

5 Machten, exponenten en logaritmen

Voorkennis Herleiden van machten

Bladzijde 10

1 a $x^2 \cdot x^3 = x^5$
 b $2p^3 \cdot 3p^2 = 6p^5$
 c $4a^2b \cdot 5a^3b^2 = 20a^5b^3$
 d $-2p^4q^3 \cdot -3pq = 6p^5q^4$
 e $5x^2y \cdot 2x - 3x^3y = 10x^3y - 3x^3y = 7x^3y$
 f $12a^4b \cdot \frac{1}{4}ab - 8ab = 3a^5b^2 - 8ab$

2 a $(p^2q)^3 = p^6q^3$
 b $(3x^2)^3 = 27x^6$
 c $(-5x^2y^3)^2 = 25x^4y^6$
 d $(-4ab^4)^2 = 16a^2b^8$
 e $(3a)^2 \cdot (2a^2)^3 = 9a^2 \cdot 8a^6 = 72a^8$
 f $(3a^3)^2 + (2a^2)^3 = 9a^6 + 8a^6 = 17a^6$

3 a $\frac{12x^6}{4x^2} = 3x^4$
 b $\frac{5x^{10}}{15x^5} = \frac{x^5}{3} = \frac{1}{3}x^5$
 c $\frac{24a^4b^2}{6ab} = 4a^3b$
 d $\frac{-15p^6q}{5p^2q} = -3p^4$
 e $\frac{10x^3y^2}{5x^2y} = 2xy$
 f $\frac{(2ab)^3}{(3ab)^2} = \frac{8a^3b^3}{9a^2b^2} = \frac{8}{9}ab$

4 a $\frac{-28a^6}{7a} = -4a^5$
 b $-(3a^4)^2 = -9a^8$
 c $(-2a^2)^5 = -32a^{10}$
 d $(-a^3)^3 = -a^9$
 e $(5a)^3 \cdot -3a = 125a^3 \cdot -3a = -375a^4$
 f $\left(\frac{9a^4}{a}\right)^2 = (9a^3)^2 = 81a^6$

5 a $(ab)^4 \cdot a = a^4b^4 \cdot a = a^5b^4$
 b $(-2ab)^3 \cdot b = -8a^3b^3 \cdot b = -8a^3b^4$
 c $(3a)^2 + (2b)^2 = 9a^2 + 4b^2$
 d $(3a)^3 - 8a^3 = 27a^3 - 8a^3 = 19a^3$
 e $\left(\frac{1}{2}a\right)^2 + (-a)^2 = \frac{1}{4}a^2 + a^2 = 1\frac{1}{4}a^2$
 f $(5a^4)^2 + (-a^2)^4 = 25a^8 + a^8 = 26a^8$

5.1 Machten met negatieve en gebroken exponenten

Bladzijde 11

- 1 a** De exponenten worden telkens 1 minder en de getallen worden steeds door 2 gedeeld.

b $2^{-3} = \frac{1}{2^3}$

$2^{-4} = \frac{1}{2^4}$

c $x^0 = 1$

$x^{-1} = \frac{1}{x}$

$x^{-3} = \frac{1}{x^3}$

Bladzijde 12

2 a $\frac{1}{a^2} = a^{-2}$

b $a^4 \cdot \frac{1}{a^6} = a^4 \cdot a^{-6} = a^{-2}$

c $\frac{a^n}{\left(\frac{1}{a^4}\right)} = \frac{a^n}{a^{-4}} = a^{n+4}$

d $\frac{a^8}{a^0} = a^{8-0} = a^8$

e $(a^3)^{-2} = a^{-6}$

f $\frac{\left(\frac{1}{a^5}\right)}{a} = \frac{a^{-5}}{a^1} = a^{-5-1} = a^{-6}$

g $\frac{a}{a^{12}} = \frac{a^1}{a^{12}} = a^{1-12} = a^{-11}$

h $\frac{1}{a^8} \cdot (a^3)^n = a^{-8} \cdot a^{3n} = a^{-8+3n}$

i $\frac{\left(\frac{1}{a^n}\right)}{a^{-3}} = \frac{a^{-n}}{a^{-3}} = a^{-n-(-3)} = a^{-n+3}$

Bladzijde 13

3 a $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

b $\left(\frac{1}{3}\right)^{-2} = (3^{-1})^{-2} = 3^2 = 9$

c $3 \cdot 5^{-2} = 3 \cdot \frac{1}{5^2} = 3 \cdot \frac{1}{25} = \frac{3}{25}$

4 a $6a^{-5}b^3 = \frac{6b^3}{a^5}$

b $\frac{1}{3}a^{-3} = \frac{1}{3a^3}$

c $5a^{-4}b^2 = \frac{5b^2}{a^4}$

d $\frac{3}{5}a^{-4} = \frac{3}{5a^4}$

e $\left(\frac{1}{2}a\right)^{-3} = (2^{-1} \cdot a)^{-3} = 2^3 \cdot a^{-3} = 8 \cdot \frac{1}{a^3} = \frac{8}{a^3}$

d $\left(\frac{2}{5}\right)^{-2} = \frac{1}{\left(\frac{2}{5}\right)^2} = \frac{1}{\frac{4}{25}} = \frac{25}{4} = 6\frac{1}{4}$

e $4 \cdot 10^{-3} = 4 \cdot \frac{1}{10^3} = \frac{4}{1000} = \frac{1}{250}$

f $\frac{1}{2} : 6^{-2} = \frac{1}{2} \cdot 6^2 = \frac{1}{2} \cdot 36 = 18$

f $\frac{1}{6}a^{-2}b^4 = \frac{b^4}{6a^2}$

g $-4 \cdot (3a)^{-2} = -4 \cdot \frac{1}{(3a)^2} = -4 \cdot \frac{1}{9a^2} = -\frac{4}{9a^2}$

h $(3a)^{-2}b^{-3} = \frac{1}{(3a)^2} \cdot \frac{1}{b^3} = \frac{1}{9a^2b^3}$

i $\frac{3}{8}a^{-1}b = \frac{3b}{8a}$

- 5** Uit $(\sqrt[3]{x})^3 = x$ en $(x^{\frac{1}{3}})^3 = x$ volgt $x^{\frac{1}{3}} = \sqrt[3]{x}$.

Bladzijde 14

6 a $a \cdot \sqrt[3]{a} = a^1 \cdot a^{\frac{1}{3}} = a^{1\frac{1}{3}}$

b $\frac{1}{\sqrt{a}} = \frac{1}{a^{\frac{1}{2}}} = a^{-\frac{1}{2}}$

c $\frac{1}{a} = a^{-1}$

d $\frac{1}{a^3} = a^{-3}$

e $a^2 \cdot \sqrt{a} = a^2 \cdot a^{\frac{1}{2}} = a^{2\frac{1}{2}}$

f $\sqrt[3]{\frac{1}{a^2}} = \sqrt[3]{a^{-2}} = a^{-\frac{2}{3}}$

g $\sqrt[3]{a^{12}} = a^{\frac{12}{3}} = a^4$

h $\frac{1}{a^4} \cdot \sqrt[3]{a} = a^{-4} \cdot a^{\frac{1}{3}} = a^{-4+\frac{1}{3}} = a^{-3\frac{2}{3}}$

i $\frac{a^3}{\sqrt[3]{a}} = \frac{a^3}{a^{\frac{1}{3}}} = a^{3-\frac{1}{3}} = a^{2\frac{2}{3}}$

7 a $8\sqrt{2} = 2^3 \cdot 2^{\frac{1}{2}} = 2^{3\frac{1}{2}}$
 b $\frac{1}{3}\sqrt{3} = 3^{-1} \cdot 3^{\frac{1}{2}} = 3^{-1+\frac{1}{2}} = 3^{-\frac{1}{2}}$
 c $\frac{8}{\sqrt{2}} = \frac{2^3}{2^{\frac{1}{2}}} = 2^{3-\frac{1}{2}} = 2^{2\frac{1}{2}}$
 d $\frac{1}{2} \cdot \sqrt[3]{2} = 2^{-1} \cdot 2^{\frac{1}{3}} = 2^{-1+\frac{1}{3}} = 2^{-\frac{2}{3}}$
 e $\frac{4\sqrt{2}}{\sqrt[3]{2}} = \frac{2^2 \cdot 2^{\frac{1}{2}}}{2^{\frac{1}{3}}} = 2^{2+\frac{1}{2}-\frac{1}{3}} = 2^{2\frac{1}{6}}$
 f $\sqrt[4]{\frac{1}{9}} = \sqrt[4]{3^{-2}} = 3^{-\frac{2}{4}} = 3^{-\frac{1}{2}}$
 g $\frac{1}{100}\sqrt{10} = 10^{-2} \cdot 10^{\frac{1}{2}} = 10^{-2+\frac{1}{2}} = 10^{-1\frac{1}{2}}$
 h $\frac{1}{8} \cdot \sqrt[3]{\frac{1}{4}} = \frac{1}{2^3} \cdot \sqrt[3]{2^{-2}} = 2^{-3} \cdot 2^{-\frac{2}{3}} = 2^{-3-\frac{2}{3}} = 2^{-3\frac{2}{3}}$
 i $10 \cdot \sqrt[3]{0,1} = 10^1 \cdot \sqrt[3]{10^{-1}} = 10^1 \cdot 10^{-\frac{1}{3}} = 10^{1-\frac{1}{3}} = 10^{\frac{2}{3}}$

8 a $5a^{3\frac{1}{3}} = 5a^3 \cdot a^{\frac{1}{3}} = 5a^3 \cdot \sqrt[3]{a}$
 b $\frac{1}{2}a^{-\frac{1}{4}}b = \frac{b}{2a^{\frac{1}{4}}} = \frac{b}{2 \cdot \sqrt[4]{a}}$
 c $3a^{-\frac{2}{3}} = \frac{3}{a^{\frac{2}{3}}} = \frac{3}{\sqrt[3]{a^2}}$
 d $\frac{2}{3}a^{-3}b^{\frac{1}{2}} = \frac{2b \cdot b^{\frac{1}{2}}}{3a^3} = \frac{2b\sqrt{b}}{3a^3}$
 e $\frac{1}{5}a^{-\frac{1}{2}}b^{\frac{1}{3}} = \frac{b^{\frac{1}{3}}}{5a^{\frac{1}{2}}} = \frac{\sqrt[3]{b}}{5\sqrt{a}}$
 f $(5a)^{-\frac{1}{2}} = \frac{1}{(5a)^{\frac{1}{2}}} = \frac{1}{\sqrt{5a}}$

9 a $\frac{x^6}{x^2 \cdot \sqrt{x}} = \frac{x^6}{x^2 \cdot x^{\frac{1}{2}}} = \frac{x^6}{x^{2\frac{1}{2}}} = x^{6-2\frac{1}{2}} = x^{3\frac{1}{2}}$
 b $x \cdot \sqrt[7]{x^3} = x^1 \cdot x^{\frac{3}{7}} = x^{1\frac{3}{7}}$
 c $\frac{x}{\sqrt[5]{x}} = \frac{x^1}{x^{\frac{1}{5}}} = x^{1-\frac{1}{5}} = x^{\frac{4}{5}}$
 d $x^4 \cdot \sqrt{x} = x^4 \cdot x^{\frac{1}{2}} = x^{4\frac{1}{2}}$
 e $\frac{\sqrt[3]{x}}{\sqrt{x}} = \frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = x^{\frac{1}{3}-\frac{1}{2}} = x^{-\frac{1}{6}}$
 f $\frac{\left(\frac{1}{x^2}\right)}{\sqrt{x}} = \frac{x^{-2}}{x^{\frac{1}{2}}} = x^{-2-\frac{1}{2}} = x^{-2\frac{1}{2}}$
 g $x^2 \cdot \frac{1}{x^3} = x^2 \cdot x^{-3} = x^{2-3} = x^{-1}$
 h $\sqrt[6]{x} \cdot \sqrt[3]{x} = x^{\frac{1}{6}} \cdot x^{\frac{1}{3}} = x^{\frac{1}{6}+\frac{1}{3}} = x^{\frac{1}{2}}$
 i $\frac{x^4 \cdot \sqrt[5]{x}}{x^5 \cdot \sqrt[4]{x}} = \frac{x^4 \cdot x^{\frac{1}{5}}}{x^5 \cdot x^{\frac{1}{4}}} = \frac{x^{4\frac{1}{5}}}{x^{5\frac{1}{4}}} = x^{4\frac{1}{5}-5\frac{1}{4}} = x^{-1\frac{1}{20}}$

10 $\sqrt[3]{x^{14}} = x^{\frac{14}{3}} = x^{4\frac{2}{3}} = x^4 \cdot x^{\frac{2}{3}} = x^4 \cdot \sqrt[3]{x^2}$

Bladzijde 15

11 De getallen als macht van 2 geschreven zijn: $2^0, 2^1, 2^{-1}, 2^2, 2^{-3}, 2^5, 2^{-8}, 2^{13}, 2^{-21}, 2^{34}, \dots$
 Het tiende getal is dus 2^{34} .

12 $y = 2(3x^2)^3 \cdot \frac{5}{6x^{5,6}} = 2 \cdot 3^3 \cdot (x^2)^3 \cdot \frac{5}{6} \cdot \frac{1}{x^{5,6}} = 45 \cdot x^6 \cdot x^{-5,6} = 45x^{0,4}$

Bij het herleiden van $(3x^2)^3$ tot $3^3 \cdot (x^2)^3$ is de regel $(ab)^p = a^p b^p$ gebruikt.

