3 Hoeken en afstanden

Voorkennis Goniometrische verhoudingen

Bladzijde 98

1 $\tan(\angle A) = \frac{BC}{AB} = \frac{3}{5}$ geeft $\angle A \approx 31{,}0°$

$\sin(\angle D) = \frac{EF}{DE} = \frac{8}{11}$ geeft $\angle D \approx 46{,}7°$

$\cos(\angle I) = \frac{HI}{GI} = \frac{4}{10}$ geeft $\angle I \approx 66{,}4°$

$\tan(\angle K) = \frac{LM}{KL} = \frac{7}{10}$ geeft $\angle K \approx 35{,}0°$

$\cos(\angle P) = \frac{\frac{1}{2}PQ}{PR} = \frac{7{,}5}{10}$ geeft $\angle P \approx 41{,}4°$

Bladzijde 99

2 $\cos(\angle C) = \frac{AC}{BC}$

$\cos(38°)$	AC
1	17

$AC = 17\cos(38°) \approx 13{,}4$

$\tan(\angle E) = \frac{DF}{DE}$

$\tan(55°)$	DF
1	5

$DF = 5\tan(55°) \approx 7{,}1$

$\sin(\angle G) = \frac{HI}{GH}$

$\sin(40°)$	7
1	GH

$GH = \frac{7}{\sin(40°)} \approx 10{,}9$

$\sin(\angle MLN) = \frac{MN}{LN}$

$\sin(60°)$	MN
1	17

$KL = MN = 17\sin(60°) \approx 14{,}7$

$PS = QS = 3$

$\tan(\angle P) = \frac{RS}{PS}$

$\tan(75°)$	RS
1	3

$RS = 3\tan(75°) \approx 11{,}2$

3 $\cos(\angle BAC) = \dfrac{AB}{AC}$

$\cos(35°)$	AB
1	10

$AB = 10\cos(35°) = 8{,}191...$

$\sin(\angle BAC) = \dfrac{BC}{AC}$

$\sin(35°)$	BC
1	10

$BC = 10\sin(35°) = 5{,}735...$

omtrek rechthoek $= 2 \cdot 8{,}191... + 2 \cdot 5{,}735... \approx 27{,}85$

4 CD is de bij zijde AB horende hoogte.

$\tan(\angle A) = \dfrac{CD}{\frac{1}{2}AB}$

$\tan(65°)$	CD
1	4

$CD = 4\tan(65°) = 8{,}578...$

opp $\triangle ABC = \frac{1}{2} \cdot 8 \cdot 8{,}578... \approx 34{,}31$

5 RS is de bij zijde PQ horende hoogte.

$$\left.\begin{array}{l}\text{opp } \triangle PQR = 20\\ \text{opp } \triangle PQR = \frac{1}{2} \cdot PQ \cdot RS\end{array}\right\} \frac{1}{2} \cdot PQ \cdot RS = 20$$

$$\frac{1}{2} \cdot 6 \cdot RS = 20$$

$$RS = 6\tfrac{2}{3}$$

$\tan(\angle P) = \dfrac{RS}{\frac{1}{2}PQ} = \dfrac{6\frac{2}{3}}{3}$ geeft $\angle P = 65{,}77...° \approx 65{,}8°$

$\angle Q = \angle P \approx 65{,}8°$

$\angle R = 180° - \angle P - \angle Q = 180° - 2 \cdot 65{,}77...° \approx 48{,}5°$

3.1 Berekeningen in driehoeken

Bladzijde 100

1 **a** In $\triangle ABC$ is $\tan(\angle A) = \dfrac{BC}{AB}$.

$\angle A = 30°$ en $AB = 6$, dus je krijgt $\tan(30°) = \dfrac{BC}{6}$.

Hieruit volgt de verhoudingstabel

$\tan(30°)$	BC
1	6

en dit geeft $BC = 6\tan(30°) = 3{,}4641... \approx 3{,}464$.

b $\tan(\angle BDC) = \dfrac{BC}{BD}$

$\tan(40°)$	$3{,}4641...$
1	BD

$BD = \dfrac{3{,}4641...}{\tan(40°)} \approx 4{,}13$

Bladzijde 101

2 **a** In $\triangle BCD$ is $\sin(70°) = \frac{BC}{5}$

$BC = 5\sin(70°) = 4{,}69...$

In $\triangle ABC$ is $\sin(\angle BAC) = \frac{BC}{AC} = \frac{4{,}69...}{10}$

$\angle BAC \approx 28{,}0°$

b $\angle ADC = 180° - \angle BDC = 180° - 70° = 110°$

$\angle ACD = 180° - \angle ADC - \angle DAC \approx 180° - 110° - 28{,}0° = 42{,}0°$

3 In $\triangle ABC$ is $\tan(28°) = \frac{4}{AB}$

$AB = \frac{4}{\tan(28°)} = 7{,}52...$

$BM = \frac{1}{2}AB = 3{,}76...$

In $\triangle BCM$ is $\tan(\angle BMC) = \frac{BC}{BM} = \frac{4}{3{,}76...}$

$\angle BMC \approx 46{,}8°$

4 In $\triangle ABC$ is $\sin(40°) = \frac{BC}{10}$

$BC = 10\sin(40°) = 6{,}42...$

$BM = \frac{1}{2}BC = 3{,}21...$

In $\triangle ABC$ is $\cos(40°) = \frac{AB}{10}$

$AB = 10\cos(40°) = 7{,}66...$

In $\triangle ABM$ is $\tan(\angle BAM) = \frac{BM}{AB} = \frac{3{,}21...}{7{,}66...}$

$\angle BAM \approx 22{,}8°$

Bladzijde 102

5 $\angle ACB = 180° - \angle CAB - \angle ABC = 180° - 31° - 90° = 59°$

In $\triangle ABC$ is $\tan(31°) = \frac{BC}{10}$

$BC = 10\tan(31°) = 6{,}00...$

In $\triangle BCD$ is $\tan(\angle BCD) = \frac{BD}{BC} = \frac{8}{6{,}00...}$

$\angle BCD \approx 53{,}1°$

$\angle ACD = \angle ACB - \angle BCD \approx 59° - 53{,}1° = 5{,}9°$

6 In $\triangle BCM$ is $\sin(50°) = \frac{BC}{6}$

$BC = 6\sin(50°) = 4{,}59...$

In $\triangle BCM$ is $\cos(50°) = \frac{BM}{6}$

$BM = 6\cos(50°) = 3{,}85...$

$AB = 2BM = 7{,}71...$

In $\triangle ABC$ is $\tan(\angle BAC) = \frac{BC}{AB} = \frac{4{,}59...}{7{,}71...}$

$\angle BAC \approx 30{,}8°$

$\angle AMC = 180° - \angle BMC = 180° - 50° = 130°$

$\angle ACM = 180° - \angle AMC - \angle MAC \approx 180° - 130° - 30{,}8° = 19{,}2°$

7 In $\triangle ABD$ is $\tan(20°) = \dfrac{BD}{10}$

$BD = 10\tan(20°) = 3{,}63...$

$BE = 2BD = 7{,}27...$

In $\triangle ABE$ is $\tan(\angle BAE) = \dfrac{BE}{AB} = \dfrac{7{,}27...}{10}$

$\angle BAE = 36{,}05...°$

$BC = 2BE = 14{,}55...$

In $\triangle ABC$ is $\tan(\angle BAC) = \dfrac{BC}{AB} = \dfrac{14{,}55...}{10}$

$\angle BAC = 55{,}51...°$

$\angle CAE = \angle BAC - \angle BAE = 55{,}51...° - 36{,}05...° \approx 19{,}5°$

8 **a** Zie de figuur hiernaast.

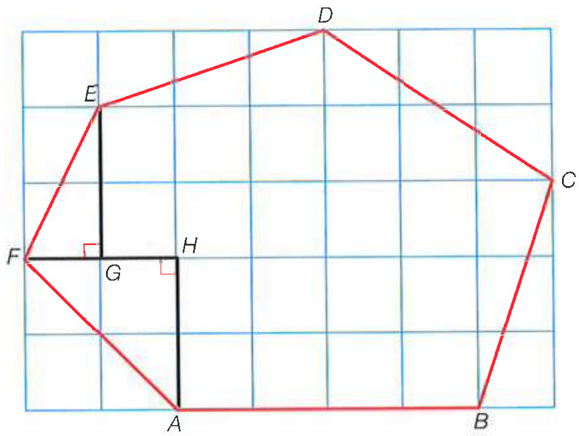
In $\triangle EFG$ is $\tan(\angle EFG) = \dfrac{EG}{FG} = \dfrac{2}{1} = 2$

$\angle EFG = 63{,}43...°$

In $\triangle AFH$ is $\tan(\angle AFH) = \dfrac{AH}{FH} = \dfrac{2}{2} = 1$

$\angle AFH = 45°$

$\angle F = \angle EFG + \angle AFH = 63{,}43...° + 45° \approx 108{,}4°$



b Zie de figuur hiernaast.

In $\triangle ADP$ is $\tan(\angle PAD) = \dfrac{DP}{AP} = \dfrac{5}{2}$

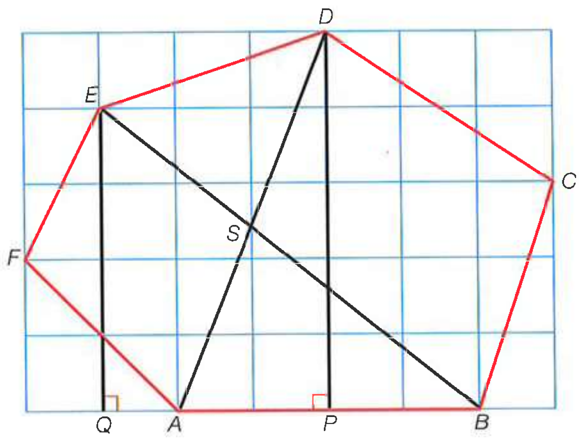
$\angle PAD = 68{,}19...°$

In $\triangle BEQ$ is $\tan(\angle QBE) = \dfrac{EQ}{BQ} = \dfrac{4}{5}$

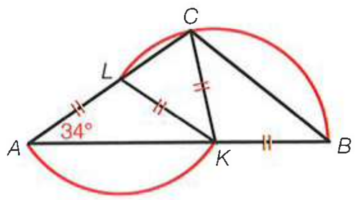
$\angle QBE = 38{,}65...°$

$\angle ASB = 180° - \angle SAB - \angle SBA$

$= 180° - 68{,}19...° - 38{,}65...° \approx 73{,}1°$



9 A en K liggen op de cirkel met middelpunt L, dus $AL = KL$.
Dus driehoek AKL is gelijkbenig.
B, C en L liggen op de cirkel met middelpunt K, dus $KB = KC = KL$.
Dus de driehoeken CKL en BCK zijn gelijkbenig.
In de berekening wordt gebruikt dat de basishoeken van een gelijkbenige driehoek gelijk zijn.



$\angle AKL = \angle KAL = 34°$

$\angle ALK = 180° - \angle AKL - \angle KAL = 180° - 2 \cdot 34° = 112°$

$\angle CLK = 180° - \angle ALK = 180° - 112° = 68°$

$\angle LCK = \angle CLK = 68°$

$\angle LKC = 180° - \angle CLK - \angle LCK = 180° - 2 \cdot 68° = 44°$

$\angle BKC = 180° - \angle LKC - \angle AKL = 180° - 44° - 34° = 102°$

$$\left.\begin{array}{l}\angle KBC = \angle KCB \\ \angle KBC + \angle KCB = 180° - 102° = 78°\end{array}\right\} \angle KBC = 39°$$

Dus $\angle B = 39°$.

Bladzijde 103

10 De vergrotingsfactor is $\frac{AB}{AP} = \frac{4+2}{4} = 1\frac{1}{2}$.

$AC = 1\frac{1}{2}AQ = 1\frac{1}{2} \cdot 3 = 4\frac{1}{2}$

$BC = 1\frac{1}{2}PQ = 1\frac{1}{2} \cdot 6 = 9$

Bladzijde 105

11 **a** $KM = \sqrt{17^2 - 15^2} = 8$

b Zie de figuur hiernaast met parallellogram $ABQD$.

