdot -12a = 1$

$54a - 108a = 1$

$-54a = 1$

$a = -\frac{1}{54}$

$a = -\frac{1}{54}$ geeft $b = -12 \cdot -\frac{1}{54} = \frac{2}{9}$

39 Stel a is het aantal rails van 7 cm, b is het aantal rails van 15 cm, en c is het aantal rails van 22 cm.
Dan geldt $7a + 15b = 95$, $7a + 22c = 189$ en $15b + 22c = 214$.

$$\begin{cases} 7a + 15b = 95 \\ 7a + 22c = 189 \end{cases} \; -$$
$$15b - 22c = -94$$

$$\begin{cases} 15b - 22c = -94 \\ 15b + 22c = 214 \end{cases} \; +$$
$$30b = 120$$

$$\left.\begin{array}{l} b = 4 \\ 15b - 22c = -94 \end{array}\right\} \begin{array}{l} 15 \cdot 4 - 22c = -94 \\ 60 - 22c = -94 \\ -22c = -154 \\ c = 7 \end{array}$$

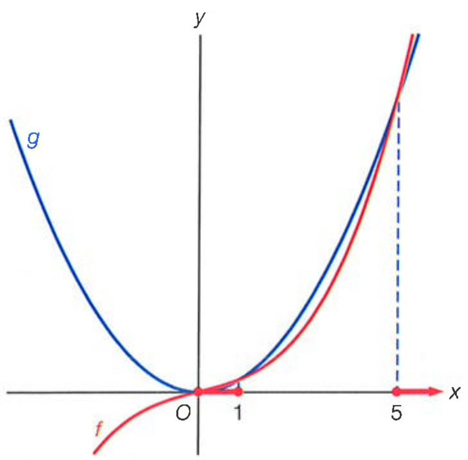
Dus Finn heeft zeven stukken rail van 22 cm.

40 **a** $17 - (2x - 1)^4 = 1$
$(2x - 1)^4 = 16$
$2x - 1 = 2 \vee 2x - 1 = -2$
$2x = 3 \vee 2x = -1$
$x = 1\frac{1}{2} \vee x = -\frac{1}{2}$

b $x^6 - 6x^3 + 5 = 0$
Stel $x^3 = u$.
$u^2 - 6u + 5 = 0$
$(u - 1)(u - 5) = 0$
$u = 1 \vee u = 5$
$x^3 = 1 \vee x^3 = 5$
$x = 1 \vee x = \sqrt[3]{5}$

c $10x^4 = 17x^2 + 657$
$10x^4 - 17x^2 - 657 = 0$
Stel $x^2 = u$.
$10u^2 - 17u - 657 = 0$
$D = (-17)^2 - 4 \cdot 10 \cdot -657 = 26\,569$
$u = \dfrac{17 + 163}{20} = 9 \vee u = \dfrac{17 - 163}{20} = -7\frac{3}{10}$
$x^2 = 9 \vee x^2 = -7\frac{3}{10}$
$x = 3 \vee x = -3$

d Stel $f(x) = x^3 + 5x$ en $g(x) = 6x^2$.
$f(x) = g(x)$ geeft $x^3 + 5x = 6x^2$
$x^3 - 6x^2 + 5x = 0$
$x(x^2 - 6x + 5) = 0$
$x(x - 1)(x - 5) = 0$
$x = 0 \vee x = 1 \vee x = 5$



$x^3 + 5x \geq 6x^2$ geeft $0 \leq x \leq 1 \vee x \geq 5$

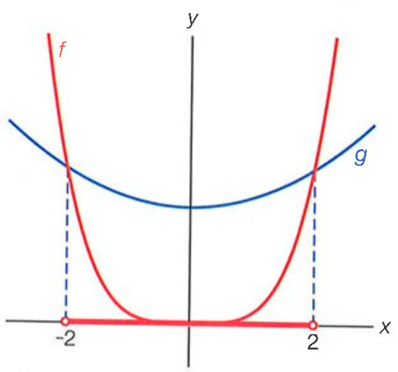
e $(2x-1)^4 - 5(2x-1)^2 + 4 = 0$
Stel $(2x-1)^2 = u$.
$u^2 - 5u + 4 = 0$
$(u-1)(u-4) = 0$
$u = 1 \vee u = 4$
$(2x-1)^2 = 1 \vee (2x-1)^2 = 4$
$2x - 1 = 1 \vee 2x - 1 = -1 \vee 2x - 1 = 2 \vee 2x - 1 = -2$
$2x = 2 \vee 2x = 0 \vee 2x = 3 \vee 2x = -1$
$x = 1 \vee x = 0 \vee x = 1\frac{1}{2} \vee x = -\frac{1}{2}$