Bij het herleiden van $(x^2)^3$ tot x^6 is de regel $(a^p)^q = a^{pq}$ gebruikt.

Bij het herleiden van $\frac{1}{x^{5,6}}$ tot $x^{-5,6}$ is de regel $a^{-p} = \frac{1}{a^p}$ gebruikt.

Bij het herleiden van $x^6 \cdot x^{-5,6}$ tot $x^{0,4}$ is de regel $a^p \cdot a^q = a^{p+q}$ gebruikt.

13 a $y = 5(2x^3)^2 \cdot \frac{3}{x^4} = 5 \cdot 4x^6 \cdot 3 \cdot x^{-4} = 60x^2$

Dus $y = 60x^2$.

b $y = \frac{1}{3}(3x^2)^3 \cdot \frac{2}{x^{10}} = \frac{1}{3} \cdot 27x^6 \cdot 2 \cdot x^{-10} = 18x^{-4}$

Dus $y = 18x^{-4}$.

$$\text{c } y = \frac{3}{x} \cdot \sqrt[4]{x^3} = 3 \cdot x^{-1} \cdot x^{\frac{3}{4}} = 3x^{-\frac{1}{4}}$$

$$\text{Dus } y = 3x^{-\frac{1}{4}}.$$

$$\text{d } y = 3(3x)^{-2} \cdot 4x^{1,2} = 3 \cdot 3^{-2} \cdot x^{-2} \cdot 4x^{1,2} = 3^{-1} \cdot 4 \cdot x^{-0,8} = \frac{1}{3} \cdot 4 \cdot x^{-0,8} = \frac{4}{3}x^{-0,8}$$

$$\text{Dus } y = \frac{4}{3}x^{-0,8}.$$

$$\text{14 a } y = \frac{5}{x\sqrt{x}} = \frac{5}{x^1 \cdot x^{\frac{1}{2}}} = \frac{5}{x^{1\frac{1}{2}}} = 5x^{-1\frac{1}{2}}$$

$$\text{Dus } y = 5x^{-1\frac{1}{2}}.$$

$$\text{b } y = 5x\sqrt{x^3} = 5x^1 \cdot x^{\frac{3}{2}} = 5x^{1+\frac{1}{2}} = 5x^{\frac{3}{2}}$$

$$\text{Dus } y = 5x^{\frac{3}{2}}.$$

$$\text{c } y = \frac{5}{x^3} \cdot 2\sqrt{x} = 5 \cdot x^{-3} \cdot 2 \cdot x^{\frac{1}{2}} = 10x^{-3+\frac{1}{2}} = 10x^{-\frac{5}{2}}$$

$$\text{Dus } y = 10x^{-\frac{5}{2}}.$$

$$\text{d } y = 3 \cdot \sqrt[4]{x^3} = 3x^{\frac{3}{4}}$$

$$\text{Dus } y = 3x^{\frac{3}{4}}.$$

$$\text{e } y = 5x^{-0,2} \cdot x^{1,3} = 5x^{-0,2+1,3} = 5x^{1,1}$$

$$\text{Dus } y = 5x^{1,1}.$$

$$\text{f } y = \frac{50x^{1,9}}{10x^{1,1}} = 5 \cdot x^{1,9-1,1} = 5x^{0,8}$$

$$\text{Dus } y = 5x^{0,8}.$$

Bladzijde 16

$$\text{15 a } h_0 = 0,6 \text{ en } d_0 = 1000 \text{ geeft } h = 0,6 \cdot \left(\frac{1000}{d}\right)^{0,25} = 0,6 \cdot \frac{1000^{0,25}}{d^{0,25}} = 3,374...d^{-0,25} \approx 3,37d^{-0,25}$$

$$\text{b } d = 300 \text{ geeft } h = 3,37 \cdot 300^{-0,25} \approx 0,8$$

Dus de golfhoogte is ongeveer 0,8 meter.

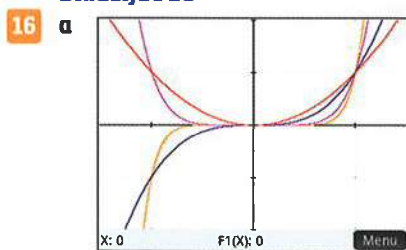
$$\text{c } \text{Voer in } y_1 = 3,37x^{-0,25} \text{ en } y_2 = 2.$$

De optie snijpunt geeft $x = 8,06...$

Dus bij waterdieptes van minder dan 8 meter.

5.2 Grafieken van machtsfuncties veranderen

Bladzijde 18

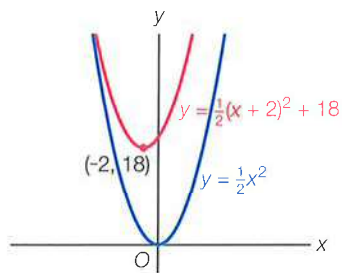


$$\text{b } (0, 0) \text{ en } (1, 1)$$

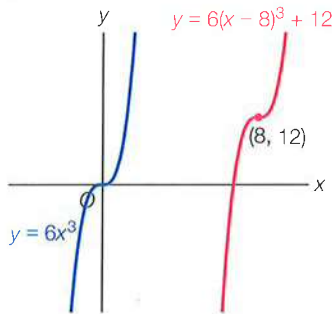
c Van de grafieken van f en h ligt geen enkel punt onder de x -as.

Bladzijde 20

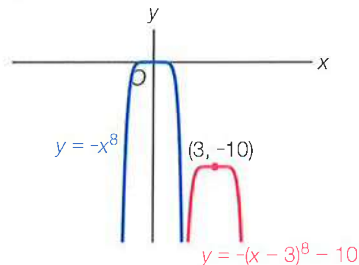
17 a $y = \frac{1}{2}x^2$
 ↓ translatie $(-2, 18)$
 $y = \frac{1}{2}(x+2)^2 + 18$



b $y = 6x^3$
 ↓ translatie $(8, 12)$
 $y = 6(x-8)^3 + 12$



c $y = -x^8$
 ↓ translatie $(3, -10)$
 $y = -(x-3)^8 - 10$



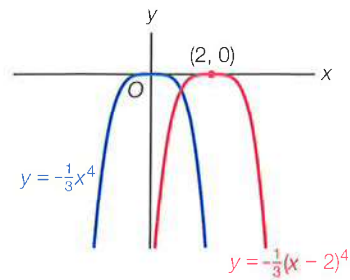
18 a $y = 3x^7 - 5$
 ↓ translatie $(2, 9)$
 $y = 3(x-2)^7 - 5 + 9$
 Dus beeldgrafiek: $y = 3(x-2)^7 + 4$.

b $y = 5(x-2)^4 - 2$
 ↓ translatie $(-5, 4)$
 $y = 5(x+5-2)^4 - 2 + 4$
 Dus beeldgrafiek: $y = 5(x+3)^4 + 2$.

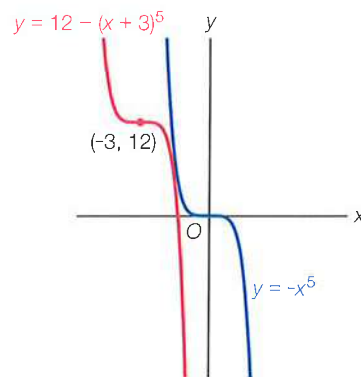
19 a $f(x) = -\frac{1}{2}x^4$
 ↓ translatie $(-2, -7)$
 $g(x) = -\frac{1}{2}(x+2)^4 - 7$
 Dus de translatie $(-2, -7)$.

b $h(x) = 5(x+3)^9 + 9$
 ↓ translatie $(7, -6)$
 $k(x) = 5(x-7+3)^9 + 9 - 6$
 oftewel $k(x) = 5(x-4)^9 + 3$
 Dus de translatie $(7, -6)$.

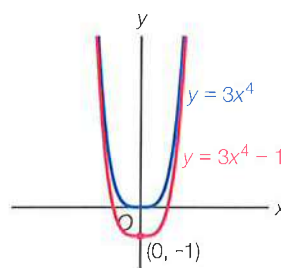
d $y = -\frac{1}{3}x^4$
 ↓ translatie $(2, 0)$
 $y = -\frac{1}{3}(x-2)^4$



e $y = -x^5$
 ↓ translatie $(-3, 12)$
 $y = 12 - (x+3)^5$



f $y = 3(x^4 - 2) + 5 = 3x^4 - 6 + 5 = 3x^4 - 1$
 $y = 3x^4$
 ↓ translatie $(0, -1)$
 $y = 3x^4 - 1$

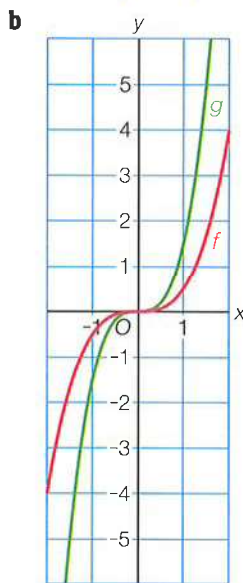


20 $y = -\frac{1}{4}(x+2)^3 + 5$
 $\downarrow$ translatie $(p, 0)$
 $y = -\frac{1}{4}(x-p+2)^3 + 5$
 $y = -\frac{1}{4}(x-p+2)^3 + 5$
 door $(-10, 21)$ $\left. \begin{array}{l} -\frac{1}{4}(-10-p+2)^3 + 5 = 21 \\ -\frac{1}{4}(-8-p)^3 = 16 \\ (-8-p)^3 = -64 \\ -8-p = -4 \\ -p = 4 \\ p = -4 \end{array} \right\}$

Dus beeldgrafiek: $y = -\frac{1}{4}(x+6)^3 + 5$.

21 a

x	-2	-1	0	1	2
$f(x)$	-4	$-\frac{1}{2}$	0	$\frac{1}{2}$	4
$g(x)$	-12	$-1\frac{1}{2}$	0	$1\frac{1}{2}$	12



- c** De functiewaarden bij de functie g zijn 3 keer zo groot als die bij de functie f .
 De y -coördinaten van punten op de grafiek van g zijn dus 3 keer zo groot als die van punten op de grafiek van f bij dezelfde x .

Bladzijde 22

22 a $y = -\frac{1}{2}x^3$
 $\downarrow$ translatie $(-3, -5)$
 $y = -\frac{1}{2}(x+3)^3 - 5$
 $\downarrow$ verm. x -as, -3
 $y = -3(-\frac{1}{2}(x+3)^3 - 5)$
 oftewel $y = 1\frac{1}{2}(x+3)^3 + 15$

- b** Het punt van symmetrie is $(-3, 15)$.

23 a $y = 0,3x^4$
 $\downarrow$ translatie $(-5, 6)$
 $y = 0,3(x+5)^4 + 6$
 $\downarrow$ verm. x -as, -4
 $y = -4(0,3(x+5)^4 + 6)$
 oftewel $y = -1,2(x+5)^4 - 24$
 De top is $(-5, -24)$.

b $y = 0,3x^4$
 $\downarrow$ verm. x -as, -4
 $y = -1,2x^4$
 $\downarrow$ translatie $(-5, 6)$
 $y = -1,2(x+5)^4 + 6$
 De top is $(-5, 6)$.

24 a $f(x) = \frac{1}{2}(x-3)^5 + 7$
 $\downarrow$ translatie $(1, 2)$
 $y = \frac{1}{2}(x-1-3)^5 + 7 + 2$
 oftewel $y = \frac{1}{2}(x-4)^5 + 9$
 $\downarrow$ verm. x -as, -1
 $y = -\frac{1}{2}(x-4)^5 - 9$
 Het punt van symmetrie is $(4, -9)$.

b $g(x) = -2\frac{1}{2}(x+4)^6 - 1$
 $\downarrow$ verm. x -as, -4
 $y = 10(x+4)^6 + 4$
 $\downarrow$ translatie $(-5, -4)$
 $h(x) = 10(x+5+4)^6 + 4 - 4$
 oftewel $h(x) = 10(x+9)^6$
 min. is $h(-9) = 0$

25 a $f(x) = -8(x+3)^7 + 4$
 $\downarrow$ verm. x -as, $-\frac{1}{2}$
 $y = 4(x+3)^7 - 2$
 $\downarrow$ translatie $(5, 13)$
 $g(x) = 4(x-2)^7 + 11$
 Dus verm. x -as, $-\frac{1}{2}$ en translatie $(5, 13)$.

b $f(x) = -8(x+3)^7 + 4$
 $\downarrow$ translatie $(5, -26)$
 $y = -8(x-2)^7 - 22$
 $\downarrow$ verm. x -as, $-\frac{1}{2}$
 $g(x) = 4(x-2)^7 + 11$
 Dus translatie $(5, -26)$ en verm. x -as, $-\frac{1}{2}$.