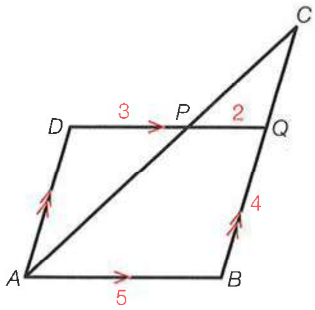
$\left.\begin{array}{l}\angle ADP = \angle CQP \text{ (Z-hoeken)}\\ \angle APD = \angle CPQ \text{ (overstaande hoeken)}\end{array}\right\} \triangle ADP \sim \triangle CQP$

AD	DP
CQ	QP

geeft

4	3
CQ	2

$CQ = \frac{2 \cdot 4}{3} = 2\frac{2}{3}$



12 $\left.\begin{array}{l}\angle PAQ = \angle BAC\\ \angle APQ = \angle ABC \text{ (rechte hoek)}\end{array}\right\} \triangle APQ \sim \triangle ABC$

De stelling van Pythagoras in $\triangle ABC$ geeft $AC = \sqrt{4^2 + 3^2} = 5$.

AP	AQ	PQ
AB	AC	BC

geeft

3	AQ	PQ
4	5	3

$PQ = \frac{3 \cdot 3}{4} = 2\frac{1}{4}$

$AQ = \frac{3 \cdot 5}{4} = 3\frac{3}{4}$, dus $CQ = 5 - 3\frac{3}{4} = 1\frac{1}{4}$.

Bladzijde 106

13 $\left.\begin{array}{l}\angle DCE = \angle FBE \text{ (rechte hoek)}\\ \angle DEC = \angle FEB \text{ (overstaande hoeken)}\end{array}\right\} \triangle CDE \sim \triangle BFE$

CD	CE
BF	BE

geeft

4	2
BF	3

$BF = \frac{3 \cdot 4}{2} = 6$

14 $\left.\begin{array}{l}\angle FAD = \angle FBE \text{ (rechte hoek)}\\ \angle AFD = \angle BFE\end{array}\right\} \triangle ADF \sim \triangle BEF$

AD	AF
BE	BF

geeft

5	9
BE	5

$BE = \frac{5 \cdot 5}{9} = 2\frac{7}{9}$

15 Zie de figuur hiernaast met hoogtelijn CD.

$\left.\begin{array}{l}\angle MBN = \angle CBD\\ \angle BNM = \angle BDC \text{ (rechte hoek)}\end{array}\right\} \triangle BMN \sim \triangle BCD$

De stelling van Pythagoras in $\triangle BCD$ geeft $CD = \sqrt{6^2 - 2^2} = \sqrt{32}$.

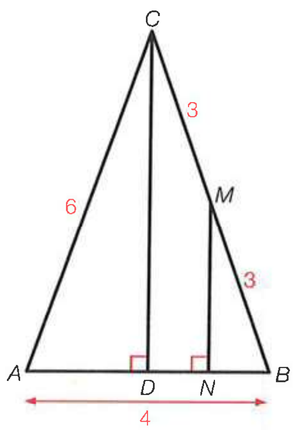
BM	MN	BN
BC	CD	BD

geeft

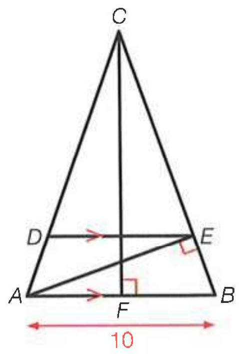
3	MN	BN
6	$\sqrt{32}$	2

$MN = \frac{3 \cdot \sqrt{32}}{6} \approx 2{,}83$

$BN = \frac{2 \cdot 3}{6} = 1$, dus $AN = 4 - 1 = 3$.



16 Zie de figuur hiernaast met hoogtelijn CF.



$$\left.\begin{array}{l} \angle FBC = \angle EBA \\ \angle BFC = \angle BEA \text{ (rechte hoek)} \end{array}\right\} \triangle BCF \sim \triangle BAE$$

$$\begin{array}{c|c} BC & BF \\ \hline AB & BE \end{array} \text{ geeft } \begin{array}{c|c} 15 & 5 \\ \hline 10 & BE \end{array}$$

$$BE = \frac{5 \cdot 10}{15} = 3\tfrac{1}{3}$$

$$CE = BC - BE = 15 - 3\tfrac{1}{3} = 11\tfrac{2}{3}$$

$$\left.\begin{array}{l} \angle ACB = \angle DCE \\ \angle CAB = \angle CDE \text{ (F-hoeken)} \end{array}\right\} \triangle ABC \sim \triangle DEC$$

$$\begin{array}{c|c} AB & BC \\ \hline DE & CE \end{array} \text{ geeft } \begin{array}{c|c} 10 & 15 \\ \hline DE & 11\tfrac{2}{3} \end{array}$$

$$DE = \frac{10 \cdot 11\tfrac{2}{3}}{15} = 7\tfrac{7}{9}$$

Bladzijde 107

17 **a** $\left.\begin{array}{l} \angle BAC = \angle DAE \\ \angle ABC = \angle ADE \text{ (rechte hoek)} \end{array}\right\} \triangle ABC \sim \triangle ADE$ dus $\begin{array}{c|c} AB & AC \\ \hline AD & AE \end{array}$

De stelling van Pythagoras in $\triangle ABC$ geeft $AC = \sqrt{20^2 + 15^2} = 25$.

$$\begin{array}{c|c} 20 & 25 \\ \hline 8 & AE \end{array} \text{ geeft } AE = \frac{8 \cdot 25}{20} = 10$$

b De stelling van Pythagoras in $\triangle ADE$ geeft $DE = \sqrt{10^2 - 8^2} = 6$.

De stelling van Pythagoras in $\triangle BCD$ geeft $CD = \sqrt{12^2 + 15^2} = \sqrt{369}$.

$\left.\begin{array}{l} \angle EDC = \angle BCD \text{ (Z-hoeken)} \\ \angle DEB = \angle CBE \text{ (Z-hoeken)} \end{array}\right\} \triangle DES \sim \triangle CBS$ dus $\begin{array}{c|c} DE & DS \\ \hline BC & CS \end{array}$

Stel $DS = x$, dan is $CS = \sqrt{369} - x$.

Dit geeft $\begin{array}{c|c} 6 & x \\ \hline 15 & \sqrt{369} - x \end{array}$

$$15x = 6(\sqrt{369} - x)$$

$$15x = 6\sqrt{369} - 6x$$

$$21x = 6\sqrt{369}$$

$$x = \frac{6\sqrt{369}}{21} \approx 5{,}49$$

Dus $DS \approx 5{,}49$.

18 **a** $\left.\begin{array}{l}\angle ABC = \angle DBE \\ \angle ACB = \angle DEB \text{ (F-hoeken)}\end{array}\right\}$ $\triangle ABC \sim \triangle DBE$ dus $\begin{array}{c|c} AB & BC \\ \hline BD & BE \end{array}$

Stel $BD = x$, dan is $AB = x + 7$.

Dit geeft $\begin{array}{c|c} x+7 & 10 \\ \hline x & 4 \end{array}$

$10x = 4(x + 7)$
$10x = 4x + 28$
$6x = 28$
$x = 4\frac{2}{3}$

Dus $BD = 4\frac{2}{3}$ en $AB = 11\frac{2}{3}$.

$\left.\begin{array}{l} AC^2 + BC^2 = 6^2 + 10^2 = 136 \\ AB^2 = (11\frac{2}{3})^2 = 136\frac{1}{9}\end{array}\right\}$ $AC^2 + BC^2 \neq AB^2$, dus $\angle C \neq 90°$.

b Uit $\triangle ABC \sim \triangle DBE$ volgt $\begin{array}{c|c} AC & BC \\ \hline DE & BE \end{array}$ oftewel $\begin{array}{c|c} 6 & 10 \\ \hline DE & 4 \end{array}$.

Dit geeft $DE = \dfrac{4 \cdot 6}{10} = 2\frac{2}{5}$.

$\left.\begin{array}{l}\angle CAE = \angle DEA \text{ (Z-hoeken)} \\ \angle ACD = \angle EDC \text{ (Z-hoeken)}\end{array}\right\}$ $\triangle ACS \sim \triangle EDS$

$\begin{array}{c|c} AC & AS \\ \hline DE & ES \end{array}$ geeft $\begin{array}{c|c} 6 & AS \\ \hline 2\frac{2}{5} & ES \end{array}$

Hieruit volgt $AS : ES = 6 : 2\frac{2}{5} = 30 : 12 = 5 : 2$.

Dus AS is $\frac{5}{7}$ deel van AE.

19 De diagonalen in een rechthoek delen elkaar middendoor, dus $DQ = \frac{1}{2}BD$.

$\left.\begin{array}{l}\angle ABD = \angle CDB \text{ (Z-hoeken)} \\ \angle APB = \angle DPM \text{ (overstaande hoeken)}\end{array}\right\}$ $\triangle ABP \sim \triangle MDP$

$\begin{array}{c|c} AB & BP \\ \hline DM & DP \end{array}$ en $DM = \frac{1}{2}CD = \frac{1}{2}AB$ geeft $DP = \frac{1}{2}BP$.

Hieruit volgt $DP : BP = 1 : 2$, en dus $DP = \frac{1}{3}BD$.

$PQ = DQ - DP = \frac{1}{2}BD - \frac{1}{3}BD = \frac{1}{6}BD$

3.2 De sinusregel en de cosinusregel

Bladzijde 109

20 **a** $\angle C = 180° - 48° - 76° = 56°$

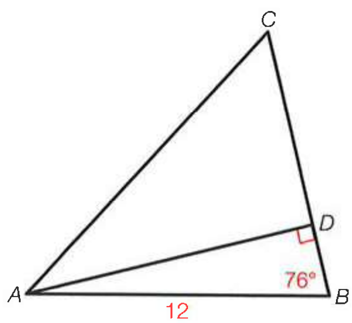
b Zie de figuur hiernaast met hoogtelijn AD.

In $\triangle ABD$ is $\sin(76°) = \dfrac{AD}{12}$

$AD = 12\sin(76°) = 11{,}64...$

In $\triangle ADC$ is $\sin(56°) = \dfrac{11{,}64...}{AC}$

$AC = \dfrac{11{,}64...}{\sin(56°)} \approx 14{,}04$



Bladzijde 110

21 **a** $\angle C = 180° - 50° - 75° = 55°$

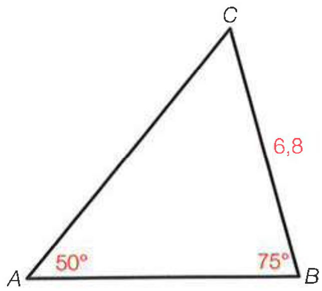
b De sinusregel geeft $\dfrac{6{,}8}{\sin(50°)} = \dfrac{AB}{\sin(55°)}$

$$AB = \frac{6{,}8\sin(55°)}{\sin(50°)} \approx 7{,}3$$

en ook

$$\frac{6{,}8}{\sin(50°)} = \frac{AC}{\sin(75°)}$$

$$AC = \frac{6{,}8\sin(75°)}{\sin(50°)} \approx 8{,}6$$



22 **a** In $\triangle ACD$ is $\sin(\angle A) = \dfrac{CD}{b}$, dus $CD = b\sin(\angle A)$.

b In $\triangle BCD$ is $\sin(\angle B) = \dfrac{CD}{a}$, dus $CD = a\sin(\angle B)$.

c Uit a en b volgt $a\sin(\angle B) = b\sin(\angle A)$.