f $x^3 - 3x\sqrt{x} - 108 = 0$
Stel $x\sqrt{x} = u$.
$u^2 - 3u - 108 = 0$
$(u+9)(u-12) = 0$
$u = -9 \vee u = 12$
$x\sqrt{x} = -9 \vee x\sqrt{x} = 12$
$x\sqrt{x} = -9$ heeft geen oplossingen.
$x\sqrt{x} = 12$ kwadrateren geeft $x^3 = 144$
$x = \sqrt[3]{144}$
$x = \sqrt[3]{144}$ geeft $\sqrt[3]{144} \cdot \sqrt{\sqrt[3]{144}} = 12$ vold.

Bladzijde 191

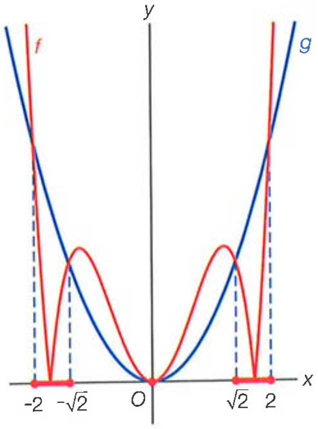
41 **a** $x^5 - 16x^3 + 28x = 0$
$x(x^4 - 16x^2 + 28) = 0$
$x = 0 \vee x^4 - 16x^2 + 28 = 0$
Stel $x^2 = u$.
$x = 0 \vee u^2 - 16u + 28 = 0$
$x = 0 \vee (u-2)(u-14) = 0$
$x = 0 \vee u = 2 \vee u = 14$
$x = 0 \vee x^2 = 2 \vee x^2 = 14$
$x = 0 \vee x = \sqrt{2} \vee x = -\sqrt{2} \vee x = \sqrt{14} \vee x = -\sqrt{14}$

b Stel $f(x) = x^4$ en $g(x) = x^2 + 12$.
$f(x) = g(x)$ geeft $x^4 = x^2 + 12$
$x^4 - x^2 - 12 = 0$
Stel $x^2 = u$.
$u^2 - u - 12 = 0$
$(u+3)(u-4) = 0$
$u = -3 \vee u = 4$
$x^2 = -3 \vee x^2 = 4$
$x = 2 \vee x = -2$



$x^4 < x^2 + 12$ geeft $-2 < x < 2$

c Stel $f(x) = |x^4 - 3x^2|$ en $g(x) = x^2$.
$f(x) = g(x)$ geeft $x^4 - 3x^2 = x^2 \vee x^4 - 3x^2 = -x^2$
$x^4 - 4x^2 = 0 \vee x^4 - 2x^2 = 0$
$x^2(x^2 - 4) = 0 \vee x^2(x^2 - 2) = 0$
$x^2 = 0 \vee x^2 = 4 \vee x^2 = 2$
$x = 0 \vee x = 2 \vee x = -2 \vee x = \sqrt{2} \vee x = -\sqrt{2}$



$|x^4 - 3x^2| \leq x^2$ geeft $-2 \leq x \leq -\sqrt{2} \vee x = 0 \vee \sqrt{2} \leq x \leq 2$

d $6x^5 + 10x^2 \cdot \sqrt{x} - 464 = 0$
$3x^5 + 5x^2 \cdot \sqrt{x} - 232 = 0$
Stel $x^2 \cdot \sqrt{x} = u$.
$3u^2 + 5u - 232 = 0$
$D = 5^2 - 4 \cdot 3 \cdot -232 = 2809$
$u = \dfrac{-5 + 53}{6} = 8 \vee u = \dfrac{-5 - 53}{6} = -9\tfrac{2}{3}$
$x^2 \cdot \sqrt{x} = 8 \vee x^2 \cdot \sqrt{x} = -9\tfrac{2}{3}$
$x^2 \cdot \sqrt{x} = -9\tfrac{2}{3}$ heeft geen oplossingen.
$x^2 \cdot \sqrt{x} = 8$ kwadrateren geeft $x^5 = 64$
$x = \sqrt[5]{64}$
$x = \sqrt[5]{64}$ geeft $(\sqrt[5]{64})^2 \cdot \sqrt{\sqrt[5]{64}} = 8$ vold.