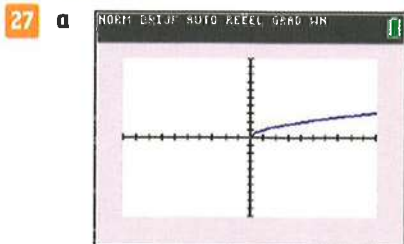
26 $f(x) = -\frac{1}{2}(x+2)^4 + 6$
 $\downarrow$ translatie (p, q)
 $y = -\frac{1}{2}(x-p+2)^4 + 6 + q$
 $\downarrow$ verm. x -as, a
 $g(x) = -\frac{1}{2}a(x-p+2)^4 + 6a + aq$
 min. is $g(5) = -10$ geeft $-p+2 = -5$ oftewel $p = 7$ en $6a + aq = -10$.

$$\begin{aligned} g(x) = -\frac{1}{2}a(x-5)^4 - 10 \Bigg\} & \begin{aligned} & -\frac{1}{2}a(1-5)^4 - 10 = 310 \\ & -\frac{1}{2}a \cdot (-4)^4 = 320 \\ & -128a = 320 \\ & a = -2\frac{1}{2} \end{aligned} \\ \text{door } A(1, 310) & \Bigg\} \begin{aligned} & 6 \cdot -2\frac{1}{2} - 2\frac{1}{2}q = -10 \\ & -15 - 2\frac{1}{2}q = -10 \\ & -2\frac{1}{2}q = 5 \\ & q = -2 \end{aligned} \end{aligned}$$

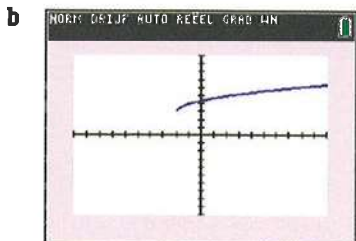
Dus $a = -2\frac{1}{2}$, $p = 7$ en $q = -2$.

5.3 Wortelfuncties

Bladzijde 24



$x \geq 0$ en $y \geq 0$



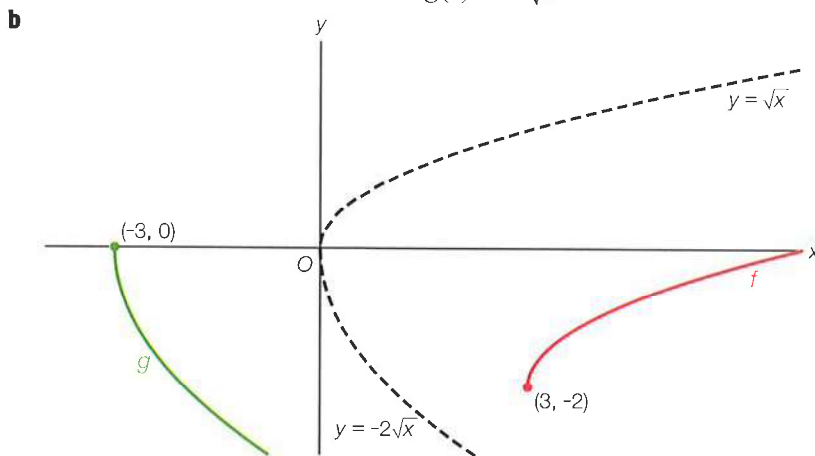
$x \geq -2$ en $y \geq 3$

c $y = \sqrt{x}$
 $\downarrow$ translatie $(-2, 3)$
 $y = \sqrt{x+2} + 3$

Bladzijde 25

28 a $y = \sqrt{x}$
 $\downarrow$ translatie $(3, -2)$
 $f(x) = \sqrt{x-3} - 2$

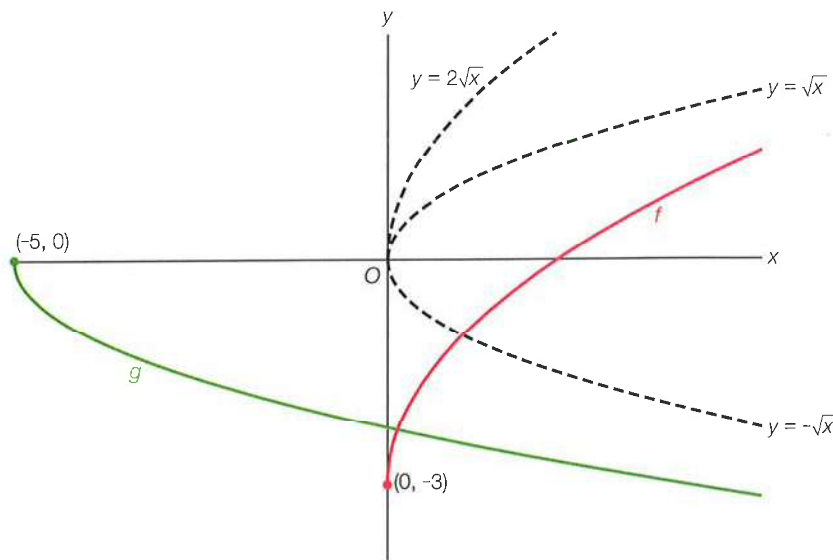
$y = \sqrt{x}$
 $\downarrow$ verm. x-as, -2
 $y = -2\sqrt{x}$
 $\downarrow$ translatie $(-3, 0)$
 $g(x) = -2\sqrt{x+3}$



c $D_f = [3, \rightarrow)$, $B_f = [-2, \rightarrow)$, $D_g = [-3, \rightarrow)$ en $B_g = \langle \leftarrow, 0 \rangle$.

29 a $y = \sqrt{x}$
 $\downarrow$ verm. x-as, 2
 $y = 2\sqrt{x}$
 $\downarrow$ translatie $(0, -3)$
 $f(x) = 2\sqrt{x} - 3$

$y = \sqrt{x}$
 $\downarrow$ verm. x-as, -1
 $y = -\sqrt{x}$
 $\downarrow$ translatie $(-5, 0)$
 $g(x) = -\sqrt{x+5}$

b

c $D_f = [0, \rightarrow)$, $B_f = [-3, \rightarrow)$, $D_g = [-5, \rightarrow)$ en $B_g = \langle \leftarrow, 0]$.

Bladzijde 26

- 30** **a** Randpunt $(-5, 3)$, $D_f = [-5, \rightarrow)$ en $B_f = [3, \rightarrow)$.
b Randpunt $(-3, -7)$, $D_g = [-3, \rightarrow)$ en $B_g = [-7, \rightarrow)$.
c Randpunt $(-1, 0)$, $D_h = [-1, \rightarrow)$ en $B_h = \langle \leftarrow, 0]$.
d Randpunt $(0, 1)$, $D_k = [0, \rightarrow)$ en $B_k = [1, \rightarrow)$.
e Randpunt $(1, -1)$, $D_l = [1, \rightarrow)$ en $B_l = \langle \leftarrow, -1]$.
f Randpunt $(0, -3)$, $D_m = [0, \rightarrow)$ en $B_m = [-3, \rightarrow)$.

- 31** **a** Voor het domein van $f(x) = 5 - \sqrt{2x - 6}$ moet gelden $2x - 6 \geq 0$
 $2x \geq 6$
 $x \geq 3$

Dus het domein is $D_f = [3, \rightarrow)$.

- b** De uitkomst van een wortel is minstens 0, dus de uitkomst van $f(x)$ is hoogstens 5.
Dus het bereik is $B_f = \langle \leftarrow, 5]$.

Bladzijde 28

- 32** Bij een wortelfunctie is niet elke x toegestaan, dus bij voorbeeld b moet je rekening houden met het domein.
Bij een wortelfunctie is niet elke uitkomst mogelijk, dus bij voorbeeld c moet je rekening houden met het bereik.

- 33** **a** $8 - 4x \geq 0$
 $-4x \geq -8$
 $x \leq 2$ dus $D_f = \langle \leftarrow, 2]$,
randpunt $(2, 3)$ en $B_f = [3, \rightarrow)$.
b $4x - 8 \geq 0$
 $4x \geq 8$
 $x \geq 2$ dus $D_g = [2, \rightarrow)$,
randpunt $(2, 3)$ en $B_g = [3, \rightarrow)$.
c $2x + 6 \geq 0$
 $2x \geq -6$
 $x \geq -3$ dus $D_h = [-3, \rightarrow)$,
randpunt $(-3, 5)$ en $B_h = \langle \leftarrow, 5]$.
d $x \geq 0$ dus $D_k = [0, \rightarrow)$,
randpunt $(0, 3)$ en $B_k = \langle \leftarrow, 3]$.

34 a $2x + 5 \geq 0$

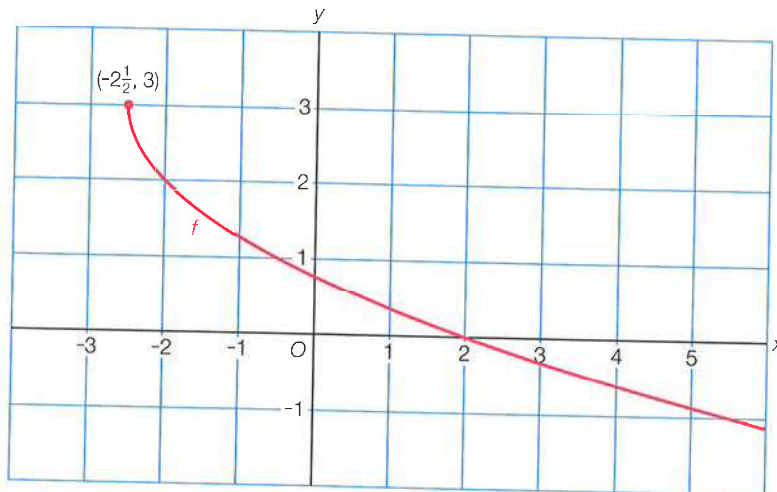
$$2x \geq -5$$

$$x \geq -2\frac{1}{2}$$

$D_f = [-2\frac{1}{2}, \rightarrow)$ en het randpunt is $(-2\frac{1}{2}, 3)$.

Voer in $y_1 = 3 - \sqrt{2x + 5}$.

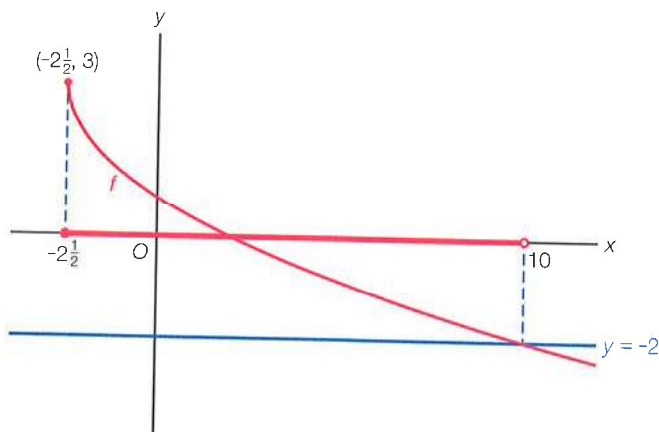
x	-2,5	-2	-1	0	1	2	3
$f(x)$	3	2	1,3	0,8	0,4	0	-0,3



$$B_f = \langle \leftarrow, 3 \rangle$$

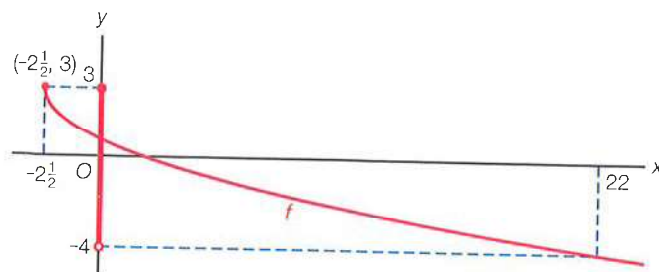
b Voer in $y_2 = -2$.

De optie snijpunt geeft $x = 10$.



$$f(x) > -2 \text{ geeft } -2\frac{1}{2} \leq x < 10$$

c $f(22) = -4$



$$x < 22 \text{ geeft } -4 < f(x) \leq 3$$

35 a $7 - 3x \geq 0$

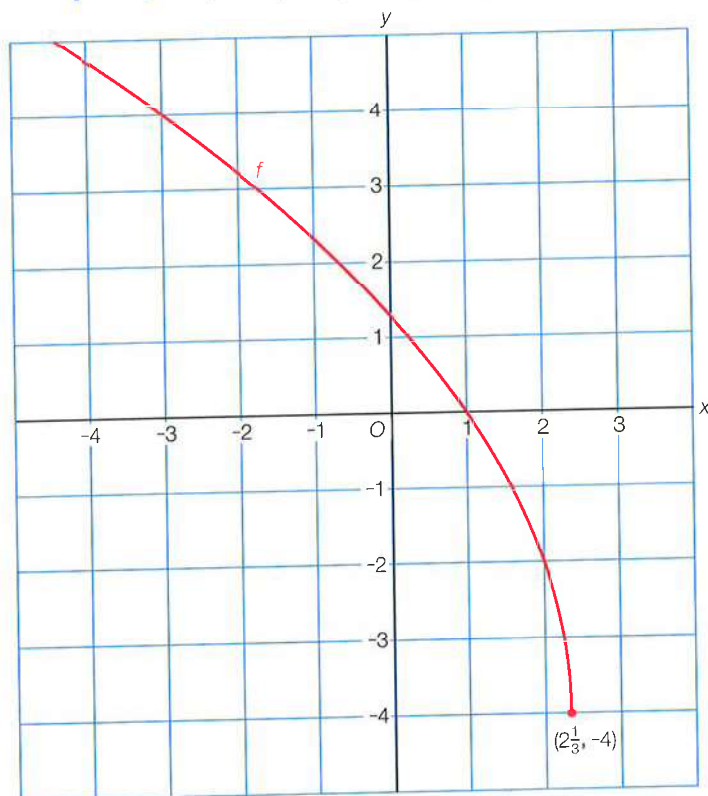
$$-3x \geq -7$$

$$x \leq 2\frac{1}{3}$$

$D_f = \langle \leftarrow, 2\frac{1}{3} \rangle$ en het randpunt is $(2\frac{1}{3}, -4)$.