Linker- en rechterlid delen door $\sin(\angle A) \cdot \sin(\angle B)$ geeft $\dfrac{a}{\sin(\angle A)} = \dfrac{b}{\sin(\angle B)}$.

d In $\triangle ABE$ is $\sin(\angle B) = \dfrac{AE}{c}$, dus $AE = c\sin(\angle B)$.

e In $\triangle ACE$ is $\sin(\angle C) = \dfrac{AE}{b}$, dus $AE = b\sin(\angle C)$.

f Uit e en f volgt $b\sin(\angle C) = c\sin(\angle B)$.

Linker- en rechterlid delen door $\sin(\angle B) \cdot \sin(\angle C)$ geeft $\dfrac{b}{\sin(\angle B)} = \dfrac{c}{\sin(\angle C)}$.

g $$\left.\begin{array}{l} \dfrac{a}{\sin(\angle A)} = \dfrac{b}{\sin(\angle B)} \\ \dfrac{b}{\sin(\angle B)} = \dfrac{c}{\sin(\angle C)} \end{array}\right\} \text{dus } \frac{a}{\sin(\angle A)} = \frac{b}{\sin(\angle B)} = \frac{c}{\sin(\angle C)}$$

23 **a** In $\triangle ACD$ is $\sin(\angle DAC) = \sin(180° - \angle BAC) = \dfrac{CD}{b}$, dus $CD = b\sin(180° - \angle BAC)$.

b In $\triangle BCD$ is $\sin(\angle B) = \dfrac{CD}{a}$, dus $CD = a\sin(\angle B)$.

c Uit a en b volgt $b\sin(180° - \angle BAC) = a\sin(\angle B)$.

Linker- en rechterlid delen door $\sin(180° - \angle BAC) \cdot \sin(\angle B)$ geeft $\dfrac{a}{\sin(180° - \angle BAC)} = \dfrac{b}{\sin(\angle B)}$.

Bladzijde 111

24 $\angle M = 180° - 20° - 110° = 50°$

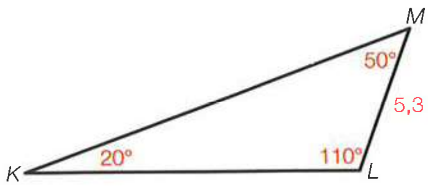
De sinusregel geeft $\dfrac{5{,}3}{\sin(20°)} = \dfrac{KL}{\sin(50°)}$

$$KL = \frac{5{,}3\sin(50°)}{\sin(20°)} \approx 11{,}9$$

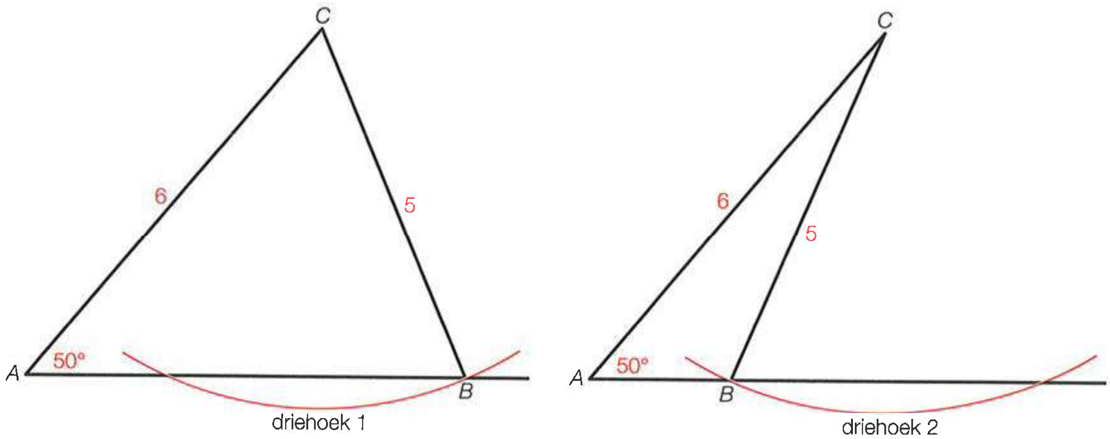
en ook

$$\frac{5{,}3}{\sin(20°)} = \frac{KM}{\sin(110°)}$$

$$KM = \frac{5{,}3\sin(110°)}{\sin(20°)} \approx 14{,}6$$



25 **a**



b De sinusregel in driehoek 1 geeft $\frac{5}{\sin(50°)} = \frac{6}{\sin(\angle B)}$

$$\sin(\angle B) = \frac{6\sin(50°)}{5} = 0{,}91...$$

$$\angle B \approx 66{,}8°$$

$\angle C \approx 180° - 50° - 66{,}8° = 63{,}2°$

De sinusregel in driehoek 1 geeft $\frac{5}{\sin(50°)} = \frac{AB}{\sin(63{,}1...°)}$

$$AB = \frac{5\sin(63{,}1...°)}{\sin(50°)} \approx 5{,}8$$

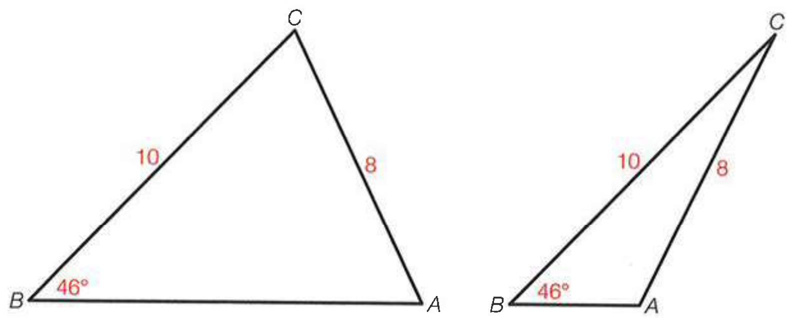
c $\angle B \approx 180° - 66{,}8° = 113{,}2°$

$\angle C \approx 180° - 50° - 113{,}2° = 16{,}8°$

De sinusregel in driehoek 2 geeft $\frac{5}{\sin(50°)} = \frac{AB}{\sin(16{,}8...°)}$

$$AB = \frac{5\sin(16{,}8...°)}{\sin(50°)} \approx 1{,}9$$

26 **a**



De sinusregel in de scherphoekige driehoek geeft $\frac{8}{\sin(46°)} = \frac{10}{\sin(\angle A)}$

$$\sin(\angle A) = \frac{10\sin(46°)}{8}$$

$$\angle A = 64{,}04...° \approx 64{,}0°$$

In de stomphoekige driehoek is $\angle A = 180° - 64{,}04...° = 115{,}95...° \approx 116{,}0°$.

b In de scherphoekige driehoek is $\angle C = 180° - 46° - 64{,}04...° = 69{,}95...°$.

De sinusregel in de scherphoekige driehoek geeft $\dfrac{8}{\sin(46°)} = \dfrac{AB}{\sin(69{,}95...°)}$

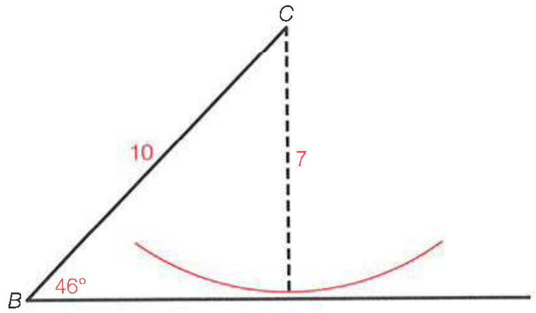
$$AB = \frac{8\sin(69{,}95...°)}{\sin(46°)} \approx 10{,}4$$

In de stomphoekige driehoek is $\angle C = 180° - 46° - 115{,}95...° = 18{,}04...°$.

De sinusregel in de stomphoekige driehoek geeft $\dfrac{8}{\sin(46°)} = \dfrac{AB}{\sin(18{,}04...°)}$

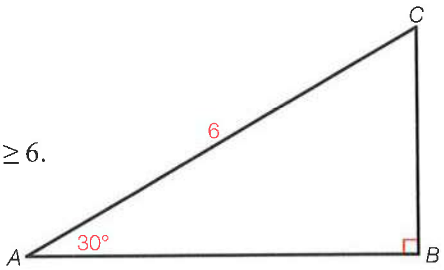
$$AB = \frac{8\sin(18{,}04...°)}{\sin(46°)} \approx 3{,}4$$

c



De cirkel met middelpunt C en straal 7 snijdt niet het andere been van hoek B.

27 **a** Zie de figuur hiernaast.



$\sin(30°) = \dfrac{BC}{6}$, dus $BC = 6\sin(30°) = 3$.

Als er geen driehoek ABC mogelijk is, dan is $BC < 3$.

b Als er precies één driehoek ABC mogelijk is, dan is $BC = 3 \vee BC \geq 6$.

c Als er twee driehoeken ABC mogelijk zijn, dan is $3 < BC < 6$.

Bladzijde 112

28 $\angle ADC = 180° - 62° - 48° = 70°$

De sinusregel in $\triangle ACD$ geeft $\dfrac{6}{\sin(62°)} = \dfrac{AD}{\sin(48°)}$

$$AD = \frac{6\sin(48°)}{\sin(62°)} = 5{,}04...$$

en ook

$$\frac{6}{\sin(62°)} = \frac{AC}{\sin(70°)}$$

$$AC = \frac{6\sin(70°)}{\sin(62°)} = 6{,}38...$$

$\angle BDC = 180° - \angle ADC = 180° - 70° = 110°$

$\angle DBC = 180° - 110° - 25° = 45°$

De sinusregel in $\triangle BCD$ geeft $\dfrac{6}{\sin(45°)} = \dfrac{BD}{\sin(25°)}$

$$BD = \frac{6\sin(25°)}{\sin(45°)} = 3{,}58...$$

en ook

$$\frac{6}{\sin(45°)} = \frac{BC}{\sin(110°)}$$

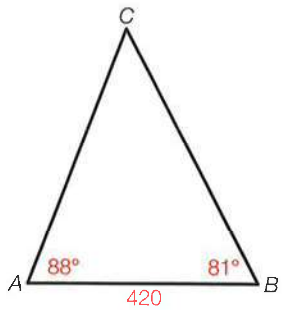
$$BC = \frac{6\sin(110°)}{\sin(45°)} = 7{,}97...$$

omtrek $\triangle ABC = AD + BD + BC + AC = 5{,}04... + 3{,}58... + 7{,}97... + 6{,}38... \approx 23{,}0$

29 $\angle ACB = 180° - 88° - 81° = 11°$

De sinusregel geeft $\dfrac{420}{\sin(11°)} = \dfrac{AC}{\sin(81°)}$

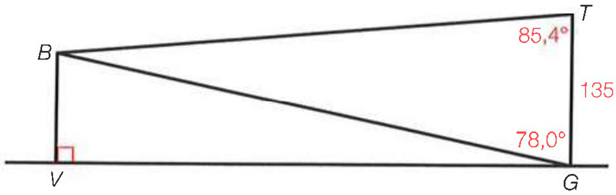
$$AC = \frac{420\sin(81°)}{\sin(11°)} \approx 2174 \text{ meter}$$



Bladzijde 113

30 **a** Zie de schets hiernaast.
BG wordt gevraagd.
$\angle GBT = 180° - 78{,}0° - 85{,}4° = 16{,}6°$
De sinusregel in $\triangle BGT$ geeft
$\dfrac{135}{\sin(16{,}6°)} = \dfrac{BG}{\sin(85{,}4°)}$

$BG = \dfrac{135\sin(85{,}4°)}{\sin(16{,}6°)} = 471{,}02...$



Bram bevindt zich op 471 meter van de top van de Big Ben als hij in het reuzenrad stapt.

b In $\triangle BGV$ is $\angle BGV = 90° - 78{,}0° = 12{,}0°$ en $\angle BVG = 90°$.
$\sin(\angle BGV) = \dfrac{BV}{BG}$ geeft $\sin(12{,}0°) = \dfrac{BV}{471{,}02...}$

$$BV = 471{,}02... \cdot \sin(12{,}0°) = 97{,}9...$$

De Big Ben is 98 meter hoog.