42 **a** inhoud kubus $= 2^3 = 8$
oppervlakte kubus $= 6 \cdot 2^2 = 24$
lengte ribben $= 12 \cdot 2 = 24$
Zo krijg je som $= 8 + 24 + 24 = 56$.

b Stel de ribbe x.
inhoud kubus $= x^3$
oppervlakte kubus $= 6x^2$
lengte ribben $= 12x$
Er geldt $x^3 + 3 \cdot 6x^2 = 144x$
$x^3 + 18x^2 - 144x = 0$
$x(x^2 + 18x - 144) = 0$
$x(x - 6)(x + 24) = 0$
$x = 0 \vee x = 6 \vee x = -24$
Dus de ribbe is 6.

c Stel de ribbe x.
Er geldt $x^3 \cdot (6x^2 + 12x) = (6x^2)^2$
$6x^5 + 12x^4 = 36x^4$
$6x^5 - 24x^4 = 0$
$6x^4(x - 4) = 0$
$x = 0 \vee x = 4$
Dus de ribbe is 4.

43 **a** $(2x^2-1)^2=x^2$
$2x^2-1=x \vee 2x^2-1=-x$
$2x^2-x-1=0 \quad \vee \quad 2x^2+x-1=0$
$D=(-1^2)-4\cdot 2\cdot -1=9 \quad D=1^2-4\cdot 2\cdot -1=9$
$x=\frac{1+3}{4}=1 \vee x=\frac{1-3}{4}=-\frac{1}{2} \vee x=\frac{-1+3}{4}=\frac{1}{2} \vee x=\frac{-1-3}{4}=-1$

b $\sqrt{2-2x}+2x=0$
$\sqrt{2-2x}=-2x$
kwadrateren geeft
$2-2x=4x^2$
$-4x^2-2x+2=0$
$2x^2+x-1=0$
$D=1^2-4\cdot 2\cdot -1=9$
$x=\frac{-1+3}{4}=\frac{1}{2} \vee x=\frac{-1-3}{4}=-1$
$x=\frac{1}{2}$ geeft $\sqrt{2-2\cdot\frac{1}{2}}=-1$ vold. niet
$x=-1$ geeft $\sqrt{2-2\cdot -1}=2$ vold.

c $\frac{2x+4}{x}=\frac{12}{x+1}$
$(2x+4)(x+1)=12x$
$2x^2+2x+4x+4=12x$
$2x^2-6x+4=0$
$x^2-3x+2=0$
$(x-1)(x-2)=0$
$x=1 \vee x=2$
vold. vold.

d $\sqrt{3x-2}+2=x$
$\sqrt{3x-2}=x-2$
kwadrateren geeft
$3x-2=(x-2)^2$
$3x-2=x^2-4x+4$
$-x^2+7x-6=0$
$x^2-7x+6=0$
$(x-1)(x-6)=0$
$x=1 \vee x=6$
$x=1$ geeft $\sqrt{3\cdot 1-2}=-1$ vold. niet
$x=6$ geeft $\sqrt{3\cdot 6-2}=4$ vold.

e $(2x-3)(x^2-3)+3=2x$
$(2x-3)(x^2-3)=2x-3$
$2x-3=0 \vee x^2-3=1$
$2x=3 \vee x^2=4$
$x=1\frac{1}{2} \vee x=2 \vee x=-2$

f $\frac{x^2-9}{2x+3}=\frac{x^2-9}{x+4}$
$x^2-9=0 \vee 2x+3=x+4$
$x^2=9 \vee x=1$
$x=3 \vee x=-3 \vee x=1$
vold. vold. vold.