Voer in $y_1 = 2\sqrt{7-3x} - 4$.

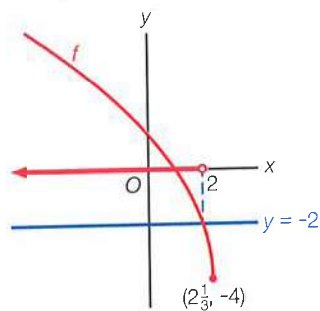
x	-3	-2	-1	0	1	2	$2\frac{1}{3}$
$f(x)$	4	3,2	2,3	1,3	0	-2	-4



$$B_f = [-4, \rightarrow)$$

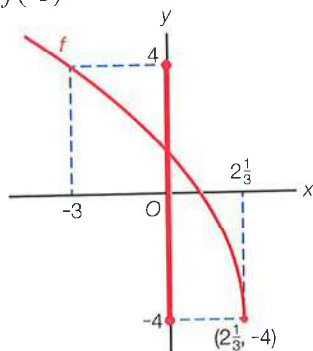
b Voer in $y_2 = -2$.

De optie snijpunt geeft $x = 2$.



$$f(x) > -2 \text{ geeft } x < 2$$

c $f(-3) = 4$



$$x \geq -3 \text{ geeft } -4 \leq f(x) \leq 4.$$

36 $5 + ax = 0$

$$ax = -5$$

$$x = -\frac{5}{a}$$

$$\left. \begin{array}{l} \text{randpunt} \left(-\frac{5}{a}, 4 \right) \\ y = 2x - 1 \end{array} \right\} \begin{array}{l} 2 \cdot -\frac{5}{a} - 1 = 4 \\ -\frac{10}{a} = 5 \\ a = -2 \end{array}$$

37 a Het linker- en rechterlid van $\sqrt{2x-5} = 3$ kwadrateren geeft

$$2x - 5 = 9$$

$$2x = 14$$

$$x = 7$$

b De uitkomst van een wortel kan niet negatief zijn, dus $\sqrt{2x-5} = -3$ heeft geen oplossingen.

Bladzijde 30

38 Bart vergeet het dubbelproduct.

Kwadrateren van linker- en rechterlid van $2x - \sqrt{x} = 10$ geeft $4x^2 - 4x\sqrt{x} + x = 100$.

39 a $x = \sqrt{5x+14}$
kwadrateren geeft

$$x^2 = 5x + 14$$

$$x^2 - 5x - 14 = 0$$

$$(x+2)(x-7) = 0$$

$$x = -2 \vee x = 7$$

$$x = -2 \text{ geeft } -2 = \sqrt{4} \text{ vold. niet}$$

$$x = 7 \text{ geeft } 7 = \sqrt{49} \text{ vold.}$$

b $3x = \sqrt{8x+20}$
kwadrateren geeft

$$9x^2 = 8x + 20$$

$$9x^2 - 8x - 20 = 0$$

$$D = (-8)^2 - 4 \cdot 9 \cdot -20 = 784$$

$$x = \frac{8+28}{18} = 2 \vee x = \frac{8-28}{18} = -1\frac{1}{9}$$

$$x = 2 \text{ geeft } 3 \cdot 2 = \sqrt{36} \text{ vold.}$$

$$x = -1\frac{1}{9} \text{ geeft } 3 \cdot -1\frac{1}{9} = \sqrt{11\frac{1}{9}} \text{ vold. niet}$$

c $5\sqrt{x} = x$
kwadrateren geeft

$$25x = x^2$$

$$x^2 - 25x = 0$$

$$x(x-25) = 0$$

$$x = 0 \vee x = 25$$

$$x = 0 \text{ geeft } 5 \cdot 0 = 0 \text{ vold.}$$

$$x = 25 \text{ geeft } 5 \cdot 5 = 25 \text{ vold.}$$

d $3x = \sqrt{18x+72}$
kwadrateren geeft

$$9x^2 = 18x + 72$$

$$9x^2 - 18x - 72 = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x+2)(x-4) = 0$$

$$x = -2 \vee x = 4$$

$$x = -2 \text{ geeft } 3 \cdot -2 = \sqrt{36} \text{ vold. niet}$$

$$x = 4 \text{ geeft } 3 \cdot 4 = \sqrt{144} \text{ vold.}$$

40 a $4 - 3\sqrt{x} = 2$

$$3\sqrt{x} = 2$$

$$\sqrt{x} = \frac{2}{3}$$

$$x = \frac{4}{9} \text{ vold.}$$

b $5\sqrt{x} - 2x = 0$

$$5\sqrt{x} = 2x$$

$$\text{kwadrateren geeft}$$

$$25x = 4x^2$$

$$4x^2 - 25x = 0$$

$$x(4x-25) = 0$$

$$x = 0 \vee 4x = 25$$

$$x = 0 \vee x = 6\frac{1}{4}$$

$$x = 0 \text{ geeft } 5\sqrt{0} - 0 = 0 \text{ vold.}$$

$$x = 6\frac{1}{4} \text{ geeft } 5\sqrt{6\frac{1}{4}} - 2 \cdot 6\frac{1}{4} = 0 \text{ vold.}$$

c $2x - 5\sqrt{x} = 3$

$$2x - 3 = 5\sqrt{x}$$

$$\text{kwadrateren geeft}$$

$$(2x-3)^2 = 25x$$

$$4x^2 - 12x + 9 = 25x$$

$$4x^2 - 37x + 9 = 0$$

$$D = (-37)^2 - 4 \cdot 4 \cdot 9 = 1225$$

$$x = \frac{37+35}{8} = 9 \vee x = \frac{37-35}{8} = \frac{1}{4}$$

$$x = 9 \text{ geeft } 2 \cdot 9 - 5\sqrt{9} = 3 \text{ vold.}$$

$$x = \frac{1}{4} \text{ geeft } 2 \cdot \frac{1}{4} - 5\sqrt{\frac{1}{4}} = 3 \text{ vold. niet}$$

d $5 - 2\sqrt{x} = 3$

$$2\sqrt{x} = 2$$

$$\sqrt{x} = 1$$

$$x = 1 \text{ vold.}$$

41 a $2x + \sqrt{x} = 10$
 $\sqrt{x} = 10 - 2x$
 kwadrateren geeft
 $x = (10 - 2x)^2$
 $x = 100 - 40x + 4x^2$
 $4x^2 - 41x + 100 = 0$
 $D = (-41)^2 - 4 \cdot 4 \cdot 100 = 81$
 $x = \frac{41 \pm 9}{8} = 6\frac{1}{4} \vee x = \frac{41 - 9}{8} = 4$
 $x = 6\frac{1}{4}$ geeft $2 \cdot 6\frac{1}{4} + \sqrt{6\frac{1}{4}} = 10$ vold. niet
 $x = 4$ geeft $2 \cdot 4 + \sqrt{4} = 10$ vold.

b $2\sqrt{3x+1} = x+3$
 kwadrateren geeft
 $4(3x+1) = (x+3)^2$
 $12x+4 = x^2+6x+9$
 $x^2-6x+5 = 0$
 $(x-1)(x-5) = 0$
 $x = 1 \vee x = 5$
 $x = 1$ geeft $2\sqrt{4} = 1+3$ vold.
 $x = 5$ geeft $2\sqrt{16} = 5+3$ vold.

42 a $\frac{\sqrt{x}}{3-\sqrt{x}} = 2$
 $\sqrt{x} = 2(3-\sqrt{x})$
 $\sqrt{x} = 6-2\sqrt{x}$
 $3\sqrt{x} = 6$
 $\sqrt{x} = 2$
 $x = 4$
 $x = 4$ geeft $\frac{\sqrt{4}}{3-\sqrt{4}} = 2$ vold.

b $\frac{2}{\sqrt{5-2x}} = \frac{1}{2-2x}$
 kruislings vermenigvuldigen geeft
 $4-4x = \sqrt{5-2x}$
 kwadrateren geeft
 $(4-4x)^2 = 5-2x$
 $16-32x+16x^2 = 5-2x$
 $16x^2-30x+11 = 0$
 $D = (-30)^2 - 4 \cdot 16 \cdot 11 = 196$
 $x = \frac{30 \pm 14}{32} = 1\frac{3}{8} \vee x = \frac{30-14}{32} = \frac{1}{2}$
 $x = 1\frac{3}{8}$ geeft $\frac{2}{\sqrt{5-2\frac{3}{4}}} = \frac{1}{2-2\frac{3}{4}}$ vold. niet
 $x = \frac{1}{2}$ geeft $\frac{2}{\sqrt{5-1}} = \frac{1}{2-1}$ vold.

c $x + 2\sqrt{6-5x} = 6$
 $2\sqrt{6-5x} = 6-x$
 kwadrateren geeft
 $4(6-5x) = (6-x)^2$
 $24-20x = 36-12x+x^2$
 $x^2+8x+12 = 0$
 $(x+2)(x+6) = 0$
 $x = -2 \vee x = -6$
 $x = -2$ geeft $-2 + 2\sqrt{6+10} = 6$ vold.
 $x = -6$ geeft $-6 + 2\sqrt{6+30} = 6$ vold.

d $10 - x\sqrt{x} = 2$
 $x\sqrt{x} = 8$
 kwadrateren geeft
 $x^3 = 64$
 $x = 4$
 $x = 4$ geeft $10 - 4\sqrt{4} = 2$ vold.

c $\sqrt{x} + \frac{1}{x} = 3\sqrt{x} - \frac{15}{x}$
 $\frac{16}{x} = 2\sqrt{x}$
 $2x\sqrt{x} = 16$
 $x\sqrt{x} = 8$
 kwadrateren geeft
 $x^3 = 64$
 $x = 4$
 $x = 4$ geeft $\sqrt{4} + \frac{1}{4} = 3\sqrt{4} - \frac{15}{4}$ vold.

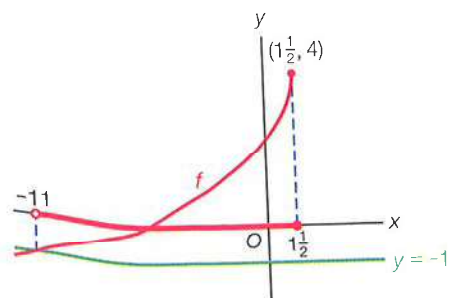
d $\frac{\sqrt{4x-3}}{x-1} = \frac{\sqrt{4x-3}}{\sqrt{3-x}}$
 $\sqrt{4x-3} = 0 \vee x-1 = \sqrt{3-x}$
 kwadrateren geeft
 $4x-3 = 0 \vee (x-1)^2 = 3-x$
 $4x = 3 \vee x^2-2x+1 = 3-x$
 $x = \frac{3}{4} \vee x^2-x-2 = 0$
 $x = \frac{3}{4} \vee (x+1)(x-2) = 0$
 $x = \frac{3}{4} \vee x = -1 \vee x = 2$
 $x = \frac{3}{4}$ geeft $\frac{0}{\frac{3}{4}-1} = \frac{0}{\sqrt{3-\frac{3}{4}}}$ vold.
 $x = -1$ geeft $\frac{\sqrt{-4-3}}{-1-1} = \frac{\sqrt{-4-3}}{\sqrt{3-(-1)}}$ vold. niet
 $x = 2$ geeft $\frac{\sqrt{8-3}}{2-1} = \frac{\sqrt{8-3}}{\sqrt{3-2}}$ vold.

43 a $3 - 2x \geq 0$

$$-2x \geq -3$$

$$x \leq 1\frac{1}{2}$$

$D_f = \langle \leftarrow, 1\frac{1}{2} \rangle$ en het randpunt is $(1\frac{1}{2}, 4)$.



b $f(x) = -1$ geeft $4 - \sqrt{3 - 2x} = -1$
 $-\sqrt{3 - 2x} = -5$
 kwadrateren geeft
 $3 - 2x = 25$
 $-2x = 22$
 $x = -11$

$x = -11$ geeft $4 - \sqrt{25} = -1$ vold.

$f(x) > -1$ geeft $-11 < x \leq 1\frac{1}{2}$

44 $7 - 2x \geq 0$

$$-2x \geq -7$$

$$x \leq 3\frac{1}{2}$$

Los op $2 + \sqrt{7 - 2x} = x$

$$\sqrt{7 - 2x} = x - 2$$

kwadrateren geeft

$$7 - 2x = (x - 2)^2$$

$$7 - 2x = x^2 - 4x + 4$$

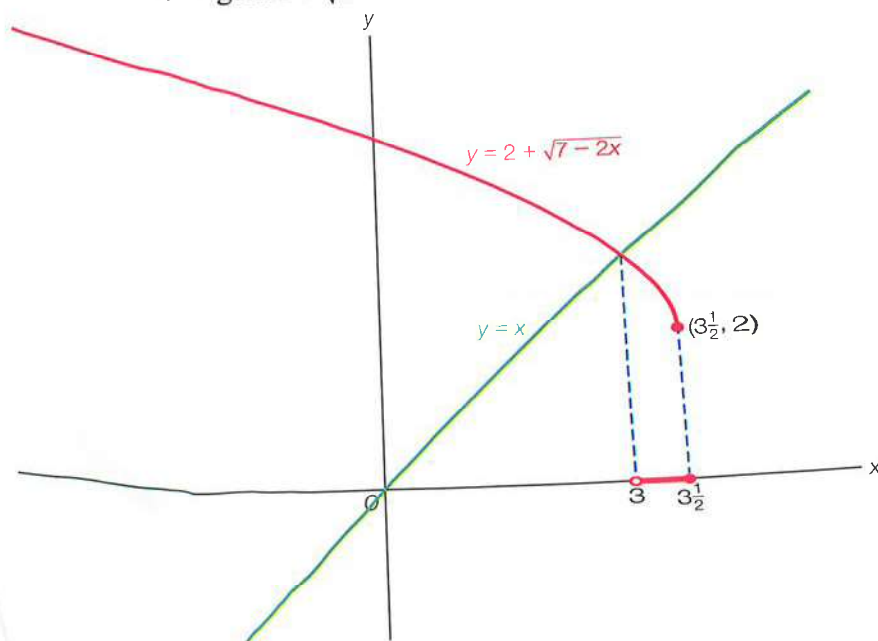
$$x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3) = 0$$

$$x = -1 \vee x = 3$$

$x = -1$ geeft $2 + \sqrt{9} = -1$ vold. niet

$x = 3$ geeft $2 + \sqrt{1} = 3$ vold.