31 $\angle BCD = 180° - \angle BDC - \angle CBD = 180° - 110° - 40° = 30°$

De sinusregel in $\triangle BCD$ geeft $\dfrac{CD}{\sin(40°)} = \dfrac{4}{\sin(30°)}$

$$CD = \frac{4\sin(40°)}{\sin(30°)} = 5{,}14...$$

$\angle ACD = \angle ACB - \angle BCD = 50° - 30° = 20°$

De sinusregel in $\triangle ACD$ geeft $\dfrac{5{,}14...}{\sin(\angle CAD)} = \dfrac{5}{\sin(20°)}$

$$\sin(\angle CAD) = \frac{5{,}14... \cdot \sin(20°)}{5} = 0{,}35...$$

$$\angle CAD = 20{,}59...°$$

$\angle ADC = 180° - \angle DAC - \angle ACD = 180° - 20{,}59...° - 20° = 139{,}40...°$

De sinusregel in $\triangle ACD$ geeft $\dfrac{AC}{\sin(139{,}40...°)} = \dfrac{5}{\sin(20°)}$

$$AC = \frac{5\sin(139{,}40...°)}{\sin(20°)} \approx 9{,}51$$

32 **a** De sinusregel geeft $\dfrac{4}{\sin(\angle A)} = \dfrac{5}{\sin(\angle B)} = \dfrac{6}{\sin(\angle C)}$.
Bij elke combinatie van twee breuken zijn er twee onbekenden.
Je kunt de sinusregel dus niet gebruiken.

b De sinusregel geeft $\dfrac{QR}{\sin(50°)} = \dfrac{5}{\sin(\angle Q)} = \dfrac{6}{\sin(\angle R)}$.
Bij elke combinatie van twee breuken zijn er twee onbekenden.
Je kunt de sinusregel dus niet gebruiken.

Bladzijde 115

33 **a** De cosinusregel geeft

$5^2 = 7^2 + 6^2 - 2 \cdot 7 \cdot 6 \cdot \cos(\angle A)$

$25 = 49 + 36 - 84\cos(\angle A)$

$84\cos(\angle A) = 60$

$\cos(\angle A) = \dfrac{60}{84}$

$\angle A = 44{,}41...^\circ \approx 44{,}4^\circ$

en ook

$6^2 = 7^2 + 5^2 - 2 \cdot 7 \cdot 5 \cdot \cos(\angle B)$

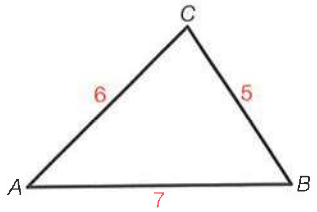
$36 = 49 + 25 - 70\cos(\angle B)$

$70\cos(\angle B) = 38$

$\cos(\angle B) = \dfrac{38}{70}$

$\angle B = 57{,}12...^\circ \approx 57{,}1^\circ$

$\angle C = 180^\circ - 44{,}41...^\circ - 57{,}12...^\circ \approx 78{,}5^\circ$



b De cosinusregel geeft

$4^2 = 5^2 + 7^2 - 2 \cdot 5 \cdot 7 \cdot \cos(\angle D)$

$16 = 25 + 49 - 70\cos(\angle D)$

$70\cos(\angle D) = 58$

$\cos(\angle D) = \dfrac{58}{70}$

$\angle D = 34{,}04...^\circ \approx 34{,}0^\circ$

en ook

$7^2 = 5^2 + 4^2 - 2 \cdot 5 \cdot 4 \cdot \cos(\angle E)$

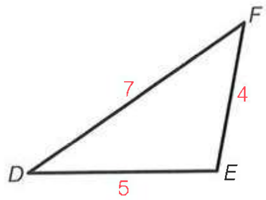
$49 = 25 + 16 - 40\cos(\angle E)$

$40\cos(\angle E) = -8$

$\cos(\angle E) = \dfrac{-8}{40}$

$\angle E = 101{,}53...^\circ \approx 101{,}5^\circ$

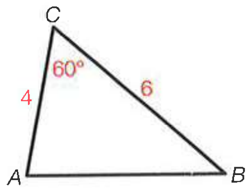
$\angle F = 180^\circ - 101{,}53...^\circ - 34{,}04...^\circ \approx 44{,}4^\circ$



34 **a** De cosinusregel geeft $AB^2 = 4^2 + 6^2 - 2 \cdot 4 \cdot 6 \cdot \cos(60^\circ)$

$AB^2 = 28$

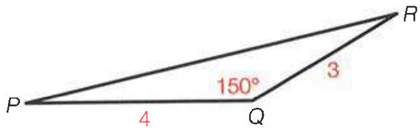
$AB \approx 5{,}29$



b De cosinusregel geeft $PR^2 = 4^2 + 3^2 - 2 \cdot 4 \cdot 3 \cdot \cos(150^\circ)$

$PR^2 = 45{,}78...$

$PR \approx 6{,}77$



35 De cosinusregel in $\triangle ABD$ geeft $BD^2 = 10^2 + 6^2 - 2 \cdot 10 \cdot 6 \cdot \cos(50^\circ)$

$BD^2 = 58{,}86...$

$BD = 7{,}67...$

De sinusregel in $\triangle BCD$ geeft $\dfrac{4}{\sin(\angle CBD)} = \dfrac{7{,}67...}{\sin(125^\circ)}$

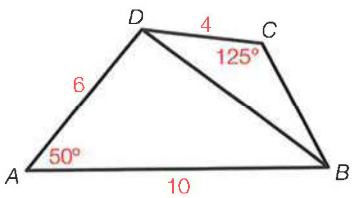
$\sin(\angle CBD) = \dfrac{4\sin(125^\circ)}{7{,}67...} = 0{,}42...$

$\angle CBD = 25{,}28...^\circ$

$\angle BDC = 180^\circ - \angle BCD - \angle CBD = 180^\circ - 125^\circ - 25{,}28...^\circ = 29{,}71...^\circ$

De sinusregel in $\triangle BCD$ geeft $\dfrac{BC}{\sin(29{,}71...^\circ)} = \dfrac{4}{\sin(25{,}28...^\circ)}$

$BC = \dfrac{4\sin(29{,}71...^\circ)}{\sin(25{,}28...^\circ)} \approx 4{,}6$



36 **a** De stelling van Pythagoras in $\triangle ACD$ geeft $AD^2 + CD^2 = AC^2$
$$x^2 + h^2 = b^2$$

b De stelling van Pythagoras in $\triangle BCD$ geeft $BC^2 = BD^2 + CD^2$
$$a^2 = (c - x)^2 + h^2$$
$$a^2 = c^2 - 2cx + x^2 + h^2$$

c Vervangen van $x^2 + h^2$ door b^2 in $a^2 = c^2 - 2cx + x^2 + h^2$ geeft $a^2 = c^2 - 2cx + b^2$, oftewel $a^2 = c^2 + b^2 - 2cx$.

d In $\triangle ACD$ is $\cos(\angle A) = \frac{x}{b}$, dus $x = b\cos(\angle A)$.

e Vervangen van x door $b\cos(\angle A)$ in $a^2 = c^2 + b^2 - 2cx$ geeft $a^2 = c^2 + b^2 - 2cb\cos(\angle A)$, oftewel $a^2 = b^2 + c^2 - 2bc\cos(\angle A)$.

Bladzijde 116

37 **a** De stelling van Pythagoras in $\triangle ACD$ geeft $AD^2 + CD^2 = AC^2$
$$x^2 + h^2 = b^2$$
De stelling van Pythagoras in $\triangle BCD$ geeft $BC^2 = BD^2 + CD^2$
$$a^2 = (c + x)^2 + h^2$$
$$a^2 = c^2 + 2cx + x^2 + h^2$$

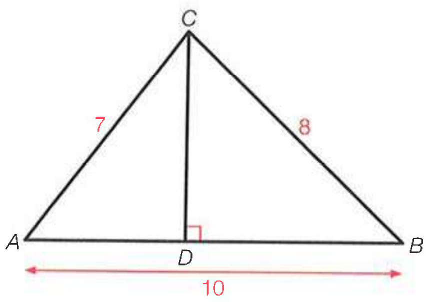
b Vervangen van $x^2 + h^2$ door b^2 in $a^2 = c^2 + 2cx + x^2 + h^2$ geeft $a^2 = c^2 + 2cx + b^2$, oftewel $a^2 = b^2 + c^2 + 2cx$.

c In $\triangle ACD$ is $\angle CAD = 180° - \angle CAB$ en $\cos(\angle CAD) = \frac{AD}{AC}$.
Dus $\cos(180° - \angle CAB) = \frac{x}{b}$ oftewel $x = b\cos(180° - \angle CAB)$.

d $\left.\begin{array}{l} a^2 = b^2 + c^2 + 2cx \\ x = -b\cos(\angle CAB) \end{array}\right\} a^2 = b^2 + c^2 - 2bc\cos(\angle CAB)$

Dus de cosinusregel geldt ook voor stomphoekige driehoeken.

38 De cosinusregel in $\triangle ABC$ geeft
$8^2 = 7^2 + 10^2 - 2 \cdot 7 \cdot 10 \cdot \cos(\angle A)$
$64 = 49 + 100 - 140\cos(\angle A)$
$140\cos(\angle A) = 85$
$\cos(\angle A) = \frac{85}{140}$
$\angle A = 52{,}61...°$
In $\triangle ACD$ is $\sin(52{,}61...°) = \frac{CD}{7}$, dus $CD = 7\sin(52{,}61...°) \approx 5{,}56$.



39 De cosinusregel in $\triangle ABD$ geeft $4^2 = 5^2 + 2^2 - 2 \cdot 5 \cdot 2 \cdot \cos(\angle B)$
$$16 = 25 + 4 - 20\cos(\angle B)$$
$$20\cos(\angle B) = 13$$
$$\cos(\angle B) = \tfrac{13}{20}$$
De cosinusregel in $\triangle ABC$ geeft $AC^2 = 5^2 + (6\tfrac{1}{2})^2 - 2 \cdot 5 \cdot 6\tfrac{1}{2} \cdot \tfrac{13}{20}$
$$AC^2 = 25$$
$$AC = 5$$

3.3 Vergelijkingen in de meetkunde

Bladzijde 118

40 II, III, IV, V, VI, VIII

Bladzijde 119

41 **a** $3a\sqrt{2} \cdot a\sqrt{7} = 3a^2\sqrt{14}$

b $\frac{5\sqrt{10}}{\sqrt{5}} = 5\sqrt{2}$

c $(3 - \sqrt{2})^2 = 9 - 2 \cdot 3 \cdot \sqrt{2} + 2 = 11 - 6\sqrt{2}$

d $\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$

e $(\frac{1}{2}x\sqrt{2})^2 = (\frac{1}{2}x)^2 \cdot (\sqrt{2})^2 = \frac{1}{4}x^2 \cdot 2 = \frac{1}{2}x^2$

f $\frac{6}{5\sqrt{2}} = \frac{6}{5\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{10} = \frac{3}{5}\sqrt{2}$

42 **a** $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1}{3}\sqrt{3}$

b $\frac{2\sqrt{14}}{3\sqrt{7}} = \frac{2}{3}\sqrt{2}$

c $\sqrt{4\frac{1}{2}} = \sqrt{\frac{9}{2}} = \frac{\sqrt{9}}{\sqrt{2}} = \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2} = 1\frac{1}{2}\sqrt{2}$

d $(\frac{1}{2}\sqrt{5})^2 = (\frac{1}{2})^2 \cdot (\sqrt{5})^2 = \frac{1}{4} \cdot 5 = 1\frac{1}{4}$

e $\frac{1}{2}a\sqrt{2} \cdot \frac{1}{2}a\sqrt{3} = \frac{1}{4}a^2\sqrt{6}$

f $(\frac{2}{3}a\sqrt{3})^2 = (\frac{2}{3}a)^2 \cdot (\sqrt{3})^2 = \frac{4}{9}a^2 \cdot 3 = 1\frac{1}{3}a^2$