44 **a** $(x^3-2x+1)(2x+1)=2x+1$
$2x+1=0 \vee x^3-2x+1=1$
$2x=-1 \vee x^3-2x=0$
$x=-\frac{1}{2} \vee x(x^2-2)=0$
$x=-\frac{1}{2} \vee x=0 \vee x^2=2$
$x=-\frac{1}{2} \vee x=0 \vee x=\sqrt{2} \vee x=-\sqrt{2}$

b $\dfrac{x^3}{x+2} = \dfrac{4x}{x+2}$
$x^3 = 4x$
$x^3 - 4x = 0$
$x(x^2 - 4) = 0$
$x = 0 \vee x^2 = 4$
$x = 0 \vee x = 2 \vee x = -2$
vold. vold. vold. niet

c $\dfrac{3(x^3-5)+21}{x^3+1} = 0$
$3(x^3 - 5) + 21 = 0$
$3(x^3 - 5) = -21$
$x^3 - 5 = -7$
$x^3 = -2$
$x = \sqrt[3]{-2}$ vold.

d $\sqrt{x^2+2x} + x = 6$
$\sqrt{x^2+2x} = 6 - x$
kwadrateren geeft
$x^2 + 2x = (6-x)^2$
$x^2 + 2x = 36 - 12x + x^2$
$14x = 36$
$x = 2\frac{4}{7}$
$x = 2\frac{4}{7}$ geeft $\sqrt{(2\frac{4}{7})^2 + 2 \cdot 2\frac{4}{7}} + 2\frac{4}{7} = 6$ vold.

e $x^3 - 26x\sqrt{x} = 27$
$x^3 - 26x\sqrt{x} - 27 = 0$
Stel $x\sqrt{x} = u$.
$u^2 - 26u - 27 = 0$
$(u+1)(u-27) = 0$
$u = -1 \vee u = 27$
$x\sqrt{x} = -1 \vee x\sqrt{x} = 27$
$x\sqrt{x} = -1$ heeft geen oplossingen.
$x\sqrt{x} = 27$ kwadrateren geeft $x^3 = 729$
$x = 9$
$x = 9$ geeft $9\sqrt{9} = 27$ vold.

f $\dfrac{x^2-1}{2x+2} = 5x$
$x^2 - 1 = 5x(2x+2)$
$x^2 - 1 = 10x^2 + 10x$
$9x^2 + 10x + 1 = 0$
$D = 10^2 - 4 \cdot 9 \cdot 1 = 64$
$x = \dfrac{-10+8}{18} = -\frac{1}{9} \vee x = \dfrac{-10-8}{18} = -1$
vold. vold. niet

45 **a** $y = \dfrac{x^4 - x^2}{x-1} = \dfrac{x^2(x^2-1)}{x-1} = \dfrac{x^2(x-1)(x+1)}{x-1} = x^2(x+1)$ mits $x \neq 1$

b $y = \dfrac{15x}{x+2} - 2x = \dfrac{15x}{x+2} - \dfrac{2x(x+2)}{x+2} = \dfrac{15x - 2x^2 - 4x}{x+2} = \dfrac{-2x^2 + 11x}{x+2}$

c $y = \dfrac{10 - \dfrac{2x}{x+3}}{5 + \dfrac{2}{x+3}} = \dfrac{\left(10 - \dfrac{2x}{x+3}\right) \cdot (x+3)}{\left(5 + \dfrac{2}{x+3}\right) \cdot (x+3)} = \dfrac{10(x+3) - 2x}{5(x+3) + 2} = \dfrac{8x + 30}{5x + 17}$ mits $x \neq -3$

d $N = \dfrac{3t^3 + 3t^2 - 6t}{t^2 + 2t} = \dfrac{3t(t^2 + t - 2)}{t(t+2)} = \dfrac{3t(t-1)(t+2)}{t(t+2)} = 3t - 3$ mits $t \neq 0 \wedge t \neq -2$

e $K=\frac{2a}{a+2}\left(\frac{a-1}{2a}-\frac{a+2}{a^2}\right)=\frac{2a}{a+2}\cdot\frac{a(a-1)-2(a+2)}{2a^2}=\frac{2a}{a+2}\cdot\frac{a^2-a-2a-4}{2a^2}$

$=\frac{2a(a^2-3a-4)}{2a^2(a+2)}=\frac{a^2-3a-4}{a(a+2)}$

f $P=\dfrac{\frac{q^2}{q^2+1}-2}{\frac{q}{q^2+1}-2q}=\dfrac{\left(\frac{q^2}{q^2+1}-2\right)\cdot(q^2+1)}{\left(\frac{q}{q^2+1}-2q\right)\cdot(q^2+1)}=\frac{q^2-2(q^2+1)}{q-2q(q^2+1)}=\frac{-q^2-2}{-2q^3-q}=\frac{q^2+2}{2q^3+q}$