$2 + \sqrt{7 - 2x} < x$ geeft $3 < x \leq 3\frac{1}{2}$

45 $\sqrt{x+1} + \sqrt{x-2} = \sqrt{x+6}$
kwadrateren geeft

$$x+1+2\sqrt{x+1} \cdot \sqrt{x-2} + x-2 = x+6$$

$$2\sqrt{x+1} \cdot \sqrt{x-2} = -x+7$$

kwadrateren geeft

$$4(x+1)(x-2) = (-x+7)^2$$

$$4(x^2 - x - 2) = x^2 - 14x + 49$$

$$4x^2 - 4x - 8 = x^2 - 14x + 49$$

$$3x^2 + 10x - 57 = 0$$

$$D = 10^2 - 4 \cdot 3 \cdot -57 = 784$$

$$x = \frac{-10 \pm 28}{6} = 3 \vee x = \frac{-10 - 28}{6} = -6\frac{1}{3}$$

$$x = 3 \text{ geeft } \sqrt{3+1} + \sqrt{3-2} = \sqrt{3+6} \text{ vold.}$$

$$x = -6\frac{1}{3} \text{ geeft } \sqrt{-6\frac{1}{3}+1} + \sqrt{-6\frac{1}{3}-2} = \sqrt{-6\frac{1}{3}+6} \text{ vold. niet}$$

46 a Je krijgt de maximale score als je alle wedstrijden wint.

$$\text{Dus de maximale score is } S = \frac{1000(16 - \sqrt{16})}{16} = 750.$$

b $\frac{1000(w - \sqrt{w})}{16} = 375$

$$1000(w - \sqrt{w}) = 6000$$

$$w - \sqrt{w} = 6$$

$$w - 6 = \sqrt{w}$$

kwadrateren geeft

$$(w - 6)^2 = w$$

$$w^2 - 12w + 36 = w$$

$$w^2 - 13w + 36 = 0$$

$$(w - 4)(w - 9) = 0$$

$$w = 4 \vee w = 9$$

$$w = 4 \text{ geeft } \frac{1000(4 - \sqrt{4})}{16} = 375 \text{ vold. niet}$$

$$w = 9 \text{ geeft } \frac{1000(9 - \sqrt{9})}{16} = 375 \text{ vold.}$$

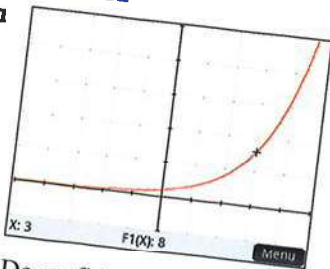
Dus Eline heeft 9 partijen gewonnen.

5.4 Exponentiële functies

Bladzijde 32

47

a



b De grafiek van f is stijgend.

c $f(-10) = 2^{-10} \approx 9,77 \cdot 10^{-4}$

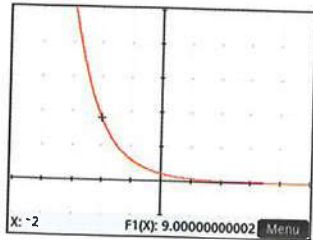
$$f(-20) = 2^{-20} \approx 9,54 \cdot 10^{-7}$$

$$f(-100) = 2^{-100} \approx 7,89 \cdot 10^{-31}$$

d $f(-500) = 2^{-500} = \frac{1}{2^{500}}$ en dit is groter dan 0.

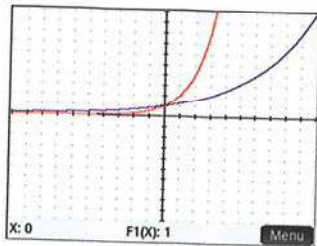
De GR geeft $2^{-500} = 0$.

e Voor elke x is $2^x > 0$, dus er is geen origineel te vinden waarvan het beeld 0 is.

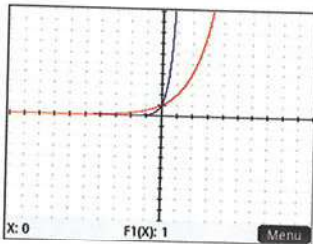
48 a

De grafiek van g is dalend.

- b** De lijn $y = 4$ snijdt de grafiek van g in één punt, dus de vergelijking $g(x) = 4$ heeft één oplossing.
De lijn $y = -4$ snijdt de grafiek van g niet, dus de vergelijking $g(x) = -4$ heeft geen oplossingen.

Bladzijde 33**49 a**

- b** De grafiek van y_2 ontstaat uit de grafiek van y_1 door de vermenigvuldiging ten opzichte van de y -as met factor 3.
c De grafiek van y_3 ontstaat uit de grafiek van y_1 door de vermenigvuldiging ten opzichte van de y -as met factor $\frac{1}{4}$.



- d** De grafiek van y_4 ontstaat uit de grafiek van y_1 door de vermenigvuldiging ten opzichte van de y -as met factor $\frac{1}{a}$.

Bladzijde 34**50 a**

- a** $y = 3^x$
↓ translatie $(-2, -1)$
 $y = 3^{x+2} - 1$
De asymptoot is de lijn $y = -1$.
b $y = 3^x$
↓ translatie $(1, 5)$
 $y = 3^{x-1} + 5$
De asymptoot is de lijn $y = 5$.
c $y = 0,5^x$
↓ verm. x -as, 2
 $y = 2 \cdot 0,5^x$
↓ translatie $(0, 3)$
 $y = 2 \cdot 0,5^x + 3$
De asymptoot is de lijn $y = 3$.

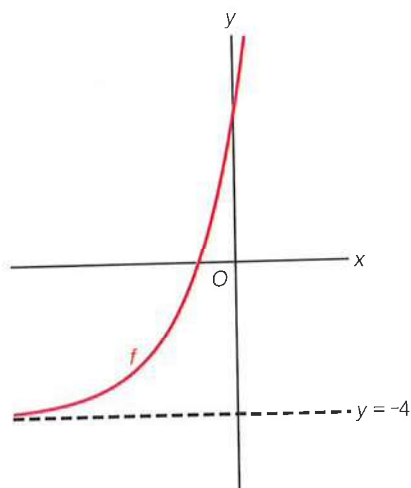
- d** $y = 0,7^x$
↓ verm. x -as, -3
 $y = -3 \cdot 0,7^x$
↓ translatie $(0, 5)$
 $y = -3 \cdot 0,7^x + 5$
De asymptoot is de lijn $y = 5$.
e $y = 2^x$
↓ verm. y -as, $\frac{1}{3}$
 $y = 2^{3x}$
↓ verm. x -as, 3
 $y = 3 \cdot 2^{3x}$
↓ translatie $(0, 4)$
 $y = 3 \cdot 2^{3x} + 4$
De asymptoot is de lijn $y = 4$.
f $y = 0,8^x$
↓ verm. y -as, 2,5
 $y = 0,8^{0,4x}$
↓ translatie $(0, -10)$
 $y = 0,8^{0,4x} - 10$
De asymptoot is de lijn $y = -10$.

Bladzijde 35

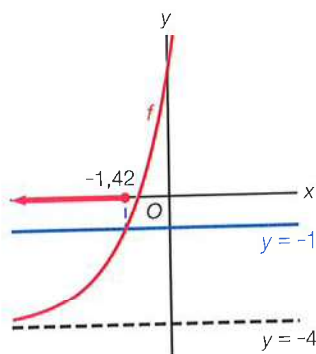
- 51** a Het bereik is $B_f = \langle -6, \rightarrow \rangle$ en de asymptoot is de lijn $y = -6$.
 b Het bereik is $B_g = \langle \leftarrow, 5 \rangle$ en de asymptoot is de lijn $y = 5$.
 c Het bereik is $B_h = \langle \leftarrow, 1000 \rangle$ en de asymptoot is de lijn $y = 1000$.
 d Het bereik is $B_k = \langle \leftarrow, 1000 \rangle$ en de asymptoot is de lijn $y = 1000$.

- 52** a $y = 3^x$
 ↓ spiegelen in de x -as
 $y = -3^x$
 ↓ translatie $(0, -1)$
 $y = -3^x - 1$
 b $y = 3^x$
 ↓ translatie $(2, 5)$
 $y = 3^{x-2} + 5$
 ↓ verm. y -as, $\frac{1}{4}$
 $y = 3^{4x-2} + 5$
 c $y = 3^x$
 ↓ translatie $(4, -5)$
 $y = 3^{x-4} - 5$
 ↓ verm. x -as, 3
 $y = 3(3^{x-4} - 5)$
 oftewel $y = 3 \cdot 3^{x-4} - 15$
 oftewel $y = 3^{x-3} - 15$
 d $y = 3^x$
 ↓ verm. x -as, 3
 $y = 3 \cdot 3^x$
 ↓ translatie $(4, -5)$
 $y = 3 \cdot 3^{x-4} - 5$
 oftewel $y = 3^{x-3} - 5$

- 53** a $y = 2^x$
 ↓ translatie $(-3, -4)$
 $f(x) = 2^{x+3} - 4$
 b $B_f = \langle -4, \rightarrow \rangle$
 De horizontale asymptoot is de lijn $y = -4$.
 c



- d $f(3) = 60$
 $x \leq 3$ geeft $-4 < f(x) \leq 60$
 e Voer in $y_1 = 2^{x+3} - 4$ en $y_2 = -1$.
 De optie snijpunt geeft $x \approx -1,42$.



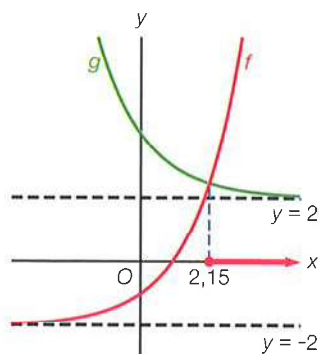
$f(x) \leq -1$ geeft $x \leq -1,42$

54

a $y = 2^x$
 $\downarrow$ translatie $(0, -2)$
 $f(x) = 2^x - 2$

$y = (\frac{1}{2})^x$
 $\downarrow$ translatie $(1, 2)$
 $g(x) = (\frac{1}{2})^{x-1} + 2$

b



$$B_f = \langle -2, \rightarrow \rangle \text{ en } B_g = \langle 2, \rightarrow \rangle$$

c Voer in $y_1 = 2^x - 2$ en $y_2 = (\frac{1}{2})^{x-1} + 2$.

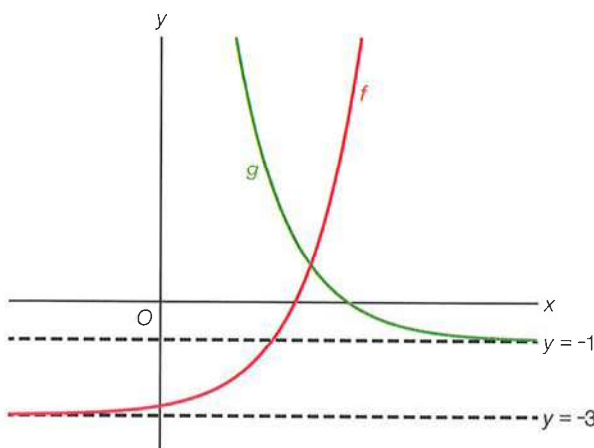
De optie snijpunt geeft $x \approx 2,15$.

$$f(x) \geq g(x) \text{ geeft } x \geq 2,15$$

d $B_f = \langle -2, \rightarrow \rangle$, dus $f(x) = p$ heeft geen oplossingen voor $p \leq -2$.

55

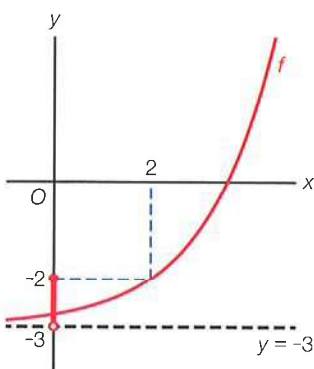
a



$$B_f = \langle -3, \rightarrow \rangle \text{ en } B_g = \langle -1, \rightarrow \rangle, \text{ dus } f(x) = p \text{ heeft één oplossing voor } p > -3 \text{ en } g(x) = p \text{ heeft geen oplossing voor } p \leq -1.$$

Dus het antwoord is voor $-3 < p \leq -1$.

b $f(2) = -2$



$$x \leq 2 \text{ geeft } -3 < f(x) \leq -2$$

c $f(1) = -2,5$ en $g(1) = 15$, dus $A(1; -2,5)$ en $B(1; 15)$.

$$AB = y_B - y_A = 15 - (-2,5) = 17,5$$

d Voer in $y_1 = 2^{x-2} - 3$, $y_2 = 4 \cdot 0,5^{x-3} - 1$ en $y_3 = 5$.