43 **a** $2\sqrt{3}(4\sqrt{3} + 2\sqrt{5}) = 8\sqrt{9} + 4\sqrt{15} = 8 \cdot 3 + 4\sqrt{15} = 24 + 4\sqrt{15}$

b $3\sqrt{2}(3\sqrt{2} - \sqrt{3}) = 9\sqrt{4} - 3\sqrt{6} = 9 \cdot 2 - 3\sqrt{6} = 18 - 3\sqrt{6}$

c $2(\sqrt{5} - \sqrt{3}) \cdot \frac{1}{2}\sqrt{2} = (\sqrt{5} - \sqrt{3}) \cdot \sqrt{2} = \sqrt{10} - \sqrt{6}$

d $(\sqrt{6} - 3)^2 = 6 - 2 \cdot \sqrt{6} \cdot 3 + 9 = 15 - 6\sqrt{6}$

e $(2 + \sqrt{2})^2 = 4 + 2 \cdot 2 \cdot \sqrt{2} + 2 = 6 + 4\sqrt{2}$

f $(5 - \sqrt{3})(5 + \sqrt{3}) = 25 - 3 = 22$

44 **a** *

b $\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$

Bladzijde 120

45 **a** $5\sqrt{28} - 2\sqrt{63} = 5 \cdot 2\sqrt{7} - 2 \cdot 3\sqrt{7} = 10\sqrt{7} - 6\sqrt{7} = 4\sqrt{7}$

b $10\sqrt{27} - \frac{1}{2}\sqrt{48} = 10 \cdot 3\sqrt{3} - \frac{1}{2} \cdot 4\sqrt{3} = 30\sqrt{3} - 2\sqrt{3} = 28\sqrt{3}$

c $\sqrt{8} - \sqrt{\frac{1}{2}} = 2\sqrt{2} - \frac{1}{2}\sqrt{2} = 1\frac{1}{2}\sqrt{2}$

d $\sqrt{80} - \frac{10}{\sqrt{5}} = 4\sqrt{5} - \frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = 4\sqrt{5} - \frac{10\sqrt{5}}{5} = 4\sqrt{5} - 2\sqrt{5} = 2\sqrt{5}$

46 **a** $\sqrt{6a^2} = \sqrt{a^2} \cdot \sqrt{6} = a\sqrt{6}$

b $\sqrt{12a^2} + a\sqrt{3} = \sqrt{4a^2} \cdot \sqrt{3} + a\sqrt{3} = 2a\sqrt{3} + a\sqrt{3} = 3a\sqrt{3}$

c $\sqrt{54a^2} - \sqrt{24a^2} = \sqrt{9a^2} \cdot \sqrt{6} - \sqrt{4a^2} \cdot \sqrt{6} = 3a\sqrt{6} - 2a\sqrt{6} = a\sqrt{6}$

d $\sqrt{80a^2} - \sqrt{20a^2} = \sqrt{16a^2} \cdot \sqrt{5} - \sqrt{4a^2} \cdot \sqrt{5} = 4a\sqrt{5} - 2a\sqrt{5} = 2a\sqrt{5}$

47 **a** $\frac{15}{\sqrt{5}} + \sqrt{20} = \frac{15}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} + 2\sqrt{5} = \frac{15\sqrt{5}}{5} + 2\sqrt{5} = 3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$

b $2\sqrt{24} - \frac{8}{\sqrt{6}} = 2 \cdot 2\sqrt{6} - \frac{8}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = 4\sqrt{6} - \frac{8\sqrt{6}}{6} = 4\sqrt{6} - 1\frac{1}{3}\sqrt{6} = 2\frac{2}{3}\sqrt{6}$

c $\sqrt{2}(\sqrt{6} - \sqrt{3}) = \sqrt{12} - \sqrt{6} = 2\sqrt{3} - \sqrt{6}$

d $\frac{21}{\sqrt{3}} - \frac{1}{2}\sqrt{12} = \frac{21}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} - \frac{1}{2} \cdot 2\sqrt{3} = \frac{21\sqrt{3}}{3} - \sqrt{3} = 7\sqrt{3} - \sqrt{3} = 6\sqrt{3}$

e $a\sqrt{3}+\sqrt{\tfrac{3}{4}a^2}=a\sqrt{3}+\sqrt{\tfrac{1}{4}a^2}\cdot\sqrt{3}=a\sqrt{3}+\tfrac{1}{2}a\sqrt{3}=1\tfrac{1}{2}a\sqrt{3}$

f $a\sqrt{2}+\sqrt{\tfrac{1}{2}a^2}=a\sqrt{2}+\sqrt{\tfrac{1}{4}a^2}\cdot\sqrt{2}=a\sqrt{2}+\tfrac{1}{2}a\sqrt{2}=1\tfrac{1}{2}a\sqrt{2}$

Bladzijde 121

48 $\left.\begin{array}{l}\text{opp } \triangle ADE=\tfrac{3}{4}\cdot\text{opp } \triangle ABC=\tfrac{3}{4}\cdot\tfrac{1}{2}\cdot 15\cdot 8=45\\ \text{opp } \triangle ADE=\tfrac{1}{2}\cdot AD\cdot AE\end{array}\right\}\begin{array}{l}\tfrac{1}{2}\cdot AD\cdot AE=45\\ AD\cdot AE=90\end{array}$

De stelling van Pythagoras in $\triangle ABC$ geeft $BC=\sqrt{15^2+8^2}=17$.
omtrek $\triangle ADE$ = omtrek $BCED$ geeft $AD+AE+DE=BC+CE+DE+BD$

$$AD+AE+DE=BC+AC-AE+DE+AB-AD$$
$$2AD+2AE=BC+AC+AB$$
$$2AD+2AE=17+8+15$$
$$AD+AE=20$$

$AD+AE=20$ oftewel $AE=20-AD$
$AD\cdot AE=90$ en $AE=20-AD$ geeft $AD(20-AD)=90$

$$20AD-AD^2=90$$
$$AD^2-20AD+90=0$$
$$D=(-20)^2-4\cdot 1\cdot 90=40,\text{ dus } \sqrt{D}=\sqrt{40}=2\sqrt{10}$$
$$AD=\frac{20+2\sqrt{10}}{2}\vee AD=\frac{20-2\sqrt{10}}{2}$$
$$AD=10+\sqrt{10}\vee AD=10-\sqrt{10}$$

$AD=10+\sqrt{10}$ geeft $AE=20-(10+\sqrt{10})=10-\sqrt{10}$
$AD=10-\sqrt{10}$ geeft $AE=20-(10-\sqrt{10})=10+\sqrt{10}$ en dit kan niet.
De stelling van Pythagoras in $\triangle ADE$ geeft $DE^2=(10+\sqrt{10})^2+(10-\sqrt{10})^2$

$$DE^2=100+2\cdot 10\cdot\sqrt{10}+10+100-2\cdot 10\cdot\sqrt{10}+10$$
$$DE^2=220$$
$$DE=\sqrt{220}=\sqrt{4}\cdot\sqrt{55}=2\sqrt{55}$$

49 **a** $2x=16-8\sqrt{3}$
$x=8-4\sqrt{3}$

b $5x-10\sqrt{2}=50$
$5x=50+10\sqrt{2}$
$x=10+2\sqrt{2}$

Bladzijde 122

50 **a** $x\sqrt{5}+2=\sqrt{10}$

$x\sqrt{5}=-2+\sqrt{10}$

$x=\dfrac{-2}{\sqrt{5}}+\dfrac{\sqrt{10}}{\sqrt{5}}$

$x=\dfrac{-2}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}}+\sqrt{2}$

$x=\dfrac{-2\sqrt{5}}{5}+\sqrt{2}$

$x=-\tfrac{2}{5}\sqrt{5}+\sqrt{2}$

b $7-x\sqrt{3}=\sqrt{2}$

$x\sqrt{3}=7-\sqrt{2}$

$x=\dfrac{7}{\sqrt{3}}-\dfrac{\sqrt{2}}{\sqrt{3}}$

$x=\dfrac{7}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}-\dfrac{\sqrt{2}}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}$

$x=\dfrac{7\sqrt{3}}{3}-\dfrac{\sqrt{6}}{3}$

$x=2\tfrac{1}{3}\sqrt{3}-\tfrac{1}{3}\sqrt{6}$

c $\frac{1}{2}x\sqrt{2}+\sqrt{10}=0$

$\frac{1}{2}x\sqrt{2}=-\sqrt{10}$

$x\sqrt{2}=-2\sqrt{10}$

$x=\dfrac{-2\sqrt{10}}{\sqrt{2}}$

$x=-2\sqrt{5}$

d $x\sqrt{8}+x\sqrt{2}=12$

$x\cdot 2\sqrt{2}+x\sqrt{2}=12$

$2x\sqrt{2}+x\sqrt{2}=12$

$3x\sqrt{2}=12$

$x=\dfrac{12}{3\sqrt{2}}$

$x=\dfrac{12}{3\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}$

$x=\dfrac{12\sqrt{2}}{6}$

$x=2\sqrt{2}$

51 **a** $8x-2x\sqrt{3}=\sqrt{6}$

$x(8-2\sqrt{3})=\sqrt{6}$

$x=\dfrac{\sqrt{6}}{8-2\sqrt{3}}$

b $\frac{1}{2}x\sqrt{2}+x\sqrt{10}=10$

$x\sqrt{2}+2x\sqrt{10}=20$

$x(\sqrt{2}+2\sqrt{10})=20$

$x=\dfrac{20}{\sqrt{2}+2\sqrt{10}}$

c $x\sqrt{5}+x\sqrt{2}=\sqrt{10}$

$x(\sqrt{5}+\sqrt{2})=\sqrt{10}$

$x=\dfrac{\sqrt{10}}{\sqrt{5}+\sqrt{2}}$

d $x-\sqrt{2}=x\sqrt{3}$

$x-x\sqrt{3}=\sqrt{2}$

$x(1-\sqrt{3})=\sqrt{2}$

$x=\dfrac{\sqrt{2}}{1-\sqrt{3}}$

52 **a** $2x\sqrt{6}-6=4\sqrt{6}$

$2x\sqrt{6}=6+4\sqrt{6}$

$x=\dfrac{6}{2\sqrt{6}}+\dfrac{4\sqrt{6}}{2\sqrt{6}}$

$x=\dfrac{6}{2\sqrt{6}}\cdot\dfrac{\sqrt{6}}{\sqrt{6}}+2$

$x=\dfrac{6\sqrt{6}}{12}+2$

$x=\frac{1}{2}\sqrt{6}+2$

b $3x\sqrt{2}-2x=6$

$x(3\sqrt{2}-2)=6$

$x=\dfrac{6}{3\sqrt{2}-2}$

c $3\sqrt{2}+x\sqrt{2}=4\sqrt{10}$

$x\sqrt{2}=4\sqrt{10}-3\sqrt{2}$

$x=\dfrac{4\sqrt{10}}{\sqrt{2}}-\dfrac{3\sqrt{2}}{\sqrt{2}}$

$x=4\sqrt{5}-3$

d $4x\sqrt{3}+6=2x$

$4x\sqrt{3}-2x=-6$

$-2x(1-2\sqrt{3})=-6$

$x(1-2\sqrt{3})=3$

$x=\dfrac{3}{1-2\sqrt{3}}$

53 Bij 51d krijg je $x-\sqrt{2}=x\sqrt{3}$

$x-x\sqrt{3}=\sqrt{2}$

$x(1-\sqrt{3})=\sqrt{2}$

$x=\dfrac{\sqrt{2}}{1-\sqrt{3}}=\dfrac{\sqrt{2}}{1-\sqrt{3}}\cdot\dfrac{1+\sqrt{3}}{1+\sqrt{3}}=\dfrac{\sqrt{2}(1+\sqrt{3})}{1-3}=\dfrac{\sqrt{2}+\sqrt{6}}{-2}=-\frac{1}{2}\sqrt{2}-\frac{1}{2}\sqrt{6}$

Bij 52b krijg je $x=\dfrac{6}{3\sqrt{2}-2}\cdot\dfrac{3\sqrt{2}+2}{3\sqrt{2}+2}=\dfrac{6(3\sqrt{2}+2)}{18-4}=\dfrac{18\sqrt{2}+12}{14}=\frac{18}{14}\sqrt{2}+\frac{12}{14}=1\frac{2}{7}\sqrt{2}+\frac{6}{7}$.