Bladzijde 192

46 **a** $T=\frac{(2t-1)(t+2)}{2t^2}=\frac{2t^2+4t-t-2}{2t^2}=\frac{2t^2+3t-2}{2t^2}=1+\frac{3}{2t}-\frac{1}{t^2}$

b $B=12a-6\cdot\dfrac{\frac{a}{a^2+1}-2}{5a}=12a-6\cdot\dfrac{\left(\frac{a}{a^2+1}-2\right)\cdot(a^2+1)}{5a\cdot(a^2+1)}=12a-6\cdot\frac{a-2(a^2+1)}{5a(a^2+1)}$

$=12a-6\cdot\frac{a-2a^2-2}{5a(a^2+1)}=12a+\frac{12a^2-6a+12}{5a(a^2+1)}$

c Substitutie van $y=\frac{4x}{x-1}$ in $K=\frac{3y-2}{2y-1}$ geeft

$K=\dfrac{3\left(\frac{4x}{x-1}\right)-2}{2\left(\frac{4x}{x-1}\right)-1}=\dfrac{\left(3\left(\frac{4x}{x-1}\right)-2\right)\cdot(x-1)}{\left(2\left(\frac{4x}{x-1}\right)-1\right)\cdot(x-1)}=\frac{3\cdot 4x-2(x-1)}{2\cdot 4x-(x-1)}=\frac{10x+2}{7x+1}$ mits $x\neq 1$.

47 **a** $f^{\text{inv}}(x)=3$ geeft $x=f(3)=\frac{3\cdot 3+1}{3-2}=10$

b $g^{\text{inv}}(x)=x^2$ geeft $x=g(x^2)$

$$\begin{aligned}
&x=1+\frac{3}{x^2-3}\\
&x-1=\frac{3}{x^2-3}\\
&(x^2-3)(x-1)=3\\
&x^3-x^2-3x+3=3\\
&x^3-x^2-3x=0\\
&x(x^2-x-3)=0\\
&x=0\vee x^2-x-3=0\\
&\text{vold.}
\end{aligned}$$

Van $x^2-x-3=0$ is $D=(-1^2)-4\cdot 1\cdot -3=13$, dus $x=\frac{1+\sqrt{13}}{2}=\frac{1}{2}+\frac{1}{2}\sqrt{13}\vee x=\frac{1}{2}-\frac{1}{2}\sqrt{13}$
vold. vold.

c Voor h geldt $y=a+\frac{3x-1}{2x+1}$, dus voor h^{inv} geldt $x=a+\frac{3y-1}{2y+1}$.

$x=a+\frac{3y-1}{2y+1}$ geeft $x(2y+1)=a(2y+1)+3y-1$

$$\begin{aligned}
&2xy+x=2ay+a+3y-1\\
&2xy-2ay-3y=-x+a-1\\
&y(2x-2a-3)=-x+a-1\\
&y=\frac{-x+a-1}{2x-2a-3}=\frac{-x+a-1}{2x-(2a+3)}
\end{aligned}$$

Dus $h^{\text{inv}}(x)=\frac{-x+a-1}{2x-(2a+3)}$.

$k(x)=2-\frac{5x-25}{2x-11}=\frac{2(2x-11)}{2x-11}-\frac{5x-25}{2x-11}=\frac{4x-22-(5x-25)}{2x-11}=\frac{-x+3}{2x-11}$

Dus er moet gelden $a-1=3$ en $2a+3=11$.
Hieruit volgt $a=4$.

48 **a** $px^3 + 2px^2 + x^2 + 2\frac{1}{4}x = 0$

$x(px^2 + 2px + x + 2\frac{1}{4}) = 0$

$x = 0 \vee px^2 + 2px + x + 2\frac{1}{4} = 0$

$x = 0 \vee px^2 + (2p + 1)x + 2\frac{1}{4} = 0$

De vergelijking heeft drie oplossingen als $px^2 + (2p + 1)x + 2\frac{1}{4} = 0$ twee oplossingen heeft.