De optie snijpunt met y_1 en y_3 geeft $x = 5$, dus $P(5; 5)$.

De optie snijpunt met y_2 en y_3 geeft $x \approx 2,415$, dus $Q(2,415; 5)$.

$$PQ = x_P - x_Q \approx 5 - 2,415 = 2,585$$

Bladzijde 36

56 a $8 \cdot 4^x = 2^3 \cdot (2^2)^x = 2^3 \cdot 2^{2x} = 2^{2x+3}$

b Bij de herleiding van $(2^2)^x$ tot 2^{2x} is de regel $(a^p)^q = a^{pq}$ gebruikt.
Bij de herleiding van $2^3 \cdot 2^{2x}$ tot 2^{2x+3} is de regel $a^p \cdot a^q = a^{p+q}$ gebruikt.

57 a $y = 15 \cdot 2^{3x+2} = 15 \cdot 2^{3x} \cdot 2^2 = 15 \cdot (2^3)^x \cdot 4 = 60 \cdot 8^x$
Dus $y = 60 \cdot 8^x$.

b $y = 50 \cdot 2^{3x-1} = 50 \cdot 2^{3x} \cdot 2^{-1} = 50 \cdot (2^3)^x \cdot \frac{1}{2} = 25 \cdot 8^x$
Dus $y = 25 \cdot 8^x$.

c $y = 260 \cdot 4^{1\frac{1}{2}x-1} = 260 \cdot 4^{1\frac{1}{2}x} \cdot 4^{-1} = 260 \cdot (4^{1\frac{1}{2}})^x \cdot \frac{1}{4} = 65 \cdot 8^x$
Dus $y = 65 \cdot 8^x$.

d $y = 8 \cdot 4^{-2x-1} = 8 \cdot 4^{-2x} \cdot 4^{-1} = 8 \cdot (4^{-2})^x \cdot \frac{1}{4} = 2 \cdot (\frac{1}{16})^x$
Dus $y = 2 \cdot (\frac{1}{16})^x$.

58 a $y = 2^x \cdot 2^{2x-3} = 2^x \cdot 2^{2x} \cdot 2^{-3} = 2^{3x} \cdot \frac{1}{8} = \frac{1}{8} \cdot (2^3)^x = \frac{1}{8} \cdot 8^x$
Dus $y = \frac{1}{8} \cdot 8^x$.

b $y = 63 \cdot 3^{\frac{1}{2}x-2} = 63 \cdot 3^{\frac{1}{2}x} \cdot 3^{-2} = 63 \cdot (3^{\frac{1}{2}})^x \cdot \frac{1}{9} = 7 \cdot (\sqrt{3})^x$
Dus $y = 7 \cdot (\sqrt{3})^x$.

c $y = 50\,000 \cdot 100^{-x-2\frac{1}{2}} = 50\,000 \cdot 100^{-x} \cdot 100^{-2\frac{1}{2}} = 50\,000 \cdot (100^{-1})^x \cdot \frac{1}{100\,000} = \frac{1}{2} \cdot (\frac{1}{100})^x$
Dus $y = \frac{1}{2} \cdot (\frac{1}{100})^x$.

d $y = \frac{3}{4^{2x-1}} = 3 \cdot 4^{-2x+1} = 3 \cdot 4^{-2x} \cdot 4^1 = 3 \cdot (4^{-2})^x \cdot 4 = 12 \cdot (\frac{1}{16})^x$
Dus $y = 12 \cdot (\frac{1}{16})^x$.

59 $f(x) = 5 - 2^{\frac{1}{2}x+4}$
↓ translatie (12, -8)

$h(x) = 5 - 2^{\frac{1}{2}(x-12)+4} - 8 = -3 - 2^{\frac{1}{2}x-6+4} = -3 - 2^{\frac{1}{2}x-2} = -3 - (2^{\frac{1}{2}})^x \cdot \frac{1}{4} = -3 - \frac{1}{4} \cdot (\sqrt{2})^x$
Dus $a = -3$, $b = -\frac{1}{4}$ en $g = \sqrt{2}$.

60 $f(x) = 2^{x-3}$
↓ translatie (-5, 0)
 $g(x) = 2^{x+5-3} = 2^{x-3} \cdot 2^5 = 32 \cdot 2^{x-3}$

De vermenigvuldiging ten opzichte van de x -as met 32 toepassen op de grafiek van f geeft de grafiek van g .
Dus $a = 32$.

61 $4\sqrt{2} = 2^2 \cdot 2^{\frac{1}{2}} = 2^{2\frac{1}{2}}$
 $2^{x-1} = 4\sqrt{2}$
 $2^{x-1} = 2^{2\frac{1}{2}}$
 $x-1 = 2\frac{1}{2}$
 $x = 3\frac{1}{2}$

Bladzijde 37

62 a $2^{x+1} = 64$
 $2^{x+1} = 2^6$
 $x+1 = 6$
 $x = 5$

b $2^{x-3} = \frac{1}{8}$
 $2^{x-3} = 2^{-3}$
 $x-3 = -3$
 $x = 0$

c $2^{2x} = 2$
 $2^{2x} = 2^1$
 $2x = 1$
 $x = \frac{1}{2}$

d $2^x = 1$
 $2^x = 2^0$
 $x = 0$

e $2^x = \frac{1}{4}\sqrt{2}$
 $2^x = 2^{-2} \cdot 2^{\frac{1}{2}}$
 $2^x = 2^{-1\frac{1}{2}}$
 $x = -1\frac{1}{2}$

f $5^{x+6} = (\frac{1}{5})^x$
 $5^{x+6} = (5^{-1})^x$
 $5^{x+6} = 5^{-x}$
 $x+6 = -x$
 $2x = -6$
 $x = -3$

63 a $3^{2x+1} = 27\sqrt{3}$
 $3^{2x+1} = 3^3 \cdot 3^{\frac{1}{2}}$
 $3^{2x+1} = 3^{3\frac{1}{2}}$
 $2x+1 = 3\frac{1}{2}$
 $2x = 2\frac{1}{2}$
 $x = 1\frac{1}{4}$

b $10^{2x+1} = 0,01$
 $10^{2x+1} = 10^{-2}$
 $2x+1 = -2$
 $2x = -3$
 $x = -1\frac{1}{2}$

c $3^x - 2 = 25$
 $3^x = 27$
 $3^x = 3^3$
 $x = 3$

64 a $3 \cdot 2^x + 4 = 28$
 $3 \cdot 2^x = 24$
 $2^x = 8$
 $2^x = 2^3$
 $x = 3$

b $5^{2x-6} = 0,04$
 $5^{2x-6} = \frac{1}{25}$
 $5^{2x-6} = 5^{-2}$
 $2x-6 = -2$
 $2x = 4$
 $x = 2$

c $3 \cdot 7^{3x+1} = 147$
 $7^{3x+1} = 49$
 $7^{3x+1} = 7^2$
 $3x+1 = 2$
 $3x = 1$
 $x = \frac{1}{3}$

Bladzijde 38

65 a $f(x) = g(x)$
 $(\sqrt{2})^{x+4} = (\frac{1}{4})^x$
 $(2^{\frac{1}{2}})^{x+4} = 4^{-x}$
 $2^{\frac{1}{2}x+2} = (2^2)^{-x}$
 $2^{\frac{1}{2}x+2} = 2^{-2x}$
 $\frac{1}{2}x+2 = -2x$
 $2\frac{1}{2}x = -2$
 $x = -\frac{4}{5}$
 $f(x) \geq g(x)$ geeft $x \geq -\frac{4}{5}$

d $5 \cdot 2^x = 80$
 $2^x = 16$
 $2^x = 2^4$
 $x = 4$

e $10 \cdot 3^x = 270$
 $3^x = 27$
 $3^x = 3^3$
 $x = 3$

f $3 \cdot 8^{2-x} = 48$
 $8^{2-x} = 16$
 $(2^3)^{2-x} = 2^4$
 $2^{6-3x} = 2^4$
 $6-3x = 4$
 $-3x = -2$
 $x = \frac{2}{3}$

d $32^{x-2} = \frac{1}{16}$
 $(2^5)^{x-2} = 2^{-4}$
 $2^{5x-10} = 2^{-4}$
 $5x-10 = -4$
 $5x = 6$
 $x = 1\frac{1}{5}$

e $5 \cdot 4^{x-1} = 2\frac{1}{2}$
 $4^{x-1} = \frac{1}{2}$
 $(2^2)^{x-1} = 2^{-1}$
 $2^{2x-2} = 2^{-1}$
 $2x-2 = -1$

$2x = 1$
 $x = \frac{1}{2}$
f $8 \cdot 2^x = 4^{x+1}$
 $2^3 \cdot 2^x = (2^2)^{x+1}$
 $2^{3+x} = 2^{2x+2}$
 $3+x = 2x+2$
 $-x = -1$
 $x = 1$

b $g(x) = \sqrt{2}$
 $(\frac{1}{4})^x = \sqrt{2}$
 $2^{-2x} = 2^{\frac{1}{2}}$
 $-2x = \frac{1}{2}$
 $x = -\frac{1}{4}$
 $g(x) \geq \sqrt{2}$ geeft $x \leq -\frac{1}{4}$

66 $f(x) = 2^x$
 $\downarrow$ verm. x -as, 6
 $y = 6 \cdot 2^x$
 $\downarrow$ translatie $(0, -10)$
 $g(x) = 6 \cdot 2^x - 10$
 $f(x) = g(x)$ geeft $2^x = 6 \cdot 2^x - 10$
 $-5 \cdot 2^x = -10$
 $2^x = 2$
 $x = 1$
 $f(1) = 2$, dus het snijpunt is $(1, 2)$.

67 $f(x) = 3^{x+2} - 5$
 $\downarrow$ translatie $(3, 7)$
 $y = 3^{x-1} + 2$
 $\downarrow$ verm. y -as, b
 $g(x) = 3^{\frac{1}{b}x-1} + 2$
 $g(x) = 3^{\frac{1}{b}x-1} + 2$
 door $A(-15, 83)$ $\left\{ \begin{array}{l} 3^{\frac{-15}{b}-1} + 2 = 83 \\ 3^{\frac{-15}{b}-1} = 81 \\ 3^{\frac{-15}{b}-1} = 3^4 \\ \frac{-15}{b} - 1 = 4 \\ \frac{-15}{b} = 5 \\ b = -3 \end{array} \right.$

5.5 Logaritmen

Bladzijde 40

68 **a** $2^3 = 8$
b $2^{-2} = \frac{1}{4}$
c $2^{\frac{1}{2}} = \sqrt{2}$
d $3^2 = 9$
e $3^{-3} = \frac{1}{27}$
f $3^{\frac{1}{5}} = \sqrt[5]{3}$

Bladzijde 41

69 **a** ${}^5\log(125) = {}^5\log(5^3) = 3$
b ${}^{10}\log(0,1) = {}^{10}\log(10^{-1}) = -1$
c ${}^2\log(4) = {}^2\log(2^2) = 2$
d ${}^7\log(49) = {}^7\log(7^2) = 2$
e ${}^2\log(\sqrt{2}) = {}^2\log(2^{\frac{1}{2}}) = \frac{1}{2}$
70 **a** ${}^2\log(64\sqrt{2}) = {}^2\log(2^6 \cdot 2^{\frac{1}{2}}) = {}^2\log(2^{6\frac{1}{2}}) = 6\frac{1}{2}$
b ${}^3\log(\frac{1}{9}\sqrt{3}) = {}^3\log(3^{-2} \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{-1\frac{1}{2}}) = -1\frac{1}{2}$
c ${}^3\log(3^{21,5}) = 21,5$
d ${}^5\log(\frac{1}{125}) = {}^5\log(5^{-3}) = -3$
e ${}^{\frac{1}{3}}\log(\frac{1}{27}) = {}^{\frac{1}{3}}\log((\frac{1}{3})^3) = 3$
f ${}^{\frac{1}{2}}\log(\frac{1}{4}) = {}^{\frac{1}{2}}\log((\frac{1}{2})^2) = 2$
g ${}^2\log(\frac{1}{32} \cdot \sqrt[3]{2}) = {}^2\log(2^{-5} \cdot 2^{\frac{1}{3}}) = {}^2\log(2^{-4\frac{2}{3}}) = -4\frac{2}{3}$
h ${}^5\log(1) = {}^5\log(5^0) = 0$
i ${}^3\log(81 \cdot \sqrt[5]{27}) = {}^3\log(3^4 \cdot 3^{\frac{3}{5}}) = {}^3\log(3^{4\frac{3}{5}}) = 4\frac{3}{5}$
f ${}^2\log(0,5) = {}^2\log(2^{-1}) = -1$
g ${}^4\log(0,25) = {}^4\log(4^{-1}) = -1$
h ${}^4\log(4) = {}^4\log(4^1) = 1$
i ${}^4\log(1) = {}^4\log(4^0) = 0$

- 71 a** ${}^3\log(9) = {}^3\log(3^2) = 2$
Dus ${}^3\log(x) = 2$ geeft $x = 9$.
- b** ${}^5\log(\frac{1}{25}) = {}^5\log(5^{-2}) = -2$
Dus ${}^5\log(x) = -2$ geeft $x = \frac{1}{25}$.