Bij 52d krijg je $x=\dfrac{3}{1-2\sqrt{3}}\cdot\dfrac{1+2\sqrt{3}}{1+2\sqrt{3}}=\dfrac{3(1+2\sqrt{3})}{1-12}=\dfrac{3+6\sqrt{3}}{-11}=-\frac{3}{11}-\frac{6}{11}\sqrt{3}$.

54 **a** In $\triangle ACD$ is $CD^2 = AC^2 - AD^2$
$CD^2 = (2a)^2 - a^2$
$CD^2 = 4a^2 - a^2$
$CD^2 = 3a^2$
$CD = \sqrt{3a^2} = \sqrt{a^2} \cdot \sqrt{3} = a\sqrt{3}$

b $\left.\begin{array}{l} CD = a\sqrt{3} \\ CD = 7\sqrt{3} \end{array}\right\} \begin{array}{l} a\sqrt{3} = 7\sqrt{3} \\ a = 7 \end{array}$
$AC = 2a = 2 \cdot 7 = 14$

Bladzijde 124

55 $AC = \dfrac{AB}{\sqrt{2}} = \dfrac{8}{\sqrt{2}} = \dfrac{8}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{8\sqrt{2}}{2} = 4\sqrt{2}$

$DF = \dfrac{EF}{2} = \dfrac{6\sqrt{3}}{2} = 3\sqrt{3}$ en $DE = \sqrt{3} \cdot DF = \sqrt{3} \cdot 3\sqrt{3} = 3 \cdot 3 = 9$

$\left.\begin{array}{l} MN = \dfrac{KM}{2} = \dfrac{10}{2} = 5 \text{ en } KN = \sqrt{3} \cdot MN = 5\sqrt{3} \\ NL = MN = 5 \end{array}\right\} KL = KN + NL = 5\sqrt{3} + 5$

$QS = RS \cdot \sqrt{2} = 3\sqrt{2}$ en $PQ = QS \cdot \sqrt{3} = 3\sqrt{2} \cdot \sqrt{3} = 3\sqrt{6}$

56 **a** $\angle BDC = 180° - \angle BCD - \angle CBD = 180° - 90° - 75° = 15°$
$\angle BDE = \angle ADC - \angle BDC = 45° - 15° = 30°$
In $\triangle BDE$ is $DE = \sqrt{3} \cdot BE = \sqrt{3} \cdot 10 = 10\sqrt{3}$.
In $\triangle ABE$ is $AE = BE = 10$.
$AD = AE + DE = 10 + 10\sqrt{3}$

b In $\triangle ACD$ is $AC = \dfrac{AD}{\sqrt{2}} = \dfrac{10 + 10\sqrt{3}}{\sqrt{2}} = \dfrac{10 + 10\sqrt{3}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{10\sqrt{2} + 10\sqrt{6}}{2} = 5\sqrt{2} + 5\sqrt{6}$.

In $\triangle ABE$ is $AB = \sqrt{2} \cdot BE = \sqrt{2} \cdot 10 = 10\sqrt{2}$.

$BC = AC - AB = 5\sqrt{2} + 5\sqrt{6} - 10\sqrt{2} = 5\sqrt{6} - 5\sqrt{2}$

57 In $\triangle BCD$ is $BD = CD = \dfrac{BC}{\sqrt{2}} = \dfrac{6}{\sqrt{2}} = \dfrac{6}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{6\sqrt{2}}{2} = 3\sqrt{2}$.

In $\triangle ACD$ is $AD = \sqrt{3} \cdot CD = \sqrt{3} \cdot 3\sqrt{2} = 3\sqrt{6}$.

$O(\triangle ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot (AD + BD) \cdot CD = \frac{1}{2} \cdot (3\sqrt{6} + 3\sqrt{2}) \cdot 3\sqrt{2} = 4\frac{1}{2}\sqrt{12} + 9 = 4\frac{1}{2} \cdot 2\sqrt{3} + 9 = 9\sqrt{3} + 9$

58 **a** In $\triangle ACD$ is $CD = AD = x$.
In $\triangle BCD$ is $BD = \sqrt{3} \cdot CD = \sqrt{3} \cdot x = x\sqrt{3}$.

b $\left.\begin{array}{l} AB = AD + BD = x + x\sqrt{3} \\ AB = 10 \end{array}\right\} x + x\sqrt{3} = 10$

$x + x\sqrt{3} = 10$

$x(1 + \sqrt{3}) = 10$

$x = \dfrac{10}{1 + \sqrt{3}}$

c $O(\triangle ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot 10 \cdot \left(\dfrac{10}{1 + \sqrt{3}}\right) = \dfrac{50}{1 + \sqrt{3}}$

Bladzijde 125

59 In $\triangle ACD$ is $CD = \sqrt{3} \cdot AD = \sqrt{3} \cdot x = x\sqrt{3}$.

In $\triangle BCD$ is $BD = CD = x\sqrt{3}$.

$$\left.\begin{array}{l} AB = AD + BD = x + x\sqrt{3} \\ AB = 12 \end{array}\right\} x + x\sqrt{3} = 12$$

$$x(1+\sqrt{3}) = 12$$

$$x = \frac{12}{1+\sqrt{3}}$$

$$O(\triangle ABC) = \tfrac{1}{2} \cdot AB \cdot CD = \tfrac{1}{2} \cdot 12 \cdot \left(\frac{12}{1+\sqrt{3}} \cdot \sqrt{3}\right) = \frac{72\sqrt{3}}{1+\sqrt{3}}$$

60 **a** In $\triangle APW$ is $PW = \sqrt{2} \cdot AP = \sqrt{2} \cdot x = x\sqrt{2}$.

b Het is een regelmatige achthoek, dus $PQ = PW = x\sqrt{2}$.
$AB = AP + PQ + BQ = x + x\sqrt{2} + x = 2x + x\sqrt{2}$

c $2x + x\sqrt{2} = 6$
$x(2+\sqrt{2}) = 6$
$x = \dfrac{6}{2+\sqrt{2}}$
De zijde van de achthoek is $x\sqrt{2} = \dfrac{6}{2+\sqrt{2}} \cdot \sqrt{2} = \dfrac{6\sqrt{2}}{2+\sqrt{2}}$.

61 Stel $AP = x$, dan is ook $BQ = x$.
In $\triangle APS$ is $PS = \sqrt{3} \cdot AP = \sqrt{3} \cdot x = x\sqrt{3}$.
$PQRS$ is een vierkant, dus $PQ = PS = x\sqrt{3}$.

$$\left.\begin{array}{l} AB = AP + PQ + BQ = x + x\sqrt{3} + x = 2x + x\sqrt{3} \\ AB = 8 \end{array}\right\} 2x + x\sqrt{3} = 8$$

$$x(2+\sqrt{3}) = 8$$

$$x = \frac{8}{2+\sqrt{3}}$$

$$PQ = x\sqrt{3} = \frac{8}{2+\sqrt{3}} \cdot \sqrt{3} = \frac{8\sqrt{3}}{2+\sqrt{3}}$$

62 Bij 58c krijg je $O(\triangle ABC) = \dfrac{50}{1+\sqrt{3}} \cdot \dfrac{1-\sqrt{3}}{1-\sqrt{3}} = \dfrac{50(1-\sqrt{3})}{1-3} = \dfrac{50-50\sqrt{3}}{-2} = -25 + 25\sqrt{3}$.

Bij 59 krijg je $O(\triangle ABC) = \dfrac{72\sqrt{3}}{1+\sqrt{3}} \cdot \dfrac{1-\sqrt{3}}{1-\sqrt{3}} = \dfrac{72\sqrt{3}(1-\sqrt{3})}{1-3} = \dfrac{72\sqrt{3}-216}{-2} = 108 - 36\sqrt{3}$.

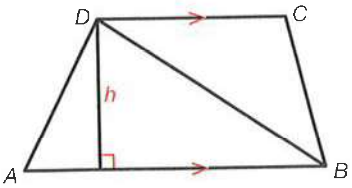
Bij 60c krijg je zijde $= \dfrac{6\sqrt{2}}{2+\sqrt{2}} \cdot \dfrac{2-\sqrt{2}}{2-\sqrt{2}} = \dfrac{6\sqrt{2}(2-\sqrt{2})}{4-2} = \dfrac{12\sqrt{2}-12}{2} = 6\sqrt{2} - 6$.

Bij 61 krijg je $PQ = \dfrac{8\sqrt{3}}{2+\sqrt{3}} \cdot \dfrac{2-\sqrt{3}}{2-\sqrt{3}} = \dfrac{8\sqrt{3}(2-\sqrt{3})}{4-3} = \dfrac{16\sqrt{3}-24}{1} = 16\sqrt{3} - 24$.

3.4 Lengte, omtrek en oppervlakte

Bladzijde 127

63 Zie de figuur hiernaast met diagonaal BD.



$$\left.\begin{array}{l} O(\triangle ABD) = \tfrac{1}{2} \cdot AB \cdot h \\ O(\triangle BCD) = \tfrac{1}{2} \cdot CD \cdot h \\ O(ABCD) = O(\triangle ABD) + O(\triangle BCD) \end{array}\right\}$$

$$O(ABCD) = \tfrac{1}{2} \cdot AB \cdot h + \tfrac{1}{2} \cdot CD \cdot h = \tfrac{1}{2}(AB + CD) \cdot h$$

Bladzijde 128

64 **a** $\sin(\angle ANC) = \dfrac{2\frac{1}{2}}{4}$

$\angle ANC = 38{,}68...°$

$\angle ANB = 2 \cdot 38{,}68...° = 77{,}36...°$

boog $AB = \dfrac{77{,}36...}{360} \cdot 2\pi \cdot 4 \approx 5{,}401$

b $O(\text{cirkelsector } MAB) = \dfrac{49{,}24...}{360} \cdot \pi \cdot 6^2 = 15{,}47...$

De stelling van Pythagoras in $\triangle ACM$ geeft $CM = \sqrt{6^2 - (2\frac{1}{2})^2} = \sqrt{29\frac{3}{4}}$.

$O(\triangle MAB) = \frac{1}{2} \cdot AB \cdot CM = \frac{1}{2} \cdot 5 \cdot \sqrt{29\frac{3}{4}} = 2\frac{1}{2}\sqrt{29\frac{3}{4}}$

$O(\text{cirkelsector } NAB) = \dfrac{77{,}36...}{360} \cdot \pi \cdot 4^2 = 10{,}80...$

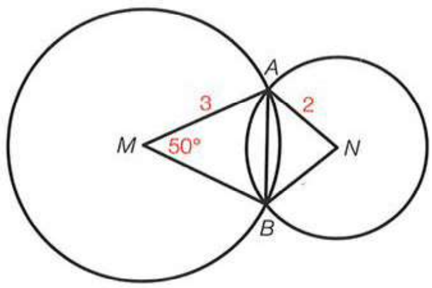
De stelling van Pythagoras in $\triangle ACN$ geeft $CN = \sqrt{4^2 - (2\frac{1}{2})^2} = \sqrt{9\frac{3}{4}}$.

$O(\triangle NAB) = \frac{1}{2} \cdot AB \cdot CN = \frac{1}{2} \cdot 5 \cdot \sqrt{9\frac{3}{4}} = 2\frac{1}{2}\sqrt{9\frac{3}{4}}$

De gevraagde oppervlakte is $15{,}47... - 2\frac{1}{2}\sqrt{29\frac{3}{4}} + 10{,}80... - 2\frac{1}{2}\sqrt{9\frac{3}{4}} \approx 4{,}83$.