$p = 0$ geeft $x + 2\frac{1}{4} = 0$, dus één oplossing.

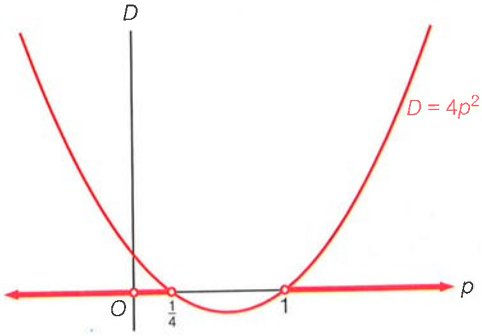
Voor $p \neq 0$ is $D = (2p + 1)^2 - 4 \cdot p \cdot 2\frac{1}{4} = 4p^2 + 4p + 1 - 9p = 4p^2 - 5p + 1$.

Twee oplossingen als $D > 0$, dus als $4p^2 - 5p + 1 > 0$.

$4p^2 - 5p + 1 = 0$

$D = (-5)^2 - 4 \cdot 4 \cdot 1 = 9$

$p = \frac{5+3}{8} = 1 \vee p = \frac{5-3}{8} = \frac{1}{4}$



De vergelijking heeft drie oplossingen voor $p < 0 \vee 0 < p < \frac{1}{4} \vee p > 1$.

b $2px^4 - px^3 + 5x^3 + 2x^2 = 0$

$x^2(2px^2 - px + 5x + 2) = 0$

$x^2 = 0 \vee 2px^2 - px + 5x + 2 = 0$

$x = 0 \vee 2px^2 + (-p + 5)x + 2 = 0$

De vergelijking heeft precies één oplossing als $2px^2 + (-p + 5)x + 2 = 0$ geen oplossingen heeft.

$p = 0$ geeft $5x + 2 = 0$, dus één oplossing.

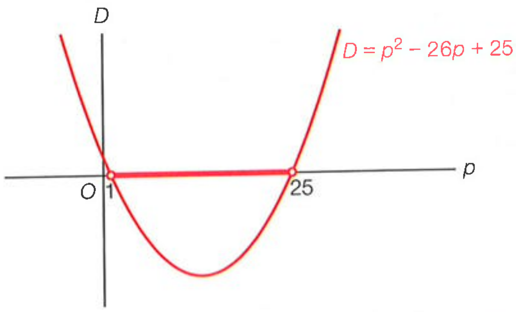
Voor $p \neq 0$ is $D = (-p + 5)^2 - 4 \cdot 2p \cdot 2 = p^2 - 10p + 25 - 16p = p^2 - 26p + 25$.

Geen oplossingen als $D < 0$, dus als $p^2 - 26p + 25 < 0$.

$p^2 - 26p + 25 = 0$

$(p - 1)(p - 25) = 0$

$p = 1 \vee p = 25$



De vergelijking heeft precies één oplossing voor $1 < p < 25$.

49 $f(x) = \frac{10x}{x^2+1}$ geeft $f'(x) = \frac{(x^2+1) \cdot 10 - 10x \cdot 2x}{(x^2+1)^2} = \frac{10x^2 + 10 - 20x^2}{(x^2+1)^2} = \frac{-10x^2+10}{(x^2+1)^2}$

$\text{rc}_{\text{raaklijn}} = -\frac{4}{5}$ geeft $\frac{-10x^2+10}{(x^2+1)^2} = \frac{-4}{5}$

$-4(x^2+1)^2 = 5(-10x^2+10)$

$-4(x^4+2x^2+1) = -50x^2+50$

$-4x^4 - 8x^2 - 4 = -50x^2 + 50$

$-4x^4 + 42x^2 - 54 = 0$

$2x^4 - 21x^2 + 27 = 0$

Stel $x^2 = u$.

$2u^2 - 21u + 27 = 0$

$D = (-21)^2 - 4 \cdot 2 \cdot 27 = 225$

$u = \frac{21+15}{4} = 9 \vee u = \frac{21-15}{4} = 1\frac{1}{2}$

$x^2 = 9 \vee x^2 = 1\frac{1}{2}$

$x = 3 \vee x = -3 \vee x = \sqrt{1\frac{1}{2}} \vee x = -\sqrt{1\frac{1}{2}}$

vold. vold. vold. vold.