- 72 a** $x = {}^5\log(0,2) = {}^5\log(\frac{1}{5}) = {}^5\log(5^{-1}) = -1$
- b** ${}^9\log(x) = \frac{1}{2}$
 $x = 9^{\frac{1}{2}} = \sqrt{9} = 3$
- c** ${}^x\log(1000) = 3$
 $x^3 = 1000$
 $x = 10$

Bladzijde 42

- 73 a** ${}^3\log(x+2) = 2$
 $x+2 = 3^2$
 $x+2 = 9$
 $x = 7$

- b** $1 + {}^{\frac{1}{2}}\log(x) = 4$
 ${}^{\frac{1}{2}}\log(x) = 3$
 $x = (\frac{1}{2})^3$
 $x = \frac{1}{8}$

- c** ${}^3\log(2x+1) = 4$
 $2x+1 = 3^4$
 $2x+1 = 81$
 $2x = 80$
 $x = 40$

- 74 a** $4 \cdot {}^3\log(x) = 2$
 ${}^3\log(x) = \frac{1}{2}$
 $x = 3^{\frac{1}{2}}$
 $x = \sqrt{3}$
- b** ${}^3\log(4x-1) = -2$
 $4x-1 = 3^{-2}$
 $4x-1 = \frac{1}{9}$
 $4x = 1\frac{1}{9}$
 $x = \frac{5}{18}$
- c** $3 + {}^2\log(x) = -1$
 ${}^2\log(x) = -4$
 $x = 2^{-4}$
 $x = \frac{1}{16}$

- 75** ${}^2\log({}^3\log(x)) = -1$
 ${}^3\log(x) = 2^{-1}$
 ${}^3\log(x) = \frac{1}{2}$
 $x = 3^{\frac{1}{2}}$
 $x = \sqrt{3}$

- d** $5 + {}^4\log(x) = 3$
 ${}^4\log(x) = -2$
 $x = 4^{-2}$
 $x = \frac{1}{16}$

- e** ${}^{\frac{1}{2}}\log(x-1) = 3$
 $x-1 = (\frac{1}{2})^3$
 $x-1 = \frac{1}{8}$
 $x = 1\frac{1}{8}$

- f** ${}^2\log(x^2-4) = 5$
 $x^2-4 = 2^5$
 $x^2-4 = 32$
 $x^2 = 36$
 $x = 6 \vee x = -6$

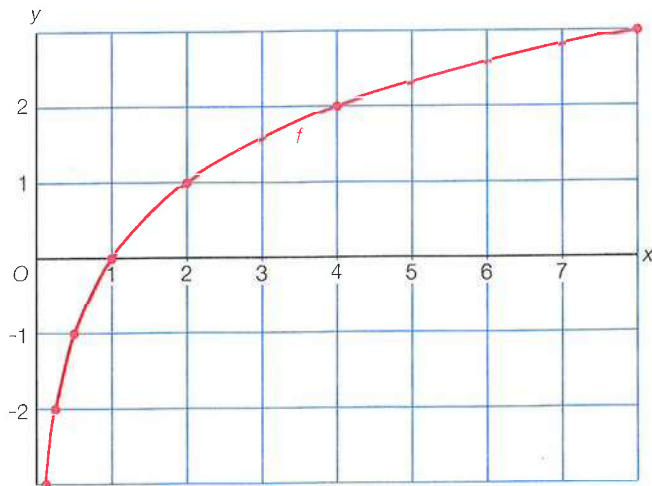
- d** ${}^5\log(3x+2) = 1$
 $3x+2 = 5^1$
 $3x+2 = 5$
 $3x = 3$
 $x = 1$

- e** ${}^3\log(0,4x-5) = 2$
 $0,4x-5 = 3^2$
 $0,4x-5 = 9$
 $0,4x = 14$
 $x = 35$

- f** $4 + 2 \cdot {}^2\log(x) = 7$
 $2 \cdot {}^2\log(x) = 3$
 ${}^2\log(x) = 1\frac{1}{2}$
 $x = 2^{1\frac{1}{2}}$
 $x = 2^1 \cdot 2^{\frac{1}{2}}$
 $x = 2\sqrt{2}$

76

a	x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
	${}^2\log(x)$	-3	-2	-1	0	1	2	3

b**c** Bij ${}^2\log(x)$ moet $x > 0$ zijn, dus $D_f = \langle 0, \rightarrow \rangle$.**d** $B_f = \mathbb{R}$ **Bladzijde 44**

77

Het effect van transformaties op de standaardgrafiek $y = {}^s\log(x)$

$y = {}^s\log(x)$ $\downarrow$ verm. x-as, a $y = a \cdot {}^s\log(x)$	Vermenigvuldig in de formule de functiewaarde met a .
$y = {}^s\log(x)$ $\downarrow$ verm. y-as, b $y = {}^s\log\left(\frac{1}{b} \cdot x\right)$	Vervang in de formule x door $\frac{1}{b} \cdot x$.
$y = {}^s\log(x)$ $\downarrow$ translatie $(c, 0)$ $y = {}^s\log(x - c)$	Vervang in de formule x door $x - c$.
$y = {}^s\log(x)$ $\downarrow$ translatie $(0, d)$ $y = {}^s\log(x) + d$	Tel in de formule d op bij de functiewaarde.

78

a $y = {}^3\log(x)$ $\downarrow$ translatie $(-2, 0)$

$$f(x) = {}^3\log(x + 2)$$

De lijn $x = -2$ is verticale asymptoot.**b** $y = {}^2\log(x)$ $\downarrow$ translatie $(1, 0)$

$$y = {}^2\log(x - 1)$$

 $\downarrow$ verm. x-as, 5

$$g(x) = 5 \cdot {}^2\log(x - 1)$$

De lijn $x = 1$ is verticale asymptoot.**c** $y = {}^{\frac{1}{2}}\log(x)$ $\downarrow$ verm. x-as, 4

$$y = 4 \cdot {}^{\frac{1}{2}}\log(x)$$

 $\downarrow$ translatie $(0, 3)$

$$h(x) = 4 \cdot {}^{\frac{1}{2}}\log(x) + 3$$

De lijn $x = 0$ is verticale asymptoot.**d** $y = {}^{\frac{1}{3}}\log(x)$ $\downarrow$ verm. x-as, -1

$$y = -{}^{\frac{1}{3}}\log(x)$$

 $\downarrow$ translatie $(-1, -2)$

$$k(x) = -{}^{\frac{1}{3}}\log(x + 1) - 2$$

De lijn $x = -1$ is verticale asymptoot.**e** $y = {}^3\log(x)$ $\downarrow$ verm. y-as, $\frac{1}{2}$

$$y = {}^3\log(2x)$$

 $\downarrow$ translatie $(0, 5)$

$$l(x) = {}^3\log(2x) + 5$$

De lijn $x = 0$ is verticale asymptoot.**f** $y = {}^{\frac{1}{4}}\log(x)$ $\downarrow$ verm. y-as, 2

$$y = {}^{\frac{1}{4}}\log\left(\frac{1}{2}x\right)$$

 $\downarrow$ verm. x-as, 3

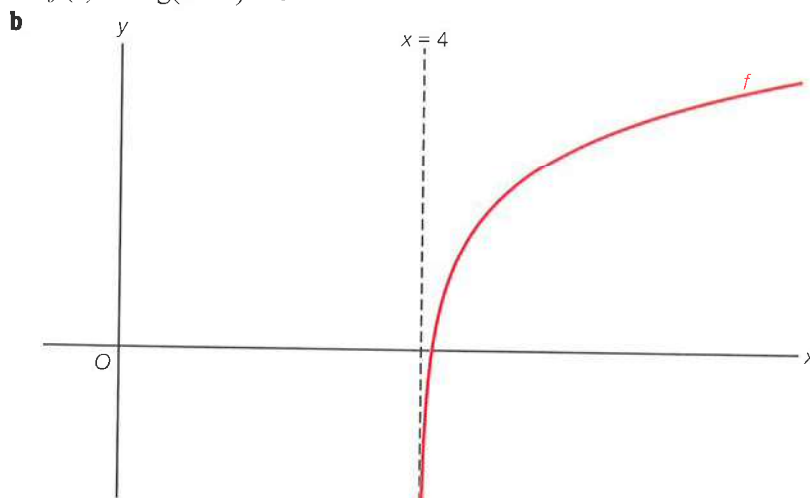
$$m(x) = 3 \cdot {}^{\frac{1}{4}}\log\left(\frac{1}{2}x\right)$$

De lijn $x = 0$ is verticale asymptoot.

Bladzijde 45

- 79 a** $y = {}^3\log(x)$
 ↓ verm. x -as, 2
 $y = 2 \cdot {}^3\log(x)$
 ↓ translatie $(0, -4)$
 $y = 2 \cdot {}^3\log(x) - 4$
- b** $y = {}^3\log(x)$
 ↓ spiegelen in de x -as
 $y = -{}^3\log(x)$
 ↓ translatie $(5, 0)$
 $y = -{}^3\log(x - 5)$
- c** $y = {}^3\log(x)$
 ↓ translatie $(-3, 2)$
 $y = {}^3\log(x + 3) + 2$
 ↓ verm. x -as, $\frac{1}{2}$
 $y = \frac{1}{2} \cdot {}^3\log(x + 3) + 1$
- d** $y = {}^3\log(x)$
 ↓ 5 omhoog
 $y = {}^3\log(x) + 5$
 ↓ verm. y -as, $\frac{1}{4}$
 $y = {}^3\log(4x) + 5$
 ↓ 3 naar links
 $y = {}^3\log(4(x + 3)) + 5$
 oftewel $y = {}^3\log(4x + 12) + 5$

- 80 a** $y = {}^3\log(x)$
 ↓ translatie $(4, 2)$
 $f(x) = {}^3\log(x - 4) + 2$



$$D_f = \langle 4, \rightarrow \rangle$$

- c** $f(x) = 6$ geeft ${}^3\log(x - 4) + 2 = 6$
 ${}^3\log(x - 4) = 4$
 $x - 4 = 3^4$
 $x - 4 = 81$
 $x = 85$

$$f(x) < 6 \text{ geeft } 4 < x < 85$$

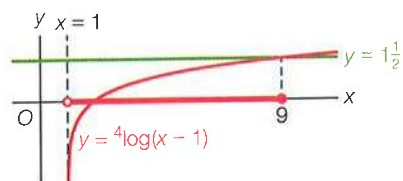
- d** $f(7) = {}^3\log(3) + 2 = 1 + 2 = 3$
 $x \leq 7$ geeft $f(x) \leq 3$

81 a ${}^4\log(x-1) = 1\frac{1}{2}$

$$x-1 = 4^{1\frac{1}{2}}$$

$$x-1 = 8$$

$$x = 9$$



$${}^4\log(x-1) \leq 1\frac{1}{2} \text{ geeft } 1 < x \leq 9$$

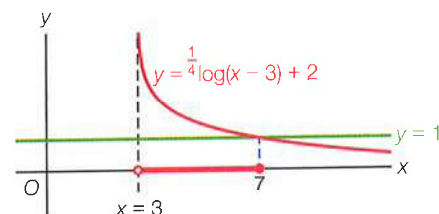
b $\frac{1}{4}\log(x-3) + 2 = 1$

$$\frac{1}{4}\log(x-3) = -1$$

$$x-3 = \left(\frac{1}{4}\right)^{-1}$$

$$x-3 = 4$$

$$x = 7$$

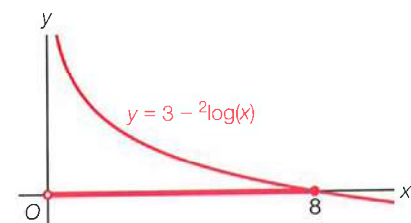


$$\frac{1}{4}\log(x-3) + 2 \geq 1 \text{ geeft } 3 < x \leq 7$$

c $3 - {}^2\log(x) = 0$

$${}^2\log(x) = 3$$

$$x = 2^3 = 8$$



$$3 - {}^2\log(x) \geq 0 \text{ geeft } 0 < x \leq 8$$

82 $y = {}^2\log(x)$
 $\downarrow$ translatie (15, 0)

$$y = {}^2\log(x-15)$$

$$\downarrow \text{verm. y-as, } \frac{1}{5}$$

$$y = {}^2\log(5x-15)$$

$$\downarrow \text{verm. x-as, } -4$$

$$y = -4 \cdot {}^2\log(5x-15)$$

$$\downarrow \text{translatie (0, 6)}$$

$$y = 6 - 4 \cdot {}^2\log(5x-15)$$

De verticale asymptoot is de lijn $x = 3$.

of $y = {}^2\log(x)$

$$\downarrow \text{verm. y-as, } \frac{1}{5}$$

$$y = {}^2\log(5x)$$

$$\downarrow \text{verm. x-as, } -4$$

$$y = -4 \cdot {}^2\log(5x)$$

$$\downarrow \text{translatie (3, 6)}$$

$$y = 6 - 4 \cdot {}^2\log(5(x-3))$$

$$\text{oftewel } y = 6 - 4 \cdot {}^2\log(5x-15)$$

Diagnostische toets**Bladzijde 48**

1 a $(a^{-\frac{1}{4}})^3 = a^{-\frac{3}{4}} = \frac{1}{a^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{a^3}}$

b $a^{-2}b^{\frac{1}{5}} = \frac{1}{a^2} \cdot \sqrt[5]{b} = \frac{\sqrt[5]{b}}{a^2}$

c $7a^{\frac{1}{3}}b^{\frac{3}{5}} = 7 \cdot \frac{1}{a^{\frac{1}{3}}} \cdot \sqrt[5]{b^3} = \frac{7 \cdot \sqrt[5]{b^3}}{\sqrt[3]{a}}$

d $6a^{4\frac{2}{3}} = 6a^4 \cdot a^{\frac{2}{3}} = 6a^4 \cdot \sqrt[3]{a^2}$

e $\frac{3}{4}a^{-2}b^{2\frac{1}{3}} = \frac{3}{4} \cdot \frac{1}{a^2} \cdot b^2 \cdot b^{\frac{1}{3}} = \frac{3b^2 \cdot \sqrt[3]{b}}{4a^2}$

f $(3a^{-2\frac{1}{2}})^{-2} = 3^{-2} \cdot a^5 = \frac{1}{9}a^5$

2 a $y = \frac{1}{5x^2 \cdot \sqrt{x}} = \frac{1}{5} \cdot \frac{1}{x^2 \cdot x^{\frac{1}{2}}} = \frac{1}{5} \cdot \frac{1}{x^{2\frac{1}{2}}} = \frac{1}{5}x^{-2\frac{1}{2}}$

Dus $y = \frac{1}{5}x^{-2\frac{1}{2}}$.

b $y = \frac{12}{5x^{-3}} \cdot \sqrt[5]{x^3} = \frac{12}{5} \cdot \frac{1}{x^{-3}} \cdot x^{\frac{3}{5}} = 2\frac{2}{5} \cdot x^3 \cdot x^{\frac{3}{5}} = 2\frac{2}{5}x^{3\frac{3}{5}}$

Dus $y = 2\frac{2}{5}x^{3\frac{3}{5}}$.

c $y = 3x^{0,3} \cdot (4x^{0,5})^4 = 3x^{0,3} \cdot 4^4 \cdot x^2 = 768x^{2,3}$
Dus $y = 768x^{2,3}$.