65 Van de cirkel met middelpunt M en straal 3 is

kortste boog $AB = \dfrac{50}{360} \cdot 2\pi \cdot 3 = 2{,}61799...$



$\sin(25°) = \dfrac{\frac{1}{2}AB}{3}$, dus $\frac{1}{2}AB = 3\sin(25°)$.

$\sin(\frac{1}{2}\angle ANB) = \dfrac{3\sin(25°)}{2}$, dus $\frac{1}{2}\angle ANB = 39{,}34...°$.

$\angle ANB = 2 \cdot 39{,}34...° = 78{,}68...°$

Van de cirkel met middelpunt N en straal 2 is

kortste boog $AB = \dfrac{78{,}68...}{360} \cdot 2\pi \cdot 2 = 2{,}74648...$

Het gevraagde verschil in lengte is $2{,}74648... - 2{,}61799... \approx 0{,}1285$.

66 Stel de straal van de rode cirkel is r.

Dan is de straal van rode cirkel en de witte ring samen $2r$, en de straal van de grootste cirkel $3r$.

$O(\text{rode cirkel}) = \pi r^2$

$O(\text{rode cirkel en witte ring samen}) = \pi(2r)^2 = 4\pi r^2$

$O(\text{grootste cirkel}) = \pi(3r)^2 = 9\pi r^2$

$O(\text{blauwe ring}) = O(\text{grootste cirkel}) - O(\text{rode cirkel en witte ring samen}) = 9\pi r^2 - 4\pi r^2 = 5\pi r^2$

Dus de oppervlakte van de blauwe ring is 5 keer zo groot als de oppervlakte van de rode cirkel.

67 Stel de straal van de grote cirkel is r.

Dan is $O(\text{grote cirkel}) = \pi r^2$.

$\triangle ABM$ is gelijkbenig (want $MA = MB$) met tophoek $= 60°$, dus $\triangle ABM$ is gelijkzijdig en dus $AB = r$.

In $\triangle ABN$ is $NA = NB = \dfrac{r}{\sqrt{2}}$.

Dus $O(\text{kleine cirkel}) = \pi \cdot \left(\dfrac{r}{\sqrt{2}}\right)^2 = \pi \cdot \dfrac{r^2}{2} = \frac{1}{2}\pi r^2$.

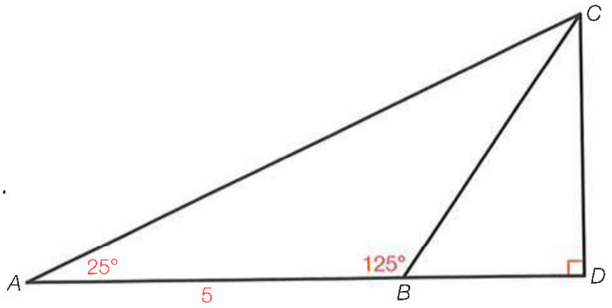
De verhouding van de oppervlakten is $\pi r^2 : \frac{1}{2}\pi r^2 = 2 : 1$.

68 Zie de figuur hiernaast.

$\angle ACB = 180° - 25° - 125° = 30°$

De sinusregel in $\triangle ABC$ geeft $\frac{5}{\sin(30°)} = \frac{BC}{\sin(25°)}$

$$BC = \frac{5\sin(25°)}{\sin(30°)} = 4{,}22...$$



$\angle CBD = 180° - \angle CBA = 180° - 125° = 55°$

In $\triangle BCD$ is $\sin(\angle CBD) = \frac{CD}{BC}$

$$\sin(55°) = \frac{CD}{4{,}22...}$$

$$CD = 4{,}22... \cdot \sin(55°) = 3{,}46...$$

$O(\triangle ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot 5 \cdot 3{,}46... \approx 8{,}65$

Bladzijde 129

69 Zie de figuur hiernaast.

De diagonalen AC en BD snijden elkaar in het punt S.

S met midden van AC en van BD.

De cosinusregel in $\triangle ABS$ geeft

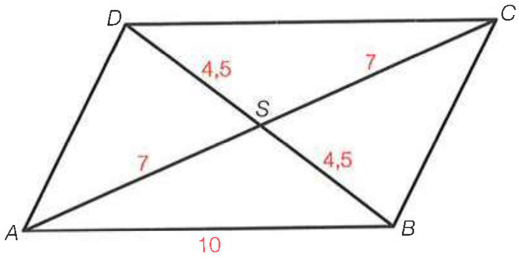
$4{,}5^2 = 10^2 + 7^2 - 2 \cdot 10 \cdot 7 \cdot \cos(\angle BAS)$

$20{,}25 = 100 + 49 - 140\cos(\angle BAS)$

$140\cos(\angle BAS) = 128{,}75$

$\cos(\angle BAS) = \frac{128{,}75}{140}$

$\angle BAS = 23{,}12...°$

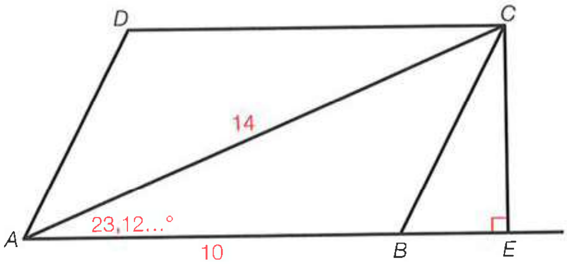


Zie de figuur hiernaast.

In $\triangle ACE$ is $\sin(23{,}12...°) = \frac{CE}{14}$

$$CE = 14\sin(23{,}12...°) = 5{,}49...$$

$O(ABCD) = AB \cdot CE = 10 \cdot 5{,}49... \approx 55{,}0$



70 Stel de hoogte van trapezium $ABCD$ is h.

$$\left.\begin{array}{l} O(ABCD) = \frac{1}{2}(AB + CD) \cdot h \\ AB = 4 \\ CD = 3 \\ O(ABCD) = 14 \end{array}\right\} \begin{array}{l} \frac{1}{2}(4+3) \cdot h = 14 \\ 3\frac{1}{2}h = 14 \\ h = 4 \end{array}$$

$AB \,//\, CD$ oftewel $BF \,//\, CG$ en $BF = CG = 2$, dus $BCGF$ is een parallellogram.

$O(BCGF) = BF \cdot h = 2 \cdot 4 = 8$

E is het midden van AD, dus de hoogte van $\triangle AEF$ is de helft van de hoogte van trapezium $ABCD$.

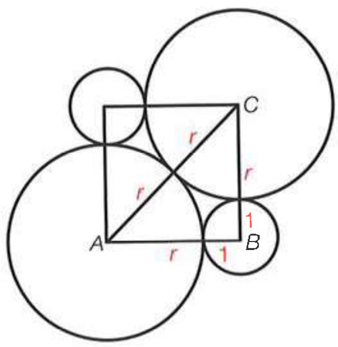
$O(\triangle AEF) = \frac{1}{2} \cdot AF \cdot \frac{1}{2}h = \frac{1}{2} \cdot 2 \cdot 2 = 2$

$O(DEFG) = O(ABCD) - O(BCGF) - O(\triangle AEF) = 14 - 8 - 2 = 4$

71 Stel de straal van een grote cirkel is r.

Zie de figuur.

In $\triangle ABC$ is $AC = 2r$ en $AB = \frac{2r}{\sqrt{2}} = \frac{2r}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2r\sqrt{2}}{2} = r\sqrt{2}$.



Ook is $AB = r + 1$.

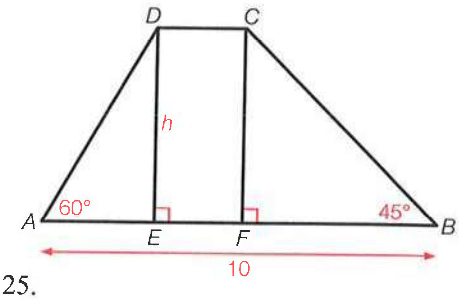
Dit geeft $r\sqrt{2} = r + 1$

$$r\sqrt{2} - r = 1$$

$$r(\sqrt{2} - 1) = 1$$

$$r = \frac{1}{\sqrt{2} - 1}$$

Dus de straal van de grote cirkels is $\frac{1}{\sqrt{2}-1}$.



72 **a** In $\triangle ADE$ is $AE = \frac{DE}{\sqrt{3}} = \frac{h}{\sqrt{3}}$.

In $\triangle BCF$ is $BF = CF = h$.

$CD = EF = AB - AE - BF = 10 - \frac{h}{\sqrt{3}} - h$

b $O(ABCD) = \frac{1}{2}(AB + CD) \cdot h = \frac{1}{2}\left(10 + 10 - \frac{h}{\sqrt{3}} - h\right) \cdot h$

Gegeven is $O(ABCD) = 25$, dus los op $\frac{1}{2}\left(10 + 10 - \frac{h}{\sqrt{3}} - h\right) \cdot h = 25$.

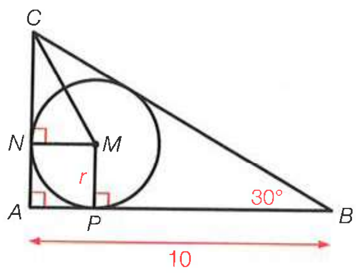
Voer in $y_1 = \frac{1}{2}\left(10 + 10 - \frac{x}{\sqrt{3}} - x\right) \cdot x$ en $y_2 = 25$.

De optie snijpunt geeft $x = 3{,}42...$ en $x = 9{,}25...$

$h = 3{,}42...$ geeft $CD = 10 - \frac{3{,}42...}{\sqrt{3}} - 3{,}42... \approx 4{,}60$

$h = 9{,}25...$ geeft $CD = 10 - \frac{9{,}25...}{\sqrt{3}} - 9{,}25... \approx -4{,}60$ kan niet

Dus $CD \approx 4{,}60$.



73 In $\triangle ABC$ is $AC = \frac{AB}{\sqrt{3}} = \frac{10}{\sqrt{3}}$.

$\angle ACB = 180° - 90° - 30° = 60°$

M is het middelpunt van de ingeschreven cirkel van $\triangle ABC$, dus CM is de bissectrice van $\angle ACB$, dus $\angle ACM = \frac{1}{2} \cdot 60° = 30°$.

Zie de figuur hiernaast met $MN \perp AC$.

In $\triangle CMN$ is $MN = r$ en $CN = r\sqrt{3}$.

$AN = MP = r$

$AC = AN + CN = r + r\sqrt{3}$

Dus moet gelden $r + r\sqrt{3} = \frac{10}{\sqrt{3}}$

$$r(1 + \sqrt{3}) = \frac{10}{\sqrt{3}}$$

$$r = \frac{\frac{10}{\sqrt{3}}}{1 + \sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{10}{\sqrt{3} + 3}$$

Diagnostische toets

Bladzijde 132

1 In $\triangle BCE$ is $\tan(55°) = \frac{5}{BE}$

$$BE = \frac{5}{\tan(55°)} = 3{,}50...$$

$AB = AE + BE = 3 + 3{,}50... = 6{,}50...$

In $\triangle ABC$ is $\tan(\angle BAC) = \frac{BC}{AB} = \frac{5}{6{,}50...}$

$\angle BAC \approx 37{,}6°$

$\angle AEC = 180° - 55° = 125°$

$\angle ACE \approx 180° - 125° - 37{,}6° = 17{,}4°$

2 **a** $\left.\begin{array}{l}\angle ABC = \angle DBE \\ \angle BAC = \angle BDE \text{ (rechte hoek)}\end{array}\right\} \triangle ABC \sim \triangle DBE$

De stelling van Pythagoras in $\triangle BDE$ geeft $DE = \sqrt{5^2 - 4^2} = 3$.