3 a $f(x) = -\frac{1}{2}(x-2)^5 + 3$

↓ verm. x-as, -2

$y = (x-2)^5 - 6$

↓ translatie (-3, 6)

$y = (x+1)^5$

Het punt van symmetrie is (-1, 0).

b $g(x) = \frac{2}{3}(x+1)^4 - 2$

↓ translatie (1, 5)

$y = \frac{2}{3}x^4 + 3$

↓ verm. x-as, 5

$h(x) = 3\frac{1}{3}x^4 + 15$

min. is $h(0) = 15$

4 a $x - 4 \geq 0$

$x \geq 4$

Dus randpunt (4, 1), $D_f = [4, \rightarrow)$ en $B_f = [1, \rightarrow)$.

b $x + 2 \geq 0$

$x \geq -2$

Dus randpunt (-2, -4), $D_g = [-2, \rightarrow)$ en $B_g = \langle \leftarrow, -4 \right]$.

c $x - 3 \geq 0$

$x \geq 3$

Dus randpunt (3, 5), $D_h = [3, \rightarrow)$ en $B_h = \langle \leftarrow, 5 \right]$.

5 a $5 - 2x \geq 0$

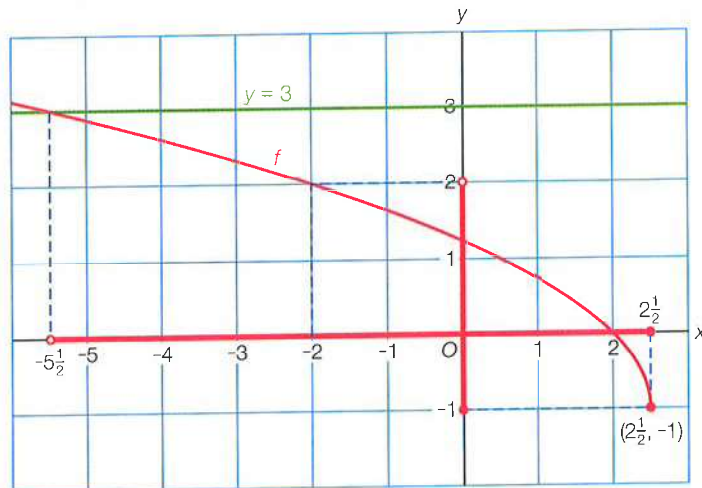
$$-2x \geq -5$$

$$x \leq 2\frac{1}{2}$$

Dus $D_f = \langle \leftarrow, 2\frac{1}{2} \rangle$ en randpunt $(2\frac{1}{2}, -1)$.

Voer in $y_1 = \sqrt{5 - 2x} - 1$.

x	-4	-3	-2	-1	0	1	2	$2\frac{1}{2}$
$f(x)$	2,6	2,3	2	1,6	1,2	0,7	0	-1



$$B_f = [-1, \rightarrow)$$

b Voer in $y_2 = 3$.

De optie snijpunt geeft $x = -5\frac{1}{2}$.

$$f(x) < 3 \text{ geeft } -5\frac{1}{2} < x \leq 2\frac{1}{2}$$

c $f(-2) = 2$

$$x > -2 \text{ geeft } -1 \leq f(x) < 2$$

6 a $2x = \sqrt{2 - 7x}$

kwadrateren geeft

$$4x^2 = 2 - 7x$$

$$4x^2 + 7x - 2 = 0$$

$$D = 7^2 - 4 \cdot 4 \cdot -2 = 81$$

$$x = \frac{-7 + 9}{8} = \frac{1}{4} \vee x = \frac{-7 - 9}{8} = -2$$

$$x = \frac{1}{4} \text{ geeft } \frac{1}{2} = \sqrt{\frac{1}{4}} \text{ vold.}$$

$$x = -2 \text{ geeft } -4 = \sqrt{16} \text{ vold. niet}$$

b $28 - x\sqrt{x} = 1$

$$-x\sqrt{x} = -27$$

kwadrateren geeft

$$x^3 = 729$$

$$x = 9 \text{ geeft } 28 - 9\sqrt{9} = 1 \text{ vold.}$$

c $2x - \sqrt{x} = 6$

$$2x - 6 = \sqrt{x}$$

kwadrateren geeft

$$(2x - 6)^2 = x$$

$$4x^2 - 24x + 36 = x$$

$$4x^2 - 25x + 36 = 0$$

$$D = 25^2 - 4 \cdot 4 \cdot 36 = 49$$

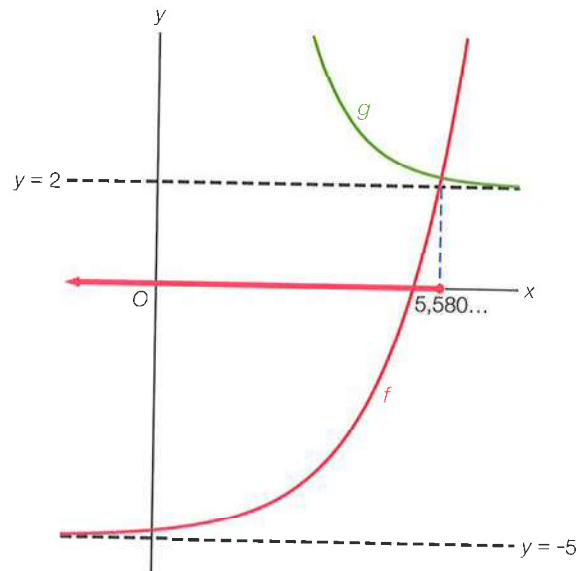
$$x = \frac{25 + 7}{8} = 4 \vee x = \frac{25 - 7}{8} = 2\frac{1}{4}$$

$$x = 4 \text{ geeft } 8 - \sqrt{4} = 6 \text{ vold.}$$

$$x = 2\frac{1}{4} \text{ geeft } 4\frac{1}{2} - \sqrt{2\frac{1}{4}} = 6 \text{ vold. niet}$$

Bladzijde 49

- 7 a** $y = 2^x$
 ↓ verm. x -as, 0,3
 $y = 0,3 \cdot 2^x$
 ↓ translatie (1, -5)
 $f(x) = 0,3 \cdot 2^{x-1} - 5$
- b** $B_f = \langle -5, \rightarrow \rangle$
 De horizontale asymptoot is de lijn $y = -5$.
- c** Voer in $y_1 = 0,3 \cdot 2^{x-1} - 5$ en $y_2 = 2 + (\frac{1}{3})^{x-4}$.
 De optie snijpunt geeft $x = 5,580...$



- $f(x) \leq g(x)$ geeft $x \leq 5,58$
- d** $f(x) = p$ heeft één oplossing als $p > -5$.
 $g(x) = p$ heeft geen oplossing als $p \leq 2$.
 Dus $-5 < p \leq 2$.
- e** $g(3) = 2 + (\frac{1}{3})^{3-4} = 2 + (\frac{1}{3})^{-1} = 2 + 3 = 5$
 $x \leq 3$ geeft $g(x) \geq 5$

- 8 a** $y = 108 \cdot 3^{4x-3} = 108 \cdot 3^{4x} \cdot 3^{-3} = 108 \cdot (3^4)^x \cdot \frac{1}{27} = 4 \cdot 81^x$
 Dus $y = 4 \cdot 81^x$.
- b** $y = \frac{250}{5^{-2x+3}} = \frac{250}{125 \cdot 5^{-2x}} = 2 \cdot 5^{2x} = 2 \cdot (5^2)^x = 2 \cdot 25^x$
 Dus $y = 2 \cdot 25^x$.
- c** $y = 2^{3x-2} \cdot 2^{-4x+3} = 2^{-x+1} = 2 \cdot 2^{-x} = 2 \cdot (\frac{1}{2})^x$
 Dus $y = 2 \cdot (\frac{1}{2})^x$.

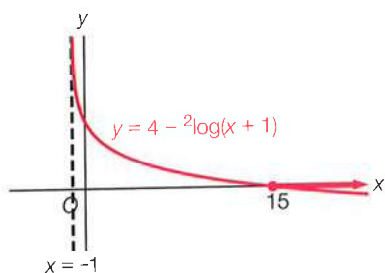
- 9 a** $5^{x-1} = 125$
 $5^{x-1} = 5^3$
 $x-1 = 3$
 $x = 4$
- b** $3^{2x-5} = \frac{1}{27}$
 $3^{2x-5} = 3^{-3}$
 $2x-5 = -3$
 $2x = 2$
 $x = 1$
- c** $2 \cdot 4^{2x-1} - 3 = 61$
 $2 \cdot 4^{2x-1} = 64$
 $4^{2x-1} = 32$
 $(2^2)^{2x-1} = 2^5$
 $2^{4x-2} = 2^5$
 $4x-2 = 5$
 $4x = 7$
 $x = 1\frac{3}{4}$

10 a ${}^2\log(\frac{1}{4}\sqrt{8}) = {}^2\log(2^{-2} \cdot 2^{\frac{1}{2}}) = {}^2\log(2^{-\frac{3}{2}}) = -\frac{3}{2}$
b ${}^3\log(27\sqrt{3}) = {}^3\log(3^3 \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{\frac{7}{2}}) = \frac{7}{2}$
c ${}^5\log(\frac{1}{125} \cdot \sqrt[3]{5}) = {}^5\log(5^{-3} \cdot 5^{\frac{1}{3}}) = {}^5\log(5^{-\frac{8}{3}}) = -\frac{8}{3}$

11 a ${}^4\log(2x-3) = 2$
 $2x-3 = 4^2$
 $2x-3 = 16$
 $2x = 19$
 $x = 9\frac{1}{2}$
b ${}^{\frac{1}{2}}\log(x-3) = -4$
 $x-3 = (\frac{1}{2})^{-4}$
 $x-3 = (2^{-1})^{-4}$
 $x-3 = 2^4$
 $x-3 = 16$
 $x = 19$
c $5 + 3 \cdot {}^2\log(x) = 20$
 $3 \cdot {}^2\log(x) = 15$
 ${}^2\log(x) = 5$
 $x = 2^5$
 $x = 32$

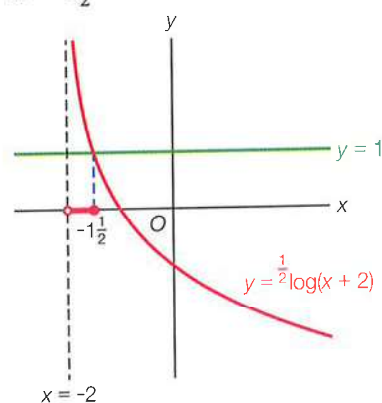
12 a $y = {}^2\log(x)$
 $\downarrow$ verm. x-as, -3
 $y = -3 \cdot {}^2\log(x)$
 $\downarrow$ translatie (4, -2)
 $y = -3 \cdot {}^2\log(x-4) - 2$
b $y = {}^2\log(x)$
 $\downarrow$ translatie (-3, 2)
 $y = {}^2\log(x+3) + 2$
 $\downarrow$ verm. x-as, $\frac{1}{2}$
 $y = \frac{1}{2} \cdot {}^2\log(x+3) + 1$

13 a $4 - {}^2\log(x+1) = 0$
 ${}^2\log(x+1) = 4$
 $x+1 = 16$
 $x = 15$



$4 - {}^2\log(x+1) \leq 0$ geeft $x \geq 15$

b ${}^{\frac{1}{2}}\log(x+2) = 1$
 $x+2 = \frac{1}{2}$
 $x = -1\frac{1}{2}$



${}^{\frac{1}{2}}\log(x+2) \geq 1$ geeft $-2 < x \leq -1\frac{1}{2}$