$\begin{array}{c|c|c} AB & AC & BC \\ \hline BD & DE & BE \end{array}$ geeft $\begin{array}{c|c|c} 6 & AC & BC \\ \hline 4 & 3 & 5 \end{array}$

$AC = \frac{6 \cdot 3}{4} = 4\frac{1}{2}$

$BC = \frac{6 \cdot 5}{4} = 7\frac{1}{2}$, dus $CE = BC - BE = 7\frac{1}{2} - 5 = 2\frac{1}{2}$.

b $\left.\begin{array}{l}\angle ASC = \angle DSE \text{ (overstaande hoeken)} \\ \angle CAS = \angle DES \text{ (Z-hoeken)}\end{array}\right\} \triangle ACS \sim \triangle EDS$ dus $\begin{array}{c|c} AC & AS \\ \hline DE & ES \end{array}$

De stelling van Pythagoras in $\triangle ABC$ geeft $AC = \sqrt{(7\frac{1}{2})^2 - 6^2} = 4\frac{1}{2}$.

De stelling van Pythagoras in $\triangle ADE$ geeft $AE = \sqrt{2^2 + 3^2} = \sqrt{13}$.

Stel $AS = x$, dan is $ES = \sqrt{13} - x$.

Dit geeft $\begin{array}{c|c} 4\frac{1}{2} & x \\ \hline 3 & \sqrt{13} - x \end{array}$

$3x = 4\frac{1}{2}(\sqrt{13} - x)$

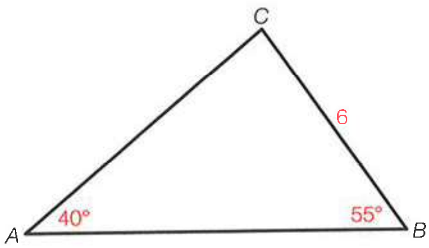
$3x = 4\frac{1}{2}\sqrt{13} - 4\frac{1}{2}x$

$7\frac{1}{2}x = 4\frac{1}{2}\sqrt{13}$

$x \approx 2{,}16$

Dus $AS \approx 2{,}16$.

3 Zie de figuur hiernaast.



$\angle C = 180° - 40° - 55° = 85°$

De sinusregel geeft $\frac{6}{\sin(40°)} = \frac{AC}{\sin(55°)}$

$AC = \frac{6\sin(55°)}{\sin(40°)} = 7{,}64...$

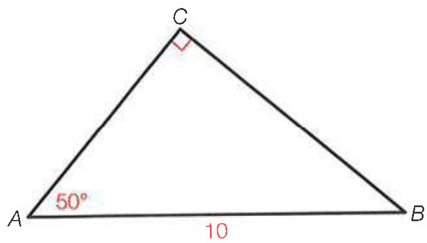
en ook

$\frac{6}{\sin(40°)} = \frac{AB}{\sin(85°)}$

$AB = \frac{6\sin(85°)}{\sin(40°)} = 9{,}29...$

De omtrek van driehoek ABC is $9{,}29... + 6 + 7{,}64... \approx 22{,}95$

4 **a** Zie de figuur hiernaast.

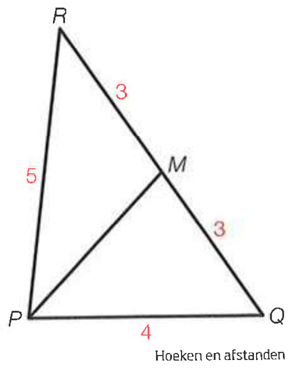


$\sin(50°) = \frac{BC}{10}$, dus $BC = 10\sin(50°) = 7{,}660...$

Als er precies één driehoek ABC mogelijk is, dan is $BC \approx 7{,}66 \vee BC \geq 10$.

b Als er twee driehoeken ABC mogelijk zijn, dan is $7{,}66 < BC < 10$.

5 Zie de figuur hiernaast.



De cosinusregel in $\triangle PQR$ geeft $5^2 = 4^2 + 6^2 - 2 \cdot 4 \cdot 6 \cdot \cos(\angle Q)$

$25 = 16 + 36 - 48\cos(\angle Q)$

$48\cos(\angle Q) = 27$

$\cos(\angle Q) = \frac{9}{16}$

De cosinusregel in $\triangle PQM$ geeft $PM^2 = 4^2 + 3^2 - 2 \cdot 4 \cdot 3 \cdot \frac{9}{16}$

$PM^2 = 11\frac{1}{2}$

$PM \approx 3{,}39$

6 De cosinusregel in $\triangle ABD$ geeft $AD^2 = 10^2 + 5^2 - 2 \cdot 10 \cdot 5 \cdot \cos(35°)$

$AD^2 = 43{,}08...$

$AD = 6{,}56...$

De sinusregel in $\triangle ABD$ geeft $\dfrac{10}{\sin(\angle ADB)} = \dfrac{6{,}56...}{\sin(35°)}$

$\sin(\angle ADB) = \dfrac{10\sin(35°)}{6{,}56...} = 0{,}87...$

De GR geeft $60{,}90...°$, dus $\angle ADB = 180° - 60{,}90...° \approx 119{,}1°$.

$\angle ADC = 180° - \angle ADB \approx 180° - 119{,}1° = 60{,}9°$.

$\angle C = 180° - \angle ADC - \angle DAC \approx 180° - 60{,}9° - 25° = 94{,}1°$

Bladzijde 133

7 **a** $\sqrt{4\frac{1}{4}} = \sqrt{\dfrac{17}{4}} = \dfrac{\sqrt{17}}{\sqrt{4}} = \dfrac{\sqrt{17}}{2} = \frac{1}{2}\sqrt{17}$

b $8a\sqrt{5} \cdot 3a\sqrt{2} = 24a^2\sqrt{10}$

c $\dfrac{6\sqrt{18}}{4\sqrt{3}} = 1\frac{1}{2}\sqrt{6}$

d $(\frac{1}{2}a\sqrt{6})^2 = (\frac{1}{2}a)^2 \cdot (\sqrt{6})^2 = \frac{1}{4}a^2 \cdot 6 = 1\frac{1}{2}a^2$

e $3\sqrt{2}(\sqrt{2} - 2\sqrt{5}) = 3\sqrt{4} - 6\sqrt{10} = 3 \cdot 2 - 6\sqrt{10} = 6 - 6\sqrt{10}$

f $(5 - \sqrt{3})^2 = 25 - 2 \cdot 5 \cdot \sqrt{3} + 3 = 28 - 10\sqrt{3}$

8 **a** $3\sqrt{24} - \sqrt{150} = 3 \cdot 2\sqrt{6} - 5\sqrt{6} = 6\sqrt{6} - 5\sqrt{6} = \sqrt{6}$

b $\sqrt{32x^2} - \sqrt{8x^2} = \sqrt{16x^2} \cdot \sqrt{2} - \sqrt{4x^2} \cdot \sqrt{2} = 4x\sqrt{2} - 2x\sqrt{2} = 2x\sqrt{2}$

c $x\sqrt{3} + \sqrt{\frac{1}{3}x^2} = x\sqrt{3} + \sqrt{\frac{1}{9}x^2} \cdot \sqrt{3} = x\sqrt{3} + \frac{1}{3}x\sqrt{3} = 1\frac{1}{3}x\sqrt{3}$

9 **a** $x\sqrt{3} + 3 = 5\sqrt{6}$

$x\sqrt{3} = 5\sqrt{6} - 3$

$x = \dfrac{5\sqrt{6}}{\sqrt{3}} - \dfrac{3}{\sqrt{3}}$

$x = 5\sqrt{2} - \dfrac{3}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$x = 5\sqrt{2} - \dfrac{3\sqrt{3}}{3}$

$x = 5\sqrt{2} - \sqrt{3}$

b $x\sqrt{2} + 2x\sqrt{5} = 5\sqrt{10}$

$x(\sqrt{2} + 2\sqrt{5}) = 5\sqrt{10}$

$x = \dfrac{5\sqrt{10}}{\sqrt{2} + 2\sqrt{5}}$

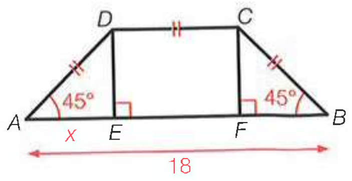
c $3x - 2\sqrt{2} = 2x\sqrt{5}$

$3x - 2x\sqrt{5} = 2\sqrt{2}$

$x(3 - 2\sqrt{5}) = 2\sqrt{2}$

$x = \dfrac{2\sqrt{2}}{3 - 2\sqrt{5}}$

10 Zie de figuur hiernaast.
Stel $AE = x$, dan is ook $BF = x$.
In $\triangle ADE$ is $AD = x\sqrt{2}$.
Dus ook $EF = CD = AD = x\sqrt{2}$.



$$\left.\begin{array}{l} AB = AE + EF + BF = x + x\sqrt{2} + x = 2x + x\sqrt{2} \\ AB = 18 \end{array}\right\} 2x + x\sqrt{2} = 18$$

$$x(2 + \sqrt{2}) = 18$$

$$x = \frac{18}{2 + \sqrt{2}}$$

$$AD = x\sqrt{2} = \frac{18}{2+\sqrt{2}} \cdot \sqrt{2} = \frac{18\sqrt{2}}{2+\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{18 \cdot 2}{2\sqrt{2}+2} = \frac{36}{2(\sqrt{2}+1)} = \frac{18}{\sqrt{2}+1}$$

11 $\triangle ABE$ is gelijkzijdig, dus $\angle EAB = \angle EBA = \angle AEB = 60°$.
$\angle DAE = 90° - \angle EAB = 90° - 60° = 30°$ en $\angle D = 90°$
Stel $AB = BE = AE = x$.
Dan geldt in $\triangle ADE$ dat $DE = \frac{x}{2} = \frac{1}{2}x$ en $AD = \frac{1}{2}x\sqrt{3}$.

$$\left.\begin{array}{l} \text{omtrek } ABCD = AB + BC + CD + AD = x + \tfrac{1}{2}x\sqrt{3} + x + \tfrac{1}{2}x\sqrt{3} = 2x + x\sqrt{3} \\ \text{omtrek } ABCD = 10 \end{array}\right\} 2x + x\sqrt{3} = 10$$

$$x(2 + \sqrt{3}) = 10$$

$$x = \frac{10}{2 + \sqrt{3}}$$

Dus de omtrek van driehoek ABE is $3 \cdot \frac{10}{2+\sqrt{3}} = \frac{30}{2+\sqrt{3}}$.

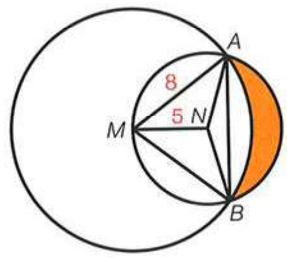
12 In $\triangle BCD$ is $CD = \frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$.

$$\left.\begin{array}{l} \text{opp } ABCD = \tfrac{1}{2}(AB + 2\sqrt{3}) \cdot 6 = 3(AB + 2\sqrt{3}) \\ \text{opp } ABCD = 30 \end{array}\right\} 3(AB + 2\sqrt{3}) = 30$$

$$AB + 2\sqrt{3} = 10$$

$$AB = 10 - 2\sqrt{3}$$

13 $\cos(\angle AMN) = \frac{\frac{1}{2}AM}{MN} = \frac{4}{5}$



$\angle AMN = 36{,}86...°$
$\angle AMB = 2 \cdot \angle AMN = 2 \cdot 36{,}86...° = 73{,}73...°$
Van de cirkel met middelpunt M en straal 8 is
kortste boog $AB = \frac{73{,}73...}{360} \cdot 2\pi \cdot 8 = 10{,}29...$
$\angle ANM = 180° - 2 \cdot 36{,}86...° = 106{,}26...°$
$\angle ANB = 360° - 2 \cdot 106{,}26...° = 147{,}47...°$
Van de cirkel met middelpunt N en straal 5 is
kortste boog $AB = \frac{147{,}47...}{360} \cdot 2\pi \cdot 5 = 12{,}87...$
De gevraagde omtrek is $10{,}20... + 12{,}87... \approx 23{,}17